

LATTICE MODEL OF NONPHONON DONOR–ACCEPTOR PHOTOLUMINESCENCE IN GERMANIUM CRYSTALS

N. A. Poklonski,^{*} I. I. Anikeev, and S. A. Vyrko

UDC 621.315.592

A formula is proposed for calculating the spectral position of the peak of the nonphonon line (zero phonon line, ZPL) of donor–acceptor (DA) photoluminescence in p- and n-type covalent semiconductors with hydrogen-like impurities at low temperatures and low levels of steady-state interband photoexcitation. The model uses a nonstoichiometric simple cubic impurity lattice formed jointly by doping (majority) and compensating (minority) impurity atoms in the crystal matrix. It is assumed that the distribution densities of energy levels of donors forming the D^0 -band and energy levels of acceptors forming the A^0 -band in the band gap of the crystal are Gaussian with equal root-mean-square fluctuations of the ionization energy. Nonphonon radiative DA-recombination is considered to occur only between nearest neighbors in the impurity lattice upon a nonequilibrium electron transition from the energy level of the first excited state of donor to the acceptor energy level in the A^0 -band, which coincides with the Fermi level in this band in a p-type semiconductor or upon a nonequilibrium hole transition from the energy level of the first excited state of acceptor to the donor energy level in the D^0 -band, which coincides with the Fermi level in the band in an n-type semiconductor. The calculated dependence of the maximum of DA-photoluminescence nonphonon line on the concentration and degree of compensation of majority impurities by minority impurities is consistent with known experimental data for neutron-transmutation doped germanium crystals.

Keywords: hydrogen-like donor and acceptors, nonstoichiometric impurity lattice, steady-state donor–acceptor photoluminescence, nonphonon line.

Introduction. Various types of photoluminescence are observed in covalent and covalent-ionic crystalline semiconductors depending on the concentration and type of hydrogen-like impurities [1, 2]. The impurities form discrete energy levels in the crystal forbidden band in weakly doped semiconductors where the distance between impurity atoms is large and Coulombic interactions between them are small. They form impurity bands, a donor D^0 -band and an acceptor A^0 -band, in the crystal energy gap at moderate impurity concentrations where the distance between them decreases and the interactions between them increase. The D^0 - or A^0 -bands expand and overlap with the closest c - or v -bands of allowed energies in highly doped degenerate semiconductors on the metallic side of the insulator–metal concentration phase transition (Mott transition).

Donor–acceptor (DA)-recombination emissive transitions are used both to create light-emitting semiconductor device structures [3, 4] and to control the homogeneity of the impurity distribution in a crystalline matrix [1, 5] for optical materials science purposes. Existing models of DA-pair emissive recombination cannot calculate dependences of the emission line maximum position on the concentration of hydrogen-like impurities that agree quantitatively with experimental data [2, 6–8]. The effect of Coulombic interaction of impurity ions that leads to the formation of impurity bands in the crystalline matrix energy gap on the position of the nonphonon photoluminescence intensity maximum is not explicitly considered in these DA-recombination models. DA-recombination spectra in gallium phosphide crystals have been studied [9, 10] and narrow photoluminescence lines situated near the intrinsic absorption edge (interband, fundamental) were observed. A model of isolated DA-pairs, where the distance between pairs was small as compared to the average distance between hydrogen-like impurities in a DA-pair, was used to describe the spectral position of these lines. The contribution of spatial fluctuations of the electrostatic potential created by impurity ions determined the spectral position of the DA-recombination

^{*}To whom correspondence should be addressed.

nonphonon line maximum while the model of isolated pairs did not work as the impurity concentration increased. Only interband (edge) photoluminescence appeared with an impurity concentration approximately an order of magnitude greater than the critical concentration necessary for the insulator–metal transition of a semiconductor at liquid He temperature [2, 11].

The aim of the present work was to propose an analytical formula for calculating the dependence of the nonphonon DA-photoluminescence peak of crystalline covalent semiconductors on the concentration of the major (doping) hydrogen-like impurity and the degree of its compensation by the minor (compensating) hydrogen-like impurity considering electrostatic fluctuations of the potential energy of impurity ions at low temperatures and levels of steady-state interband photoexcitation of the crystals. Crystals of *p*- and *n*-type Ge into which acceptors (Ga) and donors (As) were introduced homogeneously and in a controlled manner by neutron-transmutation doping were studied. The physical and technological aspects of the doping of crystalline Ge samples of various isotopic compositions after irradiation by thermal reactor neutrons followed by thorough thermal annealing (24 h at 450°C) of radiation defects have been reported [12–14].

Basic Equations of the DA-Recombination Model. Let us examine a three-dimensional crystalline *p*-type semiconductor as an example. Let the concentration of hydrogen-like acceptors in charge states (0) and (−1) be equal to $N_a = N_0 + N_{-1}$ while the concentration of hydrogen-like donors, which exist completely in charge states (+1), is $N_d = N_{+1} < N_a$. (Impurity charge states are expressed in units of the elementary charge e .) The degree of acceptor compensation by donors is $0 < K = N_d/N_a < 1$.

The absolute temperature T_j at which the minimal band electrical conductivity of holes over ν -band states is equal to the maximum hopping electrical conductivity of holes over acceptors [15] is determined from virial theory and is written [16, 17]:

$$T_j \approx \frac{0.728}{k_B} \frac{e^2}{4\pi\epsilon_r\epsilon_0} (KN_a)^{1/3}, \quad (1)$$

where k_B is Boltzmann's constant; e , elementary charge; ϵ_r , low-frequency relative dielectric permittivity (determined by ν -band electrons in ionic lattices of the crystal matrix); ϵ_0 , electrical constant; $\epsilon_r = 15.4$ for Ge crystals [18].

The ν -band hole concentration $p \ll K(1 - K)N_a$, where $K(1 - K)$ is the fraction of acceptor pairs limiting the high-temperature part of the hopping migration of holes between acceptors [15], at low temperatures ($T < T_j$) and low crystal photoexcitation levels. Then, the electrical neutrality equation of a *p*-type semiconductor is written [19]:

$$N_{-1} = p + N_{+1} \approx KN_a = N_d, \quad (2)$$

where $N_a + N_d = (1 + K)N_a$ is the summed concentration of hydrogen-like impurities of substitution of crystal-matrix atoms and $N_{-1} + N_{+1} = 2KN_a$ is the concentration of impurity ions.

Probabilities $\langle f_0 \rangle$ and $\langle f_{-1} \rangle$ that a randomly selected acceptor in the crystal matrix exists in charge state (0) (is electrically neutral) or in charge state (−1) (is a singly negatively charged ion) averaged over the crystal volume [17, 19] are:

$$\langle f_0 \rangle = N_0/N_a = \int_{-\infty}^{+\infty} G_a f_0 d(E_a - I_a) = 1 - K, \quad \langle f_{-1} \rangle = N_{-1}/N_a = \int_{-\infty}^{+\infty} G_a f_{-1} d(E_a - I_a) = K, \quad (3)$$

where G_a is a Gaussian distribution of density of hole states in the acceptor band (A^0 -band); $f_0 = 1 - f_{-1} = \{1 + [\beta_a(q_m)]\}^{-1} \times \exp[-(E_F^{(v)} + E_a)/k_B T]^{-1}$, the probability of filling a state with energy level E_a by a hole; $\beta_a(q_m)$, an effective factor of the degeneracy of hydrogen-like acceptor energy level E_a considering q_m of the levels [an unexcited state ($q = 1$) and $q_m - 1$ excited states]; $E_a = E_{a,-1} - E_{a,0} > 0$, the ionization energy of an electrically neutral acceptor from the ground state ($q = 1$) (loss of a hole from a neutral acceptor and its transition from the top of the ν -band of an undoped crystal) (Fig. 1); $E_F^{(v)} < 0$, the Fermi level; T , the absolute temperature; the top of the ν -band ($E_\nu = 0$) of an undoped crystal is selected as the reference point for $E_F^{(v)}$, I_a , and E_a .

The effective factor of the degeneracy of a hydrogen-like acceptor E_a [20–22] is:

$$\beta_a(q_m) = \beta_a \sum_{q=1}^{q_m} q^2 \exp \left[\frac{(1 - q^2)E_a}{q^2 k_B T} \right], \quad (4)$$

where β_a is the degeneracy factor of the ground energy level ($q = 1$) of the hydrogen-like acceptor in covalent and covalent-ionic crystalline semiconductors [23]; $\beta_a = 4$ for *p*-type Ge crystals doped with Ga; $q_m = (2R_{im}/a_p)^{1/2}$, the greatest number

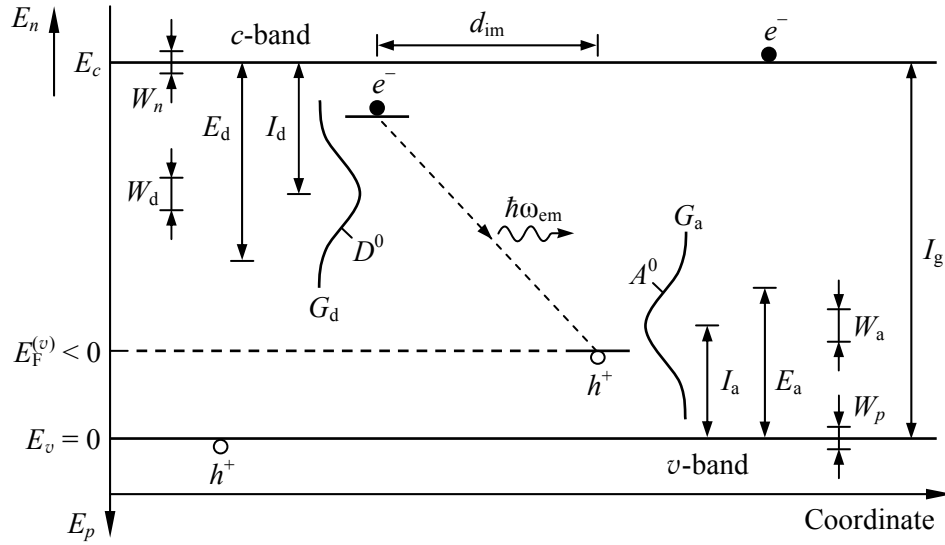


Fig. 1. Diagram of DA-recombination of electrons (e^-) and holes (h^+) in p -type semiconductor: E_n and E_p , energies of an electron and hole; E_c and E_v , bottom of c -band and top of v -band of undoped crystalline semiconductor; $E_F^{(v)} < 0$, Fermi level counted from the top of the v -band ($E_v = 0$); $I_g = E_c - E_v$, width of forbidden energy gap (band gap) of undoped semiconductor; G_d and G_a , Gaussian densities of energy level distribution of donors E_d forming the D^0 -band (relative to the ionization energy of a single donor I_d) and acceptor energy levels E_a forming the A^0 -band (relative to the ionization energy of a single acceptor I_a); $\hbar\omega_{em}$, energy of photon emitted in one act of DA-recombination of an electron on a neutral donor and a hole on a neutral acceptor; W_d and W_a , effective widths of donor D^0 and acceptor A^0 bands; W_n and W_p , root-mean-square fluctuations of potential energy of c -band electrons and v -band holes; $d_{im} = 2R_{im} \approx 1.24(N_a + N_d)^{-1/3}$, distance between impurity atoms in crystal matrix, i.e., edge length of nonstoichiometric simple cubic lattice of acceptors and donors.

of possible excited states of the acceptor in charge state (0); $R_{im} = [3/4\pi(N_d + N_a)]^{1/3} \approx 0.62[(1 + K)N_a]^{-1/3}$, the radius of the spherical region situated on one impurity atom in the crystal matrix; $a_p = e^2/8\pi\epsilon_r\epsilon_0 I_a$, the radius of the hole Bohr orbit on a solitary (single) acceptor with ionization energy I_a from the unexcited state ($q = 1$) into the v -band; and $q_m = 1$ corresponds formally to the acceptor ground state.

The distribution densities of acceptor and donor ground and excited energy states in the forbidden band (Fig. 1) are taken as normal (Gaussian) [24, 25]:

$$G_a = \frac{1}{W_a\sqrt{2\pi}} \exp\left[-\frac{(E_a - I_a)^2}{2W_a^2}\right], \quad G_d = \frac{1}{W_d\sqrt{2\pi}} \exp\left[-\frac{(E_d - I_d)^2}{2W_d^2}\right], \quad (5)$$

where W_a^2 and W_d^2 are dispersions of acceptor energy levels relative to I_a and donor energy levels relative to I_d in the semiconductor energy gap; $\int_{-\infty}^{+\infty} G_a d(E_a - I_a) = \int_{-\infty}^{+\infty} G_d d(E_d - I_d) = 1$.

Then, as before [26], the doping impurity (acceptor) with concentration $N_a = N_0 + N_{-1}$ and compensating impurity (donor) with concentration $N_d = KN_a$ form a nonstoichiometric simple cubic lattice in the semiconductor crystal matrix with translation period $d_{im} = 2R_{im} \approx 1.24[(1 + K)N_a]^{-1/3}$. The quantity d_{im} is equal to the diameter of the spherical region in the crystal per single impurity atom (both donor and acceptor). Each impurity atom in this impurity lattice has six closest neighbors (first coordination sphere). The fraction of electrically neutral acceptors in the impurity lattice is equal to $(1 - K)/(1 + K)$. The fraction of negatively charged acceptors $K/(1 + K)$ is equal to the fraction of positively charged donors

in this lattice. (Note that the quantity d_{im} is close to the average distance between impurities, which was determined before by the Voronoi–Dirichlet method [27].)

The root-mean-square (rms) fluctuation of acceptor energy levels (effective acceptor band width) W_a considering only Coulombic interaction of an acceptor in charge state (-1) with ions of the first coordination sphere of the arbitrary impurity lattice with translation period d_{im} is [28]:

$$W_a = \left(\sum_{i=1}^6 P_i U_i^2 \right)^{1/2} = \frac{e^2}{4\pi\epsilon_r\epsilon_0 d_{\text{im}}} \left(\frac{12K}{1+K} \right)^{1/2}, \quad (6)$$

where $P_i = 2K/(1+K)$ is the probability that any of the six impurity lattice sites in the first coordination sphere near an isolated impurity ion is occupied by an ionized acceptor or donor; $1/(1+K)$, the fraction of acceptors at impurity sublattice sites; $|U_i| = e^2/4\pi\epsilon_r\epsilon_0 d_{\text{im}}$, the Coulombic interaction energy modulus of an isolated ion with closest ions in the simple cubic lattice of doping and compensating impurities situated at distance $d_{\text{im}} = 2R_{\text{im}} \approx 1.24[(1+K)N_a]^{-1/3}$; here, the average interaction energy of ions in the impurity lattice is considered to be zero, i.e., $\sum_{i=1}^6 P_i U_i = 0$.

Next, the concentrations of doping impurity, its degrees of compensation K , temperatures $T \ll T_j$, and low levels of interband photoexcitation of the crystals for which the concentration of ν -band nonequilibrium holes $p \ll K(1-K)N_a$ are examined. The quantity $E_F^{(\nu)} < 0$ for this condition, according to electrical neutrality [Eq. (2)] and considering Eq. (3) for $W_a \gg k_B T$ [formally near absolute zero where $\beta_a(q_m) = \beta_a$], is given by [19, 28]:

$$2K = 1 - \operatorname{erf} \left(\frac{E_F^{(\nu)} + I_a + k_B T \ln \beta_a}{\sqrt{2}W_a} \right). \quad (7)$$

The proposed scheme of DA-photoluminescence includes five relationships between the energies of the donors, acceptors, electrons, and holes:

1) A nonequilibrium electron is raised by a photon from the ν -band to the c -band, is thermalized, and drops to the bottom of the c -band while a nonequilibrium hole is thermalized and rises to the top of the ν -band. A donor ion in charge state $(+1)$ has energy $E_{d,+1}$. An acceptor ion in charge state (-1) has energy $E_{a,-1}$. Donor and acceptor ions do not have excited states [29]. The rms fluctuations of the potential energy of donor ions W_d and acceptor ions W_a are much greater than the rms fluctuations of the potential energy of free (delocalized) electrons of the c -band W_n and ν -band holes W_p at low temperatures ($T < T_j$) [17].

2) An electron is captured nonradiatively from the bottom of the c -band on a donor in charge state $(+1)$ and forms an electrically neutral donor in an excited state ($s > 1$) with energy $E_{d,0,s}$. A hole is captured nonradiatively from the top of the ν -band on an acceptor in charge state (-1) and forms an electrically neutral acceptor in an excited state ($q > 1$) with energy $E_{a,0,q}$ [30, 31]. The indices s and q refer to the number of the electron orbit in the excited state of the donor and the hole in the acceptor excited state, respectively; $s = q = 1$ for the ground (unexcited) state of the donor and acceptor. The energy of a DA-pair in the initial state is:

$$E_{\text{in}} = E_{d,0,s} + E_{a,0,q}. \quad (8)$$

3) A nonequilibrium electron on a donor and a nonequilibrium hole on an acceptor recombine radiatively (only if the donor and acceptor are situated at distance $d_{\text{im}} \approx 1.24[(1+K)N_a]^{-1/3}$ from each other in the impurity lattice) and emit a photon of energy $\hbar\omega_{\text{em}}$. The DA-recombination results in the donor becoming an ion with energy $E_{d,+1}$; the acceptor, an ion with energy $E_{a,-1}$. The Coulombic interaction energy between the two formed ions is:

$$U_{-1,+1} = - \frac{e^2}{4\pi\epsilon_r\epsilon_0 d_{\text{im}}} < 0, \quad (9)$$

so that the energy of the final state of the DA-pair is:

$$E_{\text{out}} = E_{d,+1} + E_{a,-1} + U_{-1,+1}. \quad (10)$$

4) The rms fluctuations of the potential energy of a donor ion W_d and acceptor ion W_a are equal. A consequence of these fluctuations is an energy decrease of a hole at the top of the ν -band upon its localization on an acceptor ion in the

A^0 -band, equal to $2W_a$, and an energy decrease of an electron at the bottom of the c -band upon its localization on a donor ion in the D^0 -band, equal to $2W_d$. Then, according to the rule for addition of dispersions $4W_a^2 + 4W_d^2$ of two independent random quantities [32, 33], the energy decrease of a photon upon nonphonon radiative DA-recombination is $2\sqrt{2}W_a$, where $W_d = W_a$.

5) The energy difference of the initial and final states of a DA-pair after recombination of a donor electron with an acceptor hole is equal to the energy $\hbar\omega_{em}$ of the photon emitted by the DA-pair (without involving phonons):

$$I_g + (E_{in} - E_{out}) - 2\sqrt{2} W_a = \hbar\omega_{em},$$

where $I_g = E_c - E_v$ is the width of the forbidden energy band (gap) of an ideal (undoped) semiconductor.

Finally, the energy of a photon emitted in a single nonphonon DA-recombination considering these five relationships and Eqs. (6)–(10) is:

$$\hbar\omega_{em} = I_g - (E_{d,s} + E_{a,q}) - U_{-1,+1} - 2\sqrt{2} W_a, \quad (11)$$

where $E_{d,s} = E_{d,+1} - E_{d,0,s} > 0$ is the ionization energy of an electrically neutral donor in excited state $s > 1$ (transition of an electron from a donor at the c -band bottom with zero kinetic energy); $E_d = E_{d,+1} - E_{d,0}$, the ionization energy of a donor from the ground state (unexcited; $s = 1$); $E_{a,q} = E_{a,-1} - E_{a,0,q} > 0$, the ionization energy of an electrically neutral acceptor in excited state $q > 1$ (transition of a hole from an acceptor at the v -band bottom with zero kinetic energy); $E_a = E_{a,-1} - E_{a,0}$, the ionization energy of an acceptor from the ground state (unexcited; $q = 1$); $U_{-1,+1} < 0$, the Coulombic interaction energy of an acceptor ion and donor ion at distance d_{im} in the impurity lattice. Note that subscript "a" [acceptors in charge states (0) and (-1)] in the formulas for n -type semiconductors with hydrogen-like donors should be replaced by the subscript "d" [donors in charge states (0) and (+1)]; the symbols "p" and "v" for p -type, by symbols "n" and "c" for n -type.

The band gap $I_g = 744$ meV for Ge crystals at liquid He temperature [34]. The ionization energies of single Ga atoms (acceptors) and As atoms (donors) are $I_a = 11.32$ meV and $I_d = 14.17$ meV. The degeneracy factor of the ground energy level ($s = 1$) of a hydrogen-like donor $\beta_d = 2$. The ionization energy of a hydrogen-like acceptor in weakly doped p -Ge from the excited state $q > 1$ is $I_{a,q} = I_a/q^2$. The ionization energy of a hydrogen-like donor in weakly doped n -Ge from excited state $s > 1$ is $I_{d,s} = I_d/s^2$.

Calculated Results and Comparison of Them with Experimental Data. The results calculated using Eq. (11) were given as dependences of the position of the DA-photoluminescence maximum $\hbar\omega_{em}$ on acceptor concentration N_a for p -Ge crystals [6, 35] with degree of compensation $K = N_d/N_a = 0.4$ (Fig. 2) and on the degree of compensation of donors by acceptors $K = N_a/N_d$ for n -Ge crystals [36, 37] for donor concentration $N_d = 6 \cdot 10^{17} \text{ cm}^{-3}$ (Fig. 3). It is noteworthy that the concentration of the major impurity (Ga) was $\sim 1.85 \cdot 10^{17} \text{ cm}^{-3}$ in p -Ge:Ga,As crystals (for degree of compensation $K \approx 0.4$); the concentration of the major impurity (As), $\sim 3.61 \cdot 10^{17} \text{ cm}^{-3}$ in n -Ge:As,Ga crystals (for $K \ll 1$) [38 and references therein] for the insulator–metal concentration phase transition.

The calculations considered that a radiative DA-recombination occurred* for p -Ge:Ga,As crystals between an electron from the first donor excited state ($s = 2$) with ionization energy $I_d/4$ and an acceptor hole with energy level $E_{a,q} = -E_F^{(c)} > 0$; for n -Ge:As,Ga crystals between a donor electron with energy level $E_{d,s} = -E_F^{(v)} > 0$ and a hole from the first acceptor excited state ($q = 2$) with ionization energy $I_a/4$. Acceptor states in p -Ge:Ga,As crystals between the top of the v -band ($E_v = 0$) and the Fermi level $E_F^{(v)} < 0$ are vacant for holes. Therefore, a hole on an acceptor with energy level $E_{a,q}$ corresponding to the Fermi level participates in DA-recombination. Donor states in n -Ge:Ga,As crystal between the bottom of the c -band ($E_c = 0$) and the Fermi level $E_F^{(c)} < 0$ are vacant for electrons. Therefore, an electron on a donor with energy level $E_{d,s}$ corresponding to the Fermi level participates in DA-recombination.

Figures 2 and 3 show that the curves calculated using Eq. (11) agreed quantitatively with the experimental data. The discrepancies of the calculated curves from the experimental data upon increasing the concentration of major impurity N_a in Fig. 2 (or decreasing the degree of compensation K in Fig. 3) could be explained by the approach of the semiconductor to

* A nonequilibrium electrically neutral excited state of a compensating (minor) impurity has a larger hopping diffusion coefficient in the impurity lattice than in the ground state because of the large radius of the electron Bohr orbit on a donor (or hole for an acceptor). This increases the rate of its radiative recombination with the closest nonequilibrium electrically neutral state of the doping impurity with an energy level coinciding with the Fermi level.

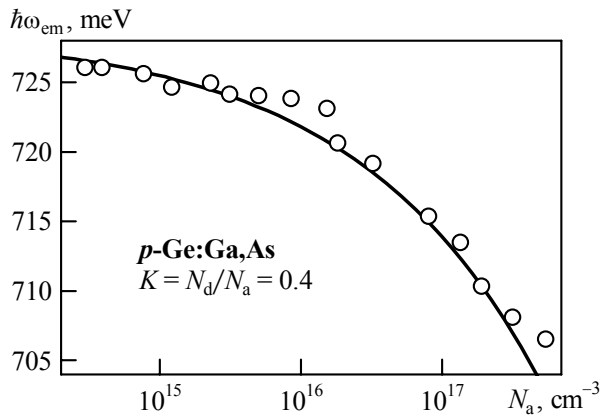


Fig. 2. Dependence of energy $\hbar\omega_{em}$ of photons emitted in DA-recombinations on concentration N_a of Ga atoms in p -type Ge crystals doped with Ga (acceptors, a) and compensated by As (donors, d); points, experimental [6, 35] at liquid He temperature; line, calculation by Eq. (11) for $E_{a,q} = -E_F^{(v)}$ and $E_{d,2} = I_d/4$ for degree of compensation $K = N_d/N_a = 0.4$.

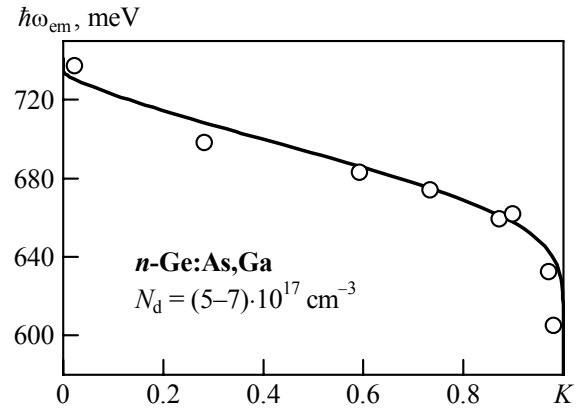


Fig. 3. Dependence of energy $\hbar\omega_{em}$ of photons emitted in DA-recombinations on degree of compensation of donors by acceptors $K = N_a/N_d$ in n -type Ge crystals doped with As (donors) and compensated by Ga (acceptors); points, experimental [36] at liquid He temperature; line, calculation by Eq. (11) for $E_{d,s} = -E_F^{(c)}$ and $E_{a,2} = I_a/4$ for donor concentration $N_d = 6 \cdot 10^{17} \text{ cm}^{-3}$.

the concentration of the insulator–metal phase transition. This led to overlap of the acceptor band with the top of the v -band (or donor band with the bottom of the c -band) and a transition from recombination of nonequilibrium impurity electrons and holes to interband (edge).

It is noteworthy that the half-widths of the nonphonon DA-photoluminescence lines ($\approx \sqrt{2}W_a$) for the Ge samples examined in Fig. 2 ($N_a > 5 \cdot 10^{15} \text{ cm}^{-3}$) were much greater than thermal energy $k_B T$ for liquid He temperature ($T = 4.2 \text{ K}$), at which the samples were held during recording of the IR-emission spectra [6]. The inhomogeneous broadening of the nonphonon lines was apparently due to random electrostatic interaction of the impurity ions during hopping migration of holes between charge states (0) and (–1) of Ga atoms.

According to the literature [35, 36], the position of the nonphonon DA-photoluminescence line maximum $\hbar\omega_{em}$ in p - and n -Ge crystals shifted to higher energies upon increasing the intensity of the steady-state interband illumination because hydrogen-like impurity ions were neutralized with photoexcitation of the crystals near the fundamental (interband) absorption [39]. This decreased the rms fluctuations of the electrostatic potential (W_a and W_d). According to Eq. (11), a decrease in $W_a = W_d$ led to an increase in $\hbar\omega_{em}$.

Conclusions. Nonphonon donor–acceptor photoluminescence was quantitatively described in terms of the proposed lattice model using semiconducting crystals of p - and n -Ge as examples. The model considered the D^0 -band of hydrogen-like donors and the A^0 -band of hydrogen-like acceptors in the energy gap of a crystalline matrix. It was assumed that the doping and compensating impurity atoms formed a nonstoichiometric simple cubic impurity lattice with period $1.24(N_a + N_d)^{-1/3}$ in the crystalline matrix, where $N_a + N_d$ is the summed concentration of acceptors and donors. The density distribution of acceptor and donor energy levels in the semiconductor band gap was assumed to be normal (Gaussian). The rms fluctuations of the potential energies of donor and acceptor ions were considered equal. Nonphonon radiative donor–acceptor recombination in the p -type semiconductor occurred between a nonequilibrium electron from the first donor excited state and a nonequilibrium hole acceptor with an energy level coincident with the Fermi level in the acceptor band. Nonphonon radiative donor–acceptor recombination in the n -type semiconductor occurred between a nonequilibrium hole from the first acceptor excited state and a nonequilibrium donor electron with an energy level coincident with the Fermi level in the donor band. An analytical formula describing the dependence of the position of the nonphonon donor–acceptor photoluminescence line maximum on the impurity concentration at liquid He temperature and low levels of steady-state interband photoexcitation of the semiconductor was obtained. A numerical calculation in terms of the proposed model

agreed quantitatively with known experimental data for the position of the nonphonon donor–acceptor recombination line maximum in neutron-transmutation doped *p*- and *n*-Ge doped crystals.

Acknowledgments. The work was supported by the State Program for Scientific Research of the Republic of Belarus "Materials science, new materials and technologies."

REFERENCES

1. M. Tajima, H. Toyota, and A. Ogura, *Jpn. J. Appl. Phys.*, **61**, No. 8, Article ID 080101 (2022).
2. A. P. Levanyuk and V. V. Osipov, *Sov. Phys. Usp.*, **24**, No. 3, 187–215 (1981).
3. S. Nakamura, T. Mukai, and M. Senoh, *Jpn. J. Appl. Phys.*, **30**, No. 12A, L1998–L2001 (1991).
4. U. Kaiser, A. N. Gruzintsev, I. I. Rhodos, and W. Richter, *Inorg. Mater.*, **36**, No. 6, 595–598 (2000).
5. V. Yu. Nekrasov, L. V. Belyakov, O. M. Sreseli, and N. N. Zinov'ev, *Semiconductors*, **33**, No. 12, 1284–1290 (1999).
6. V. P. Dobrego and I. S. Shlimak, *Sov. Phys. Semicond.*, **1**, No. 10, 1231–1236 (1968).
7. F. Williams, *Phys. Status Solidi*, **25**, No. 2, 493–512 (1968).
8. L. E. Stys and M. G. Foigel', *Sov. Phys. Semicond.*, **19**, No. 2, 135–142 (1985).
9. J. J. Hopfield, D. G. Thomas, and M. Gershenson, *Phys. Rev. Lett.*, **10**, No. 5, 162–164 (1963).
10. E. F. Gross and D. S. Nedzvetskii, *Sov. Phys. Dokl.*, **8**, No. 9, 896–900 (1964).
11. N. A. Poklonskii and S. A. Vyrko, *J. Appl. Spectrosc.*, **69**, No. 3, 434–443 (2002).
12. A. G. Zabrodskii and M. V. Alekseenko, in: *Proc. 23rd Int. Conf. on the Physics of Semiconductors*, Berlin, Germany, July 21–26, 1996, Vol. 4, World Scientific, Singapore (1996), pp. 2681–2684.
13. K. M. Itoh, E. E. Haller, J. W. Beeman, W. L. Hansen, J. Emes, L. A. Reichertz, E. Kreysa, T. Shutt, A. Cummings, W. Stockwell, B. Sadoulet, J. Muto, J. W. Farmer, and V. I. Ozhogin, *Phys. Rev. Lett.*, **77**, No. 19, 4058–4061 (1996).
14. I. S. Shlimak, *Phys. Solid State*, **41**, No. 5, 716–719 (1999).
15. N. A. Poklonski, S. A. Vyrko, I. I. Anikeev, and A. G. Zabrodskii, *Semiconductors*, **56**, No. 11, 823–830 (2022).
16. N. A. Poklonski, S. A. Vyrko, O. N. Poklonskaya, and A. G. Zabrodskii, *J. Appl. Phys.*, **110**, No. 12, Article ID 123702 (2011).
17. N. A. Poklonski, S. A. Vyrko, O. N. Poklonskaya, and A. G. Zabrodskii, *Semiconductors*, **50**, No. 6, 722–734 (2016).
18. T. G. Castner, N. K. Lee, H. S. Tan, L. Moberly, and O. Symko, *J. Low Temp. Phys.*, **38**, Nos. 3–4, 447–473 (1980).
19. N. A. Poklonski, S. A. Vyrko, A. I. Kovalev, and A. N. Dzeraviahia, *J. Phys. Commun.*, **2**, No. 1, Article ID 015013 (2018).
20. K. Seeger, *Semiconductor Physics. An Introduction*, Springer, Berlin (2004).
21. N. A. Poklonskii, S. A. Vyrko, A. G. Zabrodskii, and S. V. Egorov, *Phys. Solid State*, **45**, No. 11, 2053–2059 (2003).
22. N. A. Poklonski and S. Yu. Lopatin, *Phys. Solid State*, **43**, No. 12, 2219–2228 (2001).
23. C. Hamaguchi, *Basic Semiconductor Physics*, Springer, Cham (2023).
24. V. P. Gribkovskii and V. K. Kononenko, *J. Appl. Spectrosc.*, **12**, No. 1, 39–48 (1970).
25. E. O. Kane, *Solid-State Electron.*, **28**, Nos. 1–2, 3–10 (1985).
26. N. A. Poklonski, S. A. Vyrko, and A. G. Zabrodskii, *Solid State Commun.*, **149**, Nos. 31–32, 1248–1253 (2009).
27. N. L. Lavrik and V. P. Voloshin, *J. Chem. Phys.*, **114**, No. 21, 9489–9491 (2001).
28. N. A. Poklonski, S. A. Vyrko, and A. G. Zabrodskii, *Semicond. Sci. Technol.*, **25**, No. 8, Article ID 085006 (2010).
29. J. S. Blakemore, *Semiconductor Statistics*, Dover, New York (2002).
30. J. Shah, R. C. C. Leite, and J. P. Gordon, *Phys. Rev.*, **176**, No. 3, 938–942 (1968).
31. Ya. E. Pokrovskii, O. I. Smirnova, and A. Khvalkovskii, *J. Exp. Theor. Phys.*, **85**, No. 1, 121–129 (1997).
32. A. M. Mathai and H. J. Haubold, *Probability and Statistics: A Course for Physicists and Engineers*, De Gruyter, Berlin (2018).
33. P. Whittle, *Probability via Expectation*, Springer, New York (2000).
34. O. Madelung, *Semiconductors: Data Handbook*, Springer, Berlin (2004).
35. V. P. Dobrego and I. S. Shlimak, *Phys. Status Solidi*, **33**, No. 2, 805–809 (1969).
36. R. Rentzsch and I. S. Shlimak, *Phys. Status Solidi A*, **43**, No. 1, 231–238 (1977).
37. I. Shlimak, *Is Hopping a Science? Selected Topics of Hopping Conductivity*, World Scientific Publ. Co. Pte. Ltd., Singapore (2015).
38. N. A. Poklonski, S. A. Vyrko, and A. G. Zabrodskii, *Phys. Solid State*, **46**, No. 6, 1101–1106 (2004).
39. L. V. Berman and A. I. Seliverstov, *Sov. Phys. Semicond.*, **23**, No. 11, 1213–1216 (1989).