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SUPERRADIANT PARAMETRIC X-RAY EMISSION

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During the propagating of modulated relativistic electron bunch formed in an undulator of X-ray free electron laser (XFEL) due to the mechanism of self amplified spontaneous emission (SASE) through a crystal located at exit from the undulator, quasimonochromatic X-ray pulse is formed – superradiant parametric X-ray quanta (SPXE) in the case of a resonance between the frequency of parametric X-ray radiation and the frequency of modulation of the electron bunch. The intensity of this pulse is proportional to the square of the number of electrons in the bunch, and its characteristics are comparable with the parameter of the main XFEL pulse directed along the electron velocity. Meanwhile, the pulse of the SPXE will be directed at a large angle to the electron velocity, which expands the applicability of the XFEL.

Parametric X-ray radiation (PXR) is a well-known mechanism of radiation of charged particles propagating in a periodic medium [1]. Its qualitative properties are the emission of quasimonochromatic X-ray beams at a large angle to the electron velocity and the possibility of continuous tuning of the radiation frequency by simple rotation of the crystal. We consider that the charged particles are a bunch of electrons emitted from the XFEL channel. Due to the SASE mechanism, the electron bunch, whose density was initially uniformly distributed, is transformed into a sequence of minibunches, which leads to spatial modulation of the electron density with a period d_0. Subsequently, when the bunch propagates inside the undulator, coherent radiation is formed with a frequency $\omega_0 = 2\pi / d_0$ and intensity proportional to the square of the number of electrons in the bunch. Coherent radiation propagates in a small cone along the electron velocity. The resonance between the bunch modulation frequency and the emitted photon frequency occurs automatically, since the beam modulation and the emission frequency are determined by the same undulator radiation mechanism.

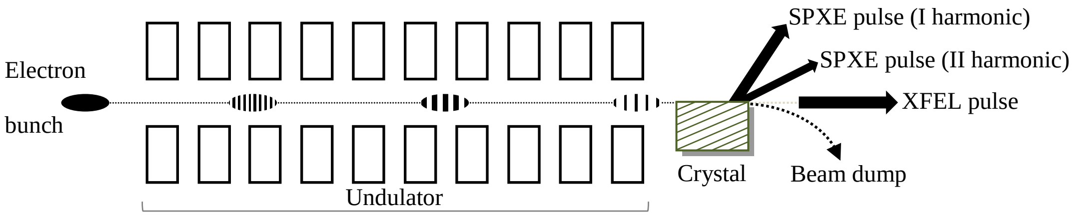

Figure 1 – Qualitative scheme of the processes leading to the generation of SPXE pulses in the XFEL channel.

Now let us consider the case of SPXE, when we assume that the electron bunch is modulated in density inside the undulator and at the end of the exit hits the crystal target (Figure 1), where PXR is generated with a frequency ω_B depending on the crystal structure and the angle θ_B between the crystallographic planes and the electron velocity. The angle θ_B can be chosen such that the resonance condition $\omega_0 \approx \omega_B$. As a result, in addition to the main XFEL pulse, the SPXE pulse will be generated with intensity proportional to the square of the number of electrons in the bunch and directed at a large angle to the electron velocity direction. The characteristics of the SPXE are comparable with the parameter of the main pulse of the XFEL, and the higher harmonics of the SPXE can be used to generate pulses of harder X-ray radiation without requiring a change in the energy of the electron bunch.

A simple and effective method for describing the electromagnetic processes of relativistic charged particles interacting with a medium, namely the method of equivalent photons (pseudophotons, the Weizsäcker-Williams approximation) is used to analyze the SPXE according to its qualitative estimation. Accordingly, the intrinsic field of a relativistic charged particle is equivalent in its characteristics to a beam of pseudophotons. In the frequency range much lower than the particle energy E, i.e. $\omega \ll E$, the spectrum of pseudophotons can be obtained by the classical description [2]. Maxwell equations will take the following form:

$$\Box\vec{A} = -4\pi e_0 \sum_a^N \vec{v}_a \delta(\vec{r} - \vec{v}_a t - \vec{r}_a) \tag{1}$$

$$\Box\varphi = -4\pi e_0 \sum_a^N \delta(\vec{r} - \vec{v}_a t - \vec{r}_a) \tag{2}$$

wher $\Box$ – d'Alembert operator, the sum applies to all particles, each of which is in the initial position $\vec{r}_a$ and has the velocity $\vec{v}_a$.

Using the Fourier transform, one can calculate the electromagnetic fields $\vec{E}(\vec{r},t)$ and $\vec{H}(\vec{r},t)$. The projection of the electromagnetic field energy flux on the x axis along the average velocity $\vec{v}$ is determined by the following expression [3]:

$$\Pi = \frac{1}{4\pi}\int_{-\infty}^{\infty} dzdydt[\vec{E}\vec{H}]_x = \int_{-\infty}^{\infty} dxdydt[\vec{E}_\perp]^2$$
$$\approx \frac{e_0^2}{2\pi^2 v}\sum_a\sum_b\int d\vec{k}\,\frac{(\vec{k}_\perp - \vec{\theta}_a k_x)(\vec{k}_\perp - \vec{\theta}_b k_x)e^{i\vec{k}(\vec{r}_b - \vec{r}_a)}e^{ix\vec{k}(\vec{v}'_a - \vec{v}'_b)}}{[k_x^2\gamma^{-2} + (\vec{k}_\perp - \vec{\theta}_a k_x)^2][k_x^2\gamma^{-2} + (\vec{k}_\perp - \vec{\theta}_b k_x)^2]} \tag{3}$$

where relativistic gamma factor of the particle γ = E/m, photon with frequency ω and wave vector $\vec{k} = (\omega\vec{v}/v, \vec{k}_\perp)$, the angle θ_a determines the small angular divergence of the electron bunch, so that $\vec{v}_a = \vec{v} + \vec{v}'_a;\quad \vec{v}'_a \ll \vec{v}$.

This projection can be split into the sum of two parts:

$$\Pi = \Pi_{sp} + \Pi_{coh} \tag{4}$$

The incoherent (spontaneous) flux Π_{sp} corresponds to the case when the indices are the same, i.e., $a = b$

$$\Pi_{sp} = \frac{e_0^2}{2\pi^2 v} N\int d\vec{k}\,\frac{k_\perp^2}{[k_x^2\gamma^{-2} + \vec{k}_\perp^2]^2} = \int \omega n_{sp}(\omega)d\omega\ ;$$
$$n_{sp}(\omega) = N\frac{2e_0^2}{\pi\omega}\ln\frac{m\gamma}{\omega} \tag{5}$$

The coherent part Π_{coh} is given by the following expression:

$$\Pi_{coh} = \frac{e_0^2}{2\pi^2 v}\int d\vec{k}\left|\vec{F}(\vec{k})\right|^2,\quad \vec{F}(\vec{k}) = \sum_a^N \frac{(\vec{k}_\perp - \vec{\theta}_a k_x)}{k_x^2\gamma^{-2} + (\vec{k}_\perp - \vec{\theta}_a k_x)^2}e^{-i\vec{k}\vec{r}_a}e^{ix\vec{k}\vec{v}'_a};$$
$$n_{coh}(\omega) \approx \frac{N^2 e_0^2}{2\pi\omega v^2}\frac{d_0^2}{L_b^2}[-e^{a^2\gamma^{-2}}Ei(-a^2\gamma^{-2})(1 + a^2\gamma^{-2}) - 1]\left|\frac{1 - e^{iL_b\omega/v}}{1 - e^{id_0\omega/v}}\right|^2 \exp\left[-\frac{\omega^2\sigma_c^2}{2v^2}\right] \tag{6}$$

where d_0 – the oscillation period of the modulated bunch of length $L_b = Kd_0$, parameter $\sigma_c \ll d_0$ determining fluctuations of the oscillation period, and the number of the minibunches $K \gg 1$.

Let us compare the contributions of the coherent N_{coh} and incoherent N_{sp} parts with the chosen parameters as follows: the typical electron energy $E = 6.7\text{GeV}$; $\gamma = 13111$. Parameters $\sigma_a = 10^{-4}$ and $\sigma_b = 2\times 10^{-5}\text{cm}$, the parameter $a^2\gamma^{-2} \approx 0.2$. Let the bunch charge $Q = 0.2$ nC, which corresponds to $N = 1.2\times10^{9}$ electrons. The duration of the photon pulse can be chosen equal to 25 fs, which corresponds to the length of the modulated bunch $L_b = 8.3\times10^{-5}\,\text{cm}$ and the modulation period $d_0 = 10^{-8}\,\text{cm}$ with the parameter $\sigma_c = 10^{-9}\,\text{cm}$. Figure 2 shows the incoherent and two harmonics of the coherent spectral density of pseudophotons of the modulated bunch with these parameters [4].

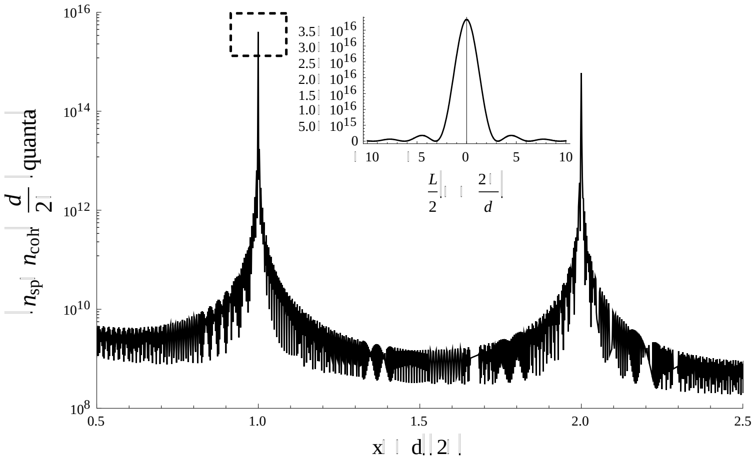

Figure 2 – The incoherent and two harmonics of the coherent pseudophotons spectral densities.

Finally, the typical frequency spread for the XFEL pulse $\Delta\omega/\omega_0 \approx 10^{-3}$ and the frequency $\omega_0 = 6.28\times10^{8}\,\text{cm}^{-1}$. Thus, we can estimate the number of incoherent photons: $N_{sp} = n_{sp}(\omega_0)\Delta\omega = 1.1\times10^{5}$, this number is significantly less than the corresponding number of photons emitted by the XFEL pulse. At the same time, the number of coherent photons near the resonant frequency $\omega_l = 2\pi l/d_0 : N_{coh} \approx 5.9\times10^{11}$ this number is comparable to the number of photons in the undulator XFEL pulse. In addition, the number of pseudophotons corresponding to the second harmonic (l = 2) is 10 times less.

We have shown that SPXE results from the reflection of coherent pseudophotons from crystallographic planes [4]. The frequencies of these pseudophotons are located near the resonance frequency $\omega_0 = 2\pi/d_0$.

$$N_{\text{SPXQ}} \approx \frac{\pi}{2\sin 2\theta_B} R \frac{\Delta\theta_B}{\Delta\theta_{ps}} N_{coh}; \quad \Delta\theta_B \approx (\omega_0 L)^{-1}, \tag{7}$$

where $\Delta\theta_{ps}$ – angular spread of pseudophotons. Thus, real photons are emitted at a large angle $2\theta_B$ to the electron velocity. Then, by choosing the orientation of the crystal, photons can be directed to any desired location. This makes it possible to obtain additional experimental windows in XFEL experiments.

Reference

1. Coherent x-rays at MAMI / W. Lauth, H. Backe, O. Kettig, P. Kunz, A. Sharafutdinov, and T. Weber // Eur. Phys. J. A. – 2006. – Vol. 28, № S1. – P. 185–195.

2. Akhiezer, A. I. Quantum Electrodynamics / A. I. Akhiezer and V. B. Berestetskii. – Interscience Monographs and Texts in Physics and Astronomy, Interscience, New York, 1965.

3. Feranchuk, I. D. Method of the equivalent photons for modulated electron beam / I. D. Feranchuk, O. D. Skoromnik, Nguyen Quang San // Journal of the Belarusian State University. Physics. Physics. – 2020. – № 3. – P. 24–31. Russian.

4. Feranchuk, I. D. Superradiant parametric x-ray emission / I. D. Feranchuk, N. Q. San, and O. D. Skoromnik // Phys. Rev. Accel. Beams. – 2022. – Vol. 25, № 12. – P. 120702 (12 pp.).