УДК 336.763, 519.863 JEL G11, C02

# MODIFICATION OF PORTFOLIO OPTIMIZATION MODELS USING FUZZY TRIANGULAR NUMBERS

**I. V. Bolshakova, E. I. Tarletskaya**

*Belarusian State University, Minsk, Republic of Belarus*

The article is devoted to solving the problem of portfolio optimization on the example of shares of music streaming services using such classical optimization models as the Markowitz model and the Konno-Yamazaki model. Due to the imperfection of classical models and their low information content, modification is proposed using fuzzy triangular numbers.

*Keywords:* portfolio optimization; Markowitz model; Konno-Yamazaki model; fuzzy triangular numbers; music streaming.

# МОДИФИКАЦИЯ МОДЕЛЕЙ ПОРТФЕЛЬНОЙ ОПТИМИЗАЦИИ С ПОМОЩЬЮ НЕЧЕТКИХ ТРЕУГОЛЬНЫХ ЧИСЕЛ

**И. В. Большакова, Е. И. Тарлецкая**

*Белорусский государственный университет, г. Минск, Республика Беларусь*

Статья посвящена решению задачи портфельной оптимизации на примере акций музыкальных стриминговых сервисов с помощью таких классических моделей оптимизации, как модель Марковица и модель Конно-Ямазаки. Ввиду несовершенства классических моделей и их малой информативности предложена модификация с помощью нечетких треугольных чисел.

*Ключевые слова:* портфельная оптимизация; модель Марковица; модель Конно-Ямазаки; нечеткие треугольные числа; музыкальный стриминг.

Music streaming services in less than 20 years of their existence have become not only the most popular way to listen to music, but also a key source of revenue for the global music industry: at the moment, music streaming accounts for almost 67% of the revenue of the entire music industry [1]. Despite the fact that the music streaming market is only on the way to reaching a state of equilibrium, these are promising and innovative companies whose shares are deservedly becoming increasingly popular on the stock market.

The analysis of the effectiveness of investments was carried out based on the formation of an investment portfolio of shares of major music streaming companies: Spotify, Tencent Music, Apple Music, Amazon Music and Yandex.Music. Since in the last year there have been significant changes and

fluctuations in the stock market, it was decided to consider the task of optimizing the short-term portfolio and calculate the profitability of companies' shares by week. Data on the profitability of shares of the selected companies are shown in Table 1.

*Table 1*

**Profitability of shares of streaming companies by weeks**

| Period | Spotify Technology | Tencent Music Entertainment | Apple Music (Apple) | Amazon Music (Amazon) | Yandex. Music (Yandex) |
|---|---|---|---|---|---|
| 1 | 17.82 | 11.08 | 8.18 | 10.78 | –0.75 |
| 2 | –8.03 | 26.53 | 1.06 | –6.60 | 1.95 |
| 3 | 2.26 | 3.58 | –2.10 | –0.78 | –3.96 |
| 4 | 2.01 | 28.03 | –0.20 | 0.77 | –2.61 |
| 5 | –2.66 | 10.95 | –3.82 | –5.35 | –3.18 |
| 6 | –4.39 | –3.90 | –5.38 | –1.38 | –6.71 |
| 7 | 3.96 | 2.15 | –1.97 | –2.97 | –4.25 |
| 8 | 1.61 | 2.73 | –1.46 | –1.47 | –6.10 |
| 9 | 5.81 | 5.56 | –0.24 | 2.48 | –0.61 |
| 10 | 10.20 | 2.86 | 3.97 | 13.99 | 3.33 |
| 11 | 6.35 | –6.90 | 2.31 | –0.89 | 3.70 |
| 12 | 4.42 | 3.58 | 5.85 | 5.13 | –1.22 |
| 13 | 18.52 | –2.88 | 5.87 | 1.12 | 6.48 |
| 14 | 3.29 | –3.80 | –2.26 | –5.59 | –1.25 |
| 15 | 0.14 | –1.11 | 1.02 | –0.42 | –7.52 |
| 16 | –6.53 | –5.24 | –3.83 | –3.81 | –1.86 |
| 17 | 5.63 | 7.11 | 2.94 | 1.50 | –0.05 |
| 18 | –1.68 | –7.50 | –1.68 | –4.39 | –4.35 |
| 19 | 4.42 | 4.79 | 4.18 | 8.51 | –2.41 |
| **Average return, %** | 3.32 | 4.09 | 0.65 | 0.56 | –1.65 |

*Source:* development of authors based on data from stock markets [2–6] for the period 30.10.2022 – 12.03.2023.

Note that for Yandex.Music, the value of the average profitability has already become negative at this stage, and therefore it is inexpedient to invest in this company. The further portfolio will be made up of shares of four companies with positive average returns.

Let's try to solve the problem of optimizing this portfolio using the Markowitz model, which is based on the following statements: the main parameters determined in portfolio management are its profitability(return) and risk; when forming a portfolio, it is impossible to accurately determine the

future dynamics of its profitability and risk. Therefore, decisions are made on the basis of expected values of profitability and risk, which are estimated on the basis of statistical information for previous periods of time.

The main task set by G. Markowitz was formulated as follows: the investor wants to get a return equal to R, based on a certain set of securities. How should he build a portfolio with the least overall risk that has an average *R* return? If the portfolio is modeled by a multidimensional random variable of the expected return of its assets, then it is possible to identify the parameters of the average return (profitability), the estimated deviation from the average (risk) and build a correlation matrix of the relationship between the assets of the portfolio [7].

Then the task of optimizing the stock portfolio is either to minimize risk, or to maximize return, or to maximize the return of the portfolio at a fixed level of its risk. Risk in the Markowitz model is calculated using a covariance matrix, which can be easily calculated using the MS Excel Data Analysis package (Table 2).

*Table 2*

**Covariance Matrix Of the Companies' Returns**

| *Covariance matrix* | Spotify | Tencent Music | Apple Music | Amazon Music |
|---|---|---|---|---|
| Spotify | 46.00 | –6.98 | 18.86 | 24.76 |
| Tencent Music | –6.98 | 91.36 | 5.52 | 3.01 |
| Apple Music | 18.86 | 5.52 | 13.34 | 14.48 |
| Amazon Music | 24.76 | 3.01 | 14.48 | 29.82 |

Now let's go directly to the indicators we need: the return and risk of the portfolio. Average return of the portfolio *R* is calculated as the sum of shares of the company's stocks in the portfolio $x_j$ multiplied by average return $\overline{r_j}$:

$$R = \sum_{j=1}^{N} \overline{r_j} \cdot x_j.$$

Portfolio risk $\sigma(\mathrm{x})$ is calculated based on the resulting covariance matrix:

$$\sigma(\mathrm{x}) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot x_j \cdot \sigma_{ij} \,,$$

where the values of $\sigma_{ij}$ are substituted from the covariance matrix [7].

As a result, to minimize the risk, all the following conditions must be met:

$$\begin{cases} \sigma(\mathrm{x}) \rightarrow min, \\ R \geq 0, \\ \sum_{j=1}^{N} x_j = 1, \\ 0 \leq x_j \leq 1. \end{cases}$$

For the established risk limit $\sigma(\mathrm{x}) \leq 5$ another restriction is added to the conditions of the task of maximizing return:

$$\begin{cases} R \rightarrow max, \\ \sigma(\mathrm{x}) \leq 5, \\ \sum_{j=1}^{N} x_j = 1, \\ 0 \leq x_j \leq 1. \end{cases}$$

In order to diversify the risk of investing capital, bilateral restrictions are imposed on the share of investments in the stocks of each company: $0.1 \leq x_1 \leq 0.55, 0.15 \leq x_2 \leq 0.7,\ 0.05 \leq x_3 \leq 0.4,\ 0.05 \leq x_4 \leq 0.35,$ where $x_1$ – Spotify's share of the portfolio, and $x_2, x_3$ и $x_4$ – shares of Tencent Music, Apple Music and Amazon Music, respectively.

Having solved the problem in three interpretations with given restrictions, we get the results shown in table 3.

*Table 3*

**Solving the Optimization Problem Using the Markowitz Model**

| Company | Proportion in portfolio, % | | |
|---|---|---|---|
| | Risk minimization | Return maximization | Return maximization ($\sigma(\mathrm{x}) \leq 5\%$) |
| Spotify ($x_1$) | 10.00 | 20.00 | 54.30 |
| Tencent Music ($x_2$) | 17.51 | 70.00 | 34.53 |
| Apple ($x_3$) | 40.00 | 5.00 | 6.17 |
| Amazon ($x_4$) | 32.49 | 5.00 | 5.00 |
| **Risk** σ, % | **4.04** | **6.80** | **5.00** |
| **Return** R, % | **1.49** | **3.59** | **3.28** |

Obviously, Tencent Music's highest return company is more preferable to maximizing returns, but these stocks are quite risky, and Spotify is preferred when restricting risk. Apple Music and Amazon Music are considered more stable and less risky, since they are parts of corporations, but they will not bring high profitability to the investor.

Konno model (Mean-Absolute Deviation, MAD) – Yamazaki is to some extent an alternative to the classical Markowitz model, since it involves the use of absolute deviation instead of standard deviation σ as a risk measure for the problem of portfolio optimization [7]. For this model, portfolio risk is calculated as follows:

$$\sigma(\mathrm{x}) = \frac{1}{\mathrm{T}} \cdot \sum_{t=1}^{T} \left| \sum_{j=1}^{N} \left(r_{jt} - \overline{r_j}\right) \cdot x_j \right|,$$

where *T* is the number of periods.

All conditions of the problems look similar to the Markowitz model. Solutions to these problems using Konno-Yamazaki model are shown in table 4.

*Table 4*

**Solving the Optimization Problem Using the Konno-Yamazaki Model**

| Company | Proportion in portfolio, % | | |
|---|---|---|---|
| | Risk minimization | Return maximization | Return maximization ($\sigma(\mathrm{x}) \leq 5\%$) |
| Spotify ($x_1$) | 10.00 | 20.00 | 22.14 |
| Tencent Music ($x_2$) | 15.37 | 70.00 | 67.86 |
| Apple ($x_3$) | 40.00 | 5.00 | 5.00 |
| Amazon ($x_4$) | 34.62 | 500 | 5.00 |
| **Risk** σ, % | **3.41** | **5.12** | **5.00** |
| **Return** R, % | **1.42** | **3.59** | **3.57** |

In general, the risk value in the Konno-Yamazaki model is slightly lower than in the Markowitz model, and therefore, with a risk limit of up to 5 %, the return is higher and Tencent Music is preferred again.

But can these classic models qualitatively predict the possible income and risks of the investor? Probably not. Now the stock market has a downward trend in the shares of most large companies and corporations, and even the S&P 500 index had a negative trend in 2022, only gradually resuming growth in early 2023. Such fluctuations in the market and in the economy cannot always be predicted, and therefore in such a situation it is important to get as much information as possible about possible losses or returns on investment.

With only the value of risk and average return, it is quite difficult to make the right investment decision.

Using the method of solving the optimization problem using fuzzy triangular numbers (FTN), in addition to the value of the average return of the portfolio, it is possible to obtain the values of pessimistic, most probable and optimistic returns, which shows a clearer picture for the investor.

Fuzzy triangular number $\tilde{R} = (r_1^j, r_2^j, r_3^j)$ – the return of the *j*-th financial instrument, consisting respectively of the minimum possible, most probable and maximum possible future profitability of the *j*-th instrument [8]. Then we will present the initial data of the portfolio task in the form of table 5.

*Table 5*

**Expected return of shares of music streaming services, presented by FTN**

| Company | Expected return, % | | | | Average return, % |
|---|---|---|---|---|---|
| | FTN $\tilde{R}^j$ | min $r_1^j$ | m.pr. $r_2^j$ | max $r_3^j$ | $\tilde{R}_{av}^j = \frac{r_1 + 2r_2 + r_3}{4}$ |
| Spotify ($x_1$) | $\tilde{R}^1$ | –8.03 | 3.32 | 18.52 | 4.28 |
| Tencent Music ($x_2$) | $\tilde{R}^2$ | –7.50 | 4.09 | 28.03 | 7.17 |
| Apple ($x_3$) | $\tilde{R}^3$ | –5.38 | 0.65 | 8.18 | 1.03 |
| Amazon ($x_4$) | $\tilde{R}^4$ | –6.60 | 0.56 | 13.99 | 2.13 |

When using FTN, the average return of the portfolio is calculated differently, and its values become higher than the average return in the Markowitz or Konno-Yamazaki model. In this case, those values of their average return correspond to the most probable return $r_2^j$.

By itself, the method of using fuzzy triangular numbers to solve an optimization problem does not imply the presence of any risk value in the solution – that is, this method is also imperfect. Therefore, it is reasonable to try to combine the classical models with fuzzy triangular number method and try to solve the optimization problem in the same three interpretations, taking as the value of the average return the average return calculated using the FTN ($\tilde{R}_{av}^j$), and the risk value σ taken from the classical model. Thus, we get the following solutions (tables 6 and 7).

So, we already get several other portfolios, where the average return is generally higher, and in the case, for example, of minimizing risk, the optimistic return reaches 14.58 % or 16.32 %. Now we see that the investor can receive losses from such investments, but at the same time the absolute optimistic return value is more than twice, or even three times higher than the pessimistic one – that is, the investor is more likely to make a profit.

*Table 6*

**Modification of the Markowitz model using FTN**

| Company | Proportion in portfolio, % | | |
|---|---|---|---|
| | Risk minimization | Return maximization | Return maximization ($\sigma(x) \leq 5\%$) |
| Spotify ($x_1$) | 10.00 | 20.00 | 48.60 |
| Tencent Music ($x_2$) | 17.51 | 70.00 | 37.52 |
| Apple ($x_3$) | 40.00 | 5.00 | 5.58 |
| Amazon ($x_4$) | 32.49 | 5.00 | 8.30 |
| **Risk σ, %** | 4.04 | 6.80 | 5.00 |
| **Most probable return $r_2^j$, %** | 1.49 | 3.59 | 3.23 |
| **Pessimistic return $r_1^j$, %** | –6.41 | –7.46 | –7.56 |
| **Average return $\tilde{R}_{av}^j$, %** | **2.79** | **6.04** | **5.01** |
| **Optimistic return $r_3^j$, %** | 14.58 | 24.43 | 21.13 |

*Table 7*

**Modification of the Konno-Yamazaki model using FTN**

| Company | Proportion in portfolio, % | | |
|---|---|---|---|
| | Risk minimization | Return maximization | Return maximization ($\sigma(x) \leq 5\%$) |
| Spotify ($x_1$) | 32.28 | 20.00 | 30.94 |
| Tencent Music ($x_2$) | 22.72 | 70.00 | 59.06 |
| Apple ($x_3$) | 40.00 | 5.00 | 5.00 |
| Amazon ($x_4$) | 5.00 | 5.00 | 5.00 |
| **Risk σ, %** | 3.60 | 5.80 | 5.00 |
| **Most probable return $r_2^j$, %** | 2.29 | 3.59 | 3.50 |
| **Pessimistic return $r_1^j$, %** | –6.78 | –7.46 | –7.51 |
| **Average return $\tilde{R}_{av}^j$, %** | **3.53** | **6.04** | **5.72** |
| **Optimistic return $r_3^j$, %** | 16.32 | 24.43 | 23.39 |

The combination classic portfolio optimization models with fuzzy triangular numbers (FTN) method can give the investor more information about possible returns or losses. They complement each other, calculating either risk values and pessimistic, optimistic and most probable return values. Using such models for solving the optimization problem on music streaming services shows that the companies are good for investing (except Yandex.Music): the optimistic return goes up to 24.43 % in a week while the highest risk value is 6.8 %, of course if we diversify risk. Thus, using operations on fuzzy numbers, it is possible to generalize the main optimization algorithms of classical portfolio theory.

## References


1. Global Music Report [Электронный ресурс] / IFPI. – 2023. – 55 с. Режим доступа: https://globalmusicreport.ifpi.org/ (дата доступа: 04.04.2023).
2. Spotify Technology S. A. Price Data [Электронный ресурс] / MarketWatch. – 2023. Режим доступа: https://ru.investing.com/equities/spotify-technology-historical-data (дата доступа: 15.03.2023).
3. Tencent Music Entertainment Group ADR Price Data [Электронный ресурс] / MarketWatch. – 2023. Режим доступа: https://ru.investing.com/equities/tencent-music-entertainment-group-historical-data (дата доступа: 15.03.2023).
4. Apple Inc. Price Data [Электронный ресурс] / MarketWatch. – 2023. Режим доступа: https://ru.investing.com/equities/apple-computer-inc-historical-data (дата доступа: 15.03.2023).
5. Amazon.com Inc. Price Data [Электронный ресурс] / MarketWatch. – 2023. Режим доступа: https://ru.investing.com/equities/amazon-com-inc-historical-data (дата доступа: 15.03.2023).
6. Динамика курса акций Yandex N.V. [Электронный ресурс] / Тинькофф. – 2023. Режим доступа: https://www.tinkoff.ru/invest/stocks/YNDX/ (дата доступа: 15.03.2023)
7. *Bolshakova I.* Portfolio optimization: a survey // Journal of the Belarusian State University. Economics. – 2017. – № 2. – С. 4–15.
8. *Bolshakova I.* Fuzzy returns in portfolio theory (method of triangular fuzzy numbers) // Journal of the Belarusian State University. Economics. – 2020. – № 2. – С. 50–59.