

УДК 535.42+537.86

СВЕТОВЫЕ ПУЧКИ, ГЕНЕРИРУЕМЫЕ АКСИКОНОМ С ЗАКРУГЛЕННОЙ ВЕРШИНОЙ

С. Н. КУРИЛКИНА^{1), 2)}, Р. Ё. А. А. АЛБЛООШИ²⁾,
П. И. РОПОТ¹⁾, А. М. ВАРАНЕЦКИЙ¹⁾

¹⁾Институт физики им. Б. И. Степанова НАН Беларуси,
пр. Независимости, 68, 220072, г. Минск, Беларусь

²⁾Белорусский государственный университет, пр. Независимости, 4, 220030, г. Минск, Беларусь

Внимание сфокусировано на реальной форме аксикона, не острой, а закругленной. Рассмотрен несовершенный аксикон с закругленной вершиной, форма которого аппроксимирована гиперболоидом, и проведен теоретический и экспериментальный анализ свойств пучка, генерируемого в дальней зоне за таким аксиконом. Продемонстрировано, что если в вершине форма аксикона отклоняется от конической в пределах десятков микрометров, то поперечное распределение интенсивности формируемого светового поля имеет сильно осциллирующий характер.

Образец цитирования:

Курилкина СН, Алблооши РЁАА, Ропот ПИ, Варанецкий АМ. Световые пучки, генерируемые аксиконом с закругленной вершиной. *Журнал Белорусского государственного университета. Физика*. 2023;2:14–21 (на англ.).
<https://doi.org/10.33581/2520-2243-2023-2-14-21>

For citation:

Kurilkina SN, Alblooshi RYAA, Ropot PI, Varanetski AM. Light beams generated by oblate-tip axicon. *Journal of the Belarusian State University. Physics*. 2023;2:14–21.
<https://doi.org/10.33581/2520-2243-2023-2-14-21>

Авторы:

Светлана Николаевна Курилкина – доктор физико-математических наук, профессор; главный научный сотрудник центра «Диагностические системы»¹⁾, профессор кафедры физической оптики и прикладной информатики физического факультета²⁾.

Рашед Ёусеф Абдулла Алхаяяс Алблооши – магистрант кафедры лазерной физики и спектроскопии физического факультета. Научный руководитель – С. Н. Курилкина.

Петр Иосифович Ропот – кандидат физико-математических наук, доцент; заместитель заведующего центром «Диагностические системы».

Алексей Михайлович Варанецкий – научный сотрудник центра «Диагностические системы».

Authors:

Svetlana N. Kurilkina, doctor of science (physics and mathematics), full professor; chief researcher at the center «Diagnostic systems»^a and professor at the department of physical optics and applied computer science, faculty of physics^b.

s.kurilkina@ifanbel.bas-net.by

<https://orcid.org/0000-0002-1866-4791>

Rashed Yousef Abdulla Alhawayas Alblooshi, master's degree student at the department of laser physics and spectroscopy, faculty of physics.

rashed.alblooshi@tii.ae

Piotr I. Ropot, PhD (physics and mathematics), docent; deputy head of the center «Diagnostic systems».

p.ropot@dragon.bas-net.by

<https://orcid.org/0009-0005-6132-8952>

Aliaksei M. Varanetski, researcher at the center «Diagnostic systems».

a.varanecki@ifanbel.bas-net.by

<https://orcid.org/0009-0005-3466-5902>

При этом ширина кольца (область, в которой нормированная на максимальное значение интенсивность превышает величину 0,5) меньше таковой в случае идеального аксикона. Эти колебания возникают в результате интерференции частей падающего пучка, проходящего через круглый линзообразный наконечник аксикона и окружающую его коническую поверхность. Показано, что периодичность колебаний зависит от параметра закругления: при увеличении данного параметра периодичность колебаний (а также радиус центрального максимума) уменьшается, а их амплитуда увеличивается. Предложен и апробирован метод определения закругления реального аксикона. Полученные результаты могут быть полезны для уточнения характеристик изготавливаемых аксиконов.

Ключевые слова: аксикон; бесселевы пучки; дифракция; интерференция; интенсивность света.

LIGHT BEAMS GENERATED BY OBLATE-TIP AXICON

*S. N. KURILKINA^{a, b}, R. Y. A. A. ALBLOOSHI^b,
P. I. ROPOT^a, A. M. VARANETSKI^a*

^a*B. I. Stepanov Institute of Physics, National Academy of Sciences of Belarus,
68 Niezaliežnasci Avenue, Minsk 220072, Belarus*

^b*Belarusian State University, 4 Niezaliežnasci Avenue, Minsk 220030, Belarus*
Corresponding author: S. N. Kurilkina (s.kurilkina@ifanbel.bas-net.by)

The attention is focused on the real shape of the tip of the axicon, which is not sharp but rather oblate. The imperfect axicon with rounded tip approximated by a hyperboloid is considered, and the properties of the beam generated in far field behind such an axicon are analysed theoretically and experimentally. It has been demonstrated that if the axicon tip deviates in its apex from the ideal sharp tip in the range of tens of micrometers, the transversal intensity distribution is strongly oscillatory. Meanwhile, the ring width (area, within which normalised intensity is larger than 0.5) is smaller as compared with the case of ideal axicon. These oscillations result from the interference of parts of the incoming beam propagating through the round, lens-like axicon tip and the conical surface surrounding the tip. It is shown that the periodicity of oscillations depends on the parameter of bluntness: if this parameter increases the periodicity of oscillations (as well as the radius of the center of the light ring) decreases, and their amplitude increases. The method for determination of the bluntness of real axicon is proposed and tested. Obtained results can be useful for correction of characteristics of conventional axicons.

Keywords: axicon; Bessel beams; diffraction; interference; intensity of light.

Introduction

Axicons are a family of cylindrical symmetrical optical elements that produce a line focus rather than a point focus from incident collimated beam [1]. There are several types of axicons, working either by reflection or by transmission, and being either converging or diverging, but the most common one is probably the conical lens. Recently, new types of axicons are created using metasurfaces [2; 3]. Axicons are widely used in many applications, ranging from optical coherence tomography [4] and multi-photon imaging [5; 6] to manipulation and sorting of micro objects like biological cells [7–9] and generation of non-linear optical interactions (including plasma formation) in solids [10–12], liquids [13–15] and gases [16–19]. This interest is mainly due to the ability of axicons to form the light beams whose transversal intensity distribution is described by the zeroth-order Bessel function of the first kind. Meanwhile, this distribution is invariant along the beam propagation. Such beams are called as «quasi-non-diffracting» ones.

As a rule, considering the features of the field behind the axicon authors ignore diffraction effects on the axicon edges and assume an ideally sharp tip (see, for example, [20–25]). However, in most of the experimental realisations, the obtained pattern is more complicated than predicted such a way. Recently, it is shown that the diffraction from the axicon edges causes noticeable modulation of the on-axis optical intensity along the beam propagation [26–28]. Moreover, due to manufacturing constraints [29], the tip of the axicon deviates from the ideal cone shape and becomes rather round, causing significant aberrations in the intensity profile [30–32] (introducing modulations in the on-axis intensity). These oscillations result from the interference of parts of the incoming beam propagating through the round, lens-like axicon tip and the conical surface surrounding the tip [30]. Meanwhile, for narrow beams the axicon acts similarly as a conventional lens. Axicons with smaller base angles (longer zones of diffraction-less) are more prone to such aberrations. Results, obtained in works [30–32], demonstrated that axicon blunt profile should be taken into account for most applications,

especially those requiring high, smooth and continuous on-axis intensities. But determination of the bluntness value stays important problem because it needs a complex and expensive equipment.

In the present paper, we study the modifications induced by the bluntness of the conical lens vertex on the transverse intensity distribution of the beam. We show in particular that the round-tip axicon causes the oscillations in transversal intensity distribution. The periodicity of them can be used for estimation of bluntness and, as a consequence, optimisation of the transverse size of the incident beam.

Theory: the beam behind axicon

Axicons – conical lenses – are optical elements that have rotational symmetry about the z -axis (fig. 1, *a*). They generate a quasi-Bessel beam throughout their depth of focus (DOF) region. It is known that beyond the DOF, the beam gradually transforms into a ring of constant width and increasing the radius as it propagates. The important parameters that characterise an axicon are its front face radius R , the base angle α and the refractive index n (see fig. 1, *a*). These parameters together determine the length of the DOF of the axicon.

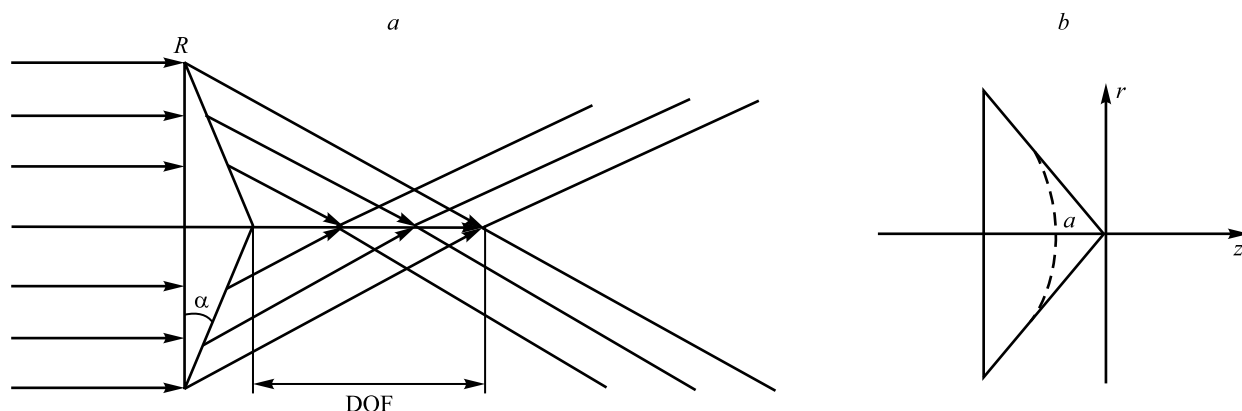


Fig. 1. Ideal refractive axicon ray tracing for the Gaussian input (*a*), DOF and approximation of its the rounded tip by hyperbola (*b*)

Let us consider now the input light beam to be a collection of rays traveling parallel to the z -axis. All these rays refract at the conical surface of the axicon towards the axis with the same angle θ . All the rays at one radial distance, come to focus at one point on the axis. The rays incident at the extreme of the axicon (i. e. the furthest radial distance) determine the DOF of the axicon, as shown in fig. 1, *a*. As follows from Snell's law, $\sin \theta = n \sin \alpha$. Thus, for the small angle α , the DOF is calculated by the formula

$$\text{DOF} = \frac{R}{(n-1)\alpha}.$$

But when the incident beam waist and the axicon radius are approximately equal, the contribution of diffraction on the edges becomes significant. It displays in oscillatory axial intensity [28]. Owing to this, as a rule, the beam waist is chosen smaller than axicon diameter $D = 2R$, and the length of focal line becomes less:

$$F = \frac{w_1}{(n-1)\alpha},$$

where w_1 is the half-width of incident beam on the front face of axicon.

Consider now the field formed by axicon with rounded tip (fig. 1, *b*). It was found that the latter can be correctly approximated by hyperbola [31] that yields a very good match between the calculations and experiments. The center of used coordinate system is combined with the tip of the perfect axicon, and z -axis is directed from the input surface.

The axicon surface is approximated by the hyperbola

$$\frac{z^2}{a^2} - \frac{r^2}{b^2} = 1, \quad (1)$$

where a and b are parameters characterised the curve. From equation (1) it follows

$$z = -\frac{a}{b} \sqrt{b^2 + r^2}. \quad (2)$$

Here $\frac{b}{a} = \operatorname{tg} \frac{\tau}{2}$, τ is the apex angle (the angle between asymptotes of hyperbola). Then we can rewrite equation (2):

$$z = -\sqrt{a^2 + r^2 \operatorname{ctg}^2 \frac{\tau}{2}} = -\sqrt{a^2 + r^2 \operatorname{tg}^2 \alpha}.$$

Parameter a (below – parameter of bluntness) characterises the shape of axicon, namely, the smaller the parameter a the closer the axicon shape to the ideal.

The transmission function of the axicon with rounded tip is determined by following relation:

$$t(r) = \exp\left(-ik_0(n-1)\sqrt{a^2 + r^2 \operatorname{tg}^2 \alpha}\right), \quad (3)$$

where k_0 denotes the free-space wavenumber associated with frequency ω . It should be noted that for $a \rightarrow 0$ and small angles α equation (3) transfers to well-known formula for transmission function of ideal axicon:

$$t_{\text{perf}}(r) = \exp[-ik_0(n-1)r \operatorname{tg} \alpha].$$

Let the field in the front face of axicon is the Gaussian beam:

$$E_{\text{in}}(r_1) = \frac{1}{\sqrt{W_g}} \exp\left(-\frac{r_1^2}{w_1^2}\right),$$

where $W_g = \frac{\pi w_1^2}{2}$. Here (as well as above) w_1 is the half-width of the beam. According to scalar wave optics, the field on distance z from the axicon is described by the integral

$$E_1(r_1, z) = \frac{-ik_0}{z\sqrt{W_g}} \int_0^R \exp\left(-\frac{r^2}{w_1^2} - ik_0(n-1)\sqrt{a^2 + r^2 \operatorname{tg}^2 \alpha} + ik_0 \frac{r_1^2 + r^2}{2z}\right) J_0\left(\frac{k_0 r r_1}{z}\right) r dr. \quad (4)$$

Let $z > F$. For this case the forming field is the conical beam. Let use equation (4) for calculation of its transversal intensity distribution. For definiteness we suppose below that $\lambda = 532$ nm (λ is the wavelength); $R = 1.77$ cm; $n = 1.4657$; $\alpha = 2^\circ$; $w_1 = 2$ mm. Thus, $F = 12$ cm. In fig. 2 the transversal intensity distribution in the plane $z = 45$ cm is represented for the case when we ignore the bluntness of the axicon tip ($a = 0$) and take it into account. It is seen that if $a = 0$ the ring field is characterised by negligible oscillations. For axicon with blunt profile the transversal intensity distribution is strongly oscillatory. Meanwhile, the ring width d (area, within which normalised intensity is larger than 0.5) is smaller as compared with the case of ideal axicon. These oscillations result from the interference of parts of the incoming beam propagating through the round, lens-like axicon tip and the conical surface surrounding the tip.

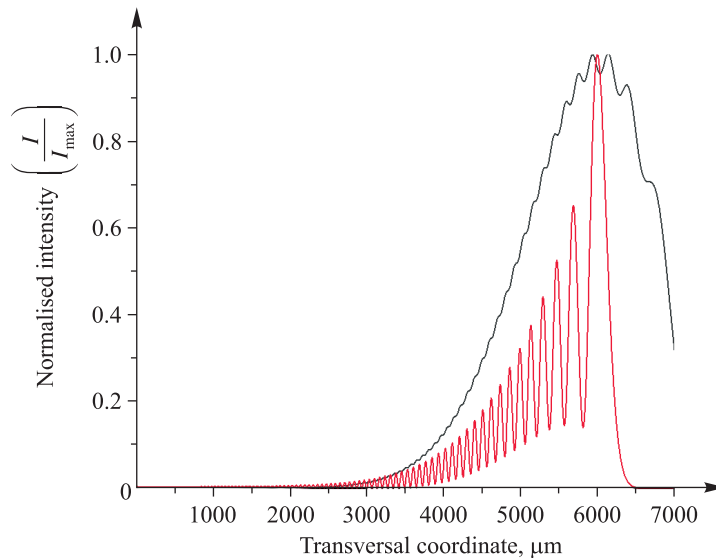


Fig. 2. Transversal intensity distribution of the field formed by ideal axicon (black curve) and axicon with blunt profile with $a = 8$ μm (red curve).
 Parameters: $\lambda = 532$ nm; $R = 1.77$ cm; $n = 1.4657$; $\alpha = 2^\circ$; $w_1 = 2$ mm; $z = 45$ cm

The periodicity of oscillations depends on the parameter of bluntness a . As illustrated in fig. 3, if parameter a increases the periodicity of oscillation (as well as the radius of the center of the light ring) decreases, and their amplitude increases.

It should be noted that diffraction pattern behind axicon is strongly dependent on the base angle α . For example, as seen from fig. 4, the change of angle α on 0.1° causes the shift of intensity maximum in the plane $z = 45$ cm on approximately $380 \mu\text{m}$. But variation of the base angle does not cause the change of the periodicity or amplitude of intensity oscillation.

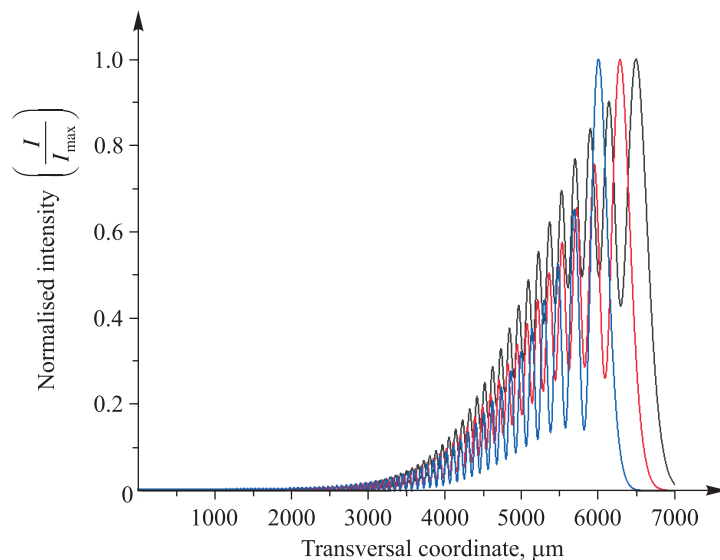


Fig. 3. Transversal intensity distribution of the field formed by axicon with blunt profile with $a = 3 \mu\text{m}$ (black curve), $a = 5 \mu\text{m}$ (red curve), $a = 8 \mu\text{m}$ (blue curve). Parameters: $\lambda = 532 \text{ nm}$; $R = 1.77 \text{ cm}$; $n = 1.4657$; $\alpha = 2^\circ$; $w_1 = 2 \text{ mm}$; $z = 45 \text{ cm}$

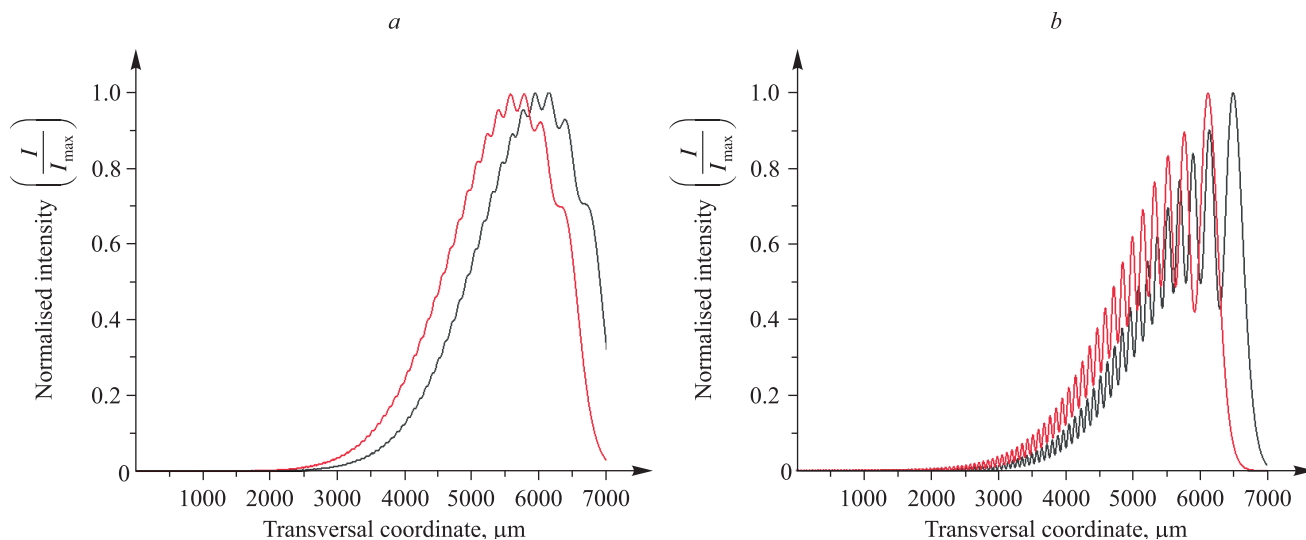


Fig. 4. Transversal intensity distribution of the field formed by ideal axicon (a) and axicon with blunt profile with $a = 3 \mu\text{m}$ (b) at base angle $\alpha = 2^\circ$ (black curve) and $\alpha = 1.9^\circ$ (red curve). Other parameters: $\lambda = 532 \text{ nm}$; $R = 1.77 \text{ cm}$; $n = 1.4657$; $w_1 = 2 \text{ mm}$; $z = 45 \text{ cm}$

Thus, position of maximum of transversal intensity distribution is the function of two parameters (parameter of bluntness a and value of the base angle α), which are determined in manufactory specification, as a rule, very roughly. Using the features of diffraction patterns in far field behind the axicon one can estimate these characteristics. In detail this problem will be consider in next section.

Experiment

In experiment the incident Gaussian beam was transformed by a commercially available axicon Thorlabs with the front face radius $R = 1.27$ cm and the base angle $\alpha = 2^\circ$ (according to manufactory specification). Transversal intensity distribution was registered by CCD camera. Measured beam waist is equal $246 \mu\text{m}$. Axicon was placed on the distance of 2.94 m from the laser. The diffraction patterns from two axicons with the same «nominal» parameters, experimentally observed in the plane $z = 45$ cm, is represented in fig. 5. As seen from fig. 5, despite equal «nominal» base angles of axicons diffraction patterns, formed by them, strongly differ as in position of the maximum of transversal intensity distribution (for the first axicon – $5265 \mu\text{m}$, and for the second axicon – $5648 \mu\text{m}$) as the amplitude of oscillation. It indicates the bluntness of their tips and difference in characteristics of axicons.

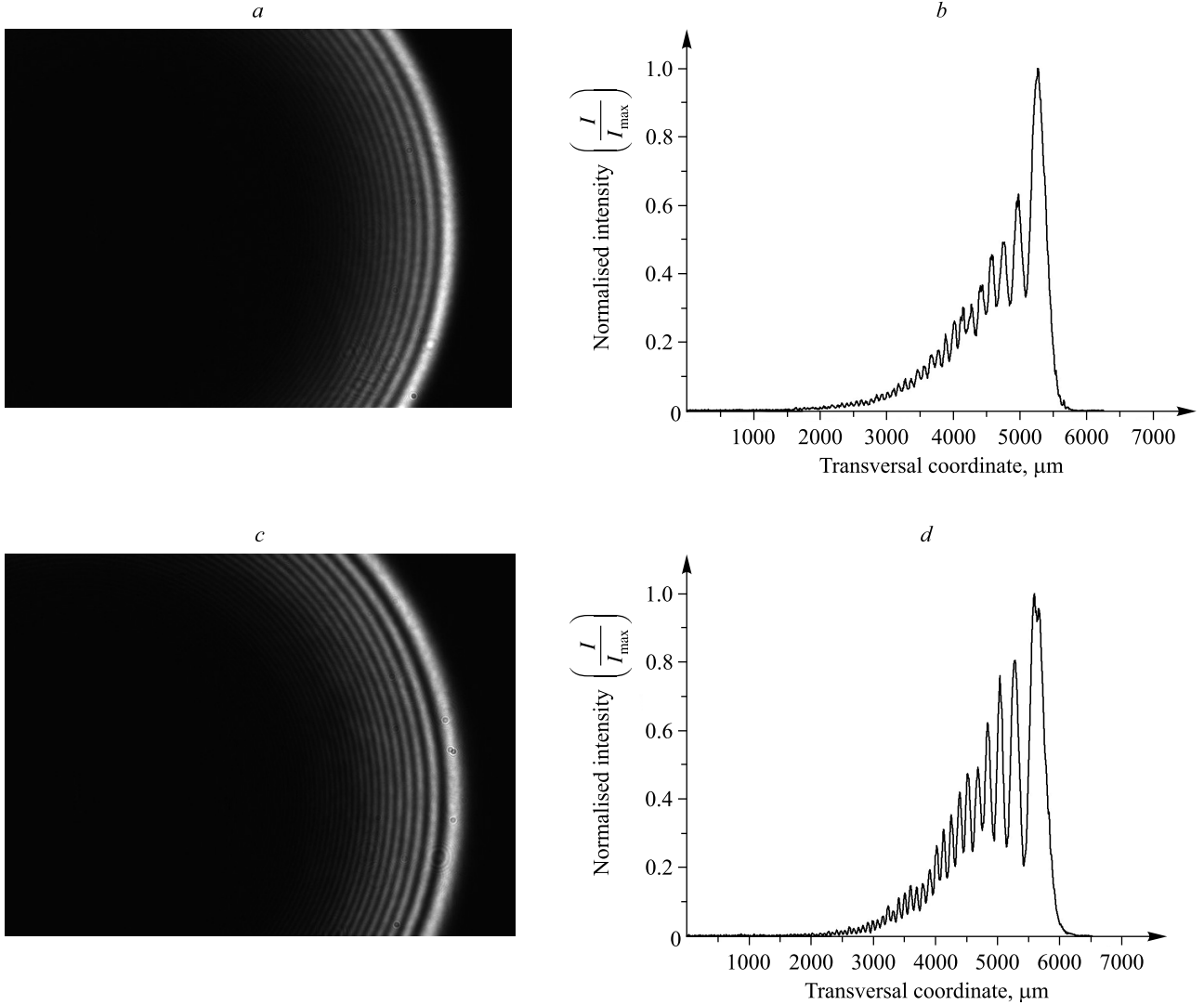


Fig. 5. Diffraction pattern and transversal intensity distribution for the case of the first (a, b) and the second (c, d) axicons

For determination of these characteristics we measure diffraction patterns from the axicon in some planes ($z = 45$ cm, $z = 60$ cm and $z = 80$ cm). Results of measurements for definite z (for example, $z = 45$ cm) are compared with the transversal intensity distributions calculated according to equation (4) for every possible pair of values α and a (value α changes from 1.8 up to 2.2° with the step 0.1° , and value a changes from 3 up to $30 \mu\text{m}$ with the step $1 \mu\text{m}$). Thus, the pair of values α_c and a_c is determined, for which coincidence of measured and calculated transversal intensity distributions takes place. Further, we verify the correlation between measured and calculated (for α_c and a_c) transversal intensity distributions for another z (for example, $z = 60$ cm and $z = 80$ cm). Satisfactory fit between curves for fixed z testifies that the pair of parameters α_c and a_c is chosen rightly.

In fig. 6, there are presented measured and calculated transversal intensity distributions for the first and the second axicons. As a result of technique described above we obtained that for the first axicon $\alpha = 1.85^\circ$ and $a = 10 \mu\text{m}$, for the second axicon $\alpha = 1.9^\circ$ and $a = 8 \mu\text{m}$.

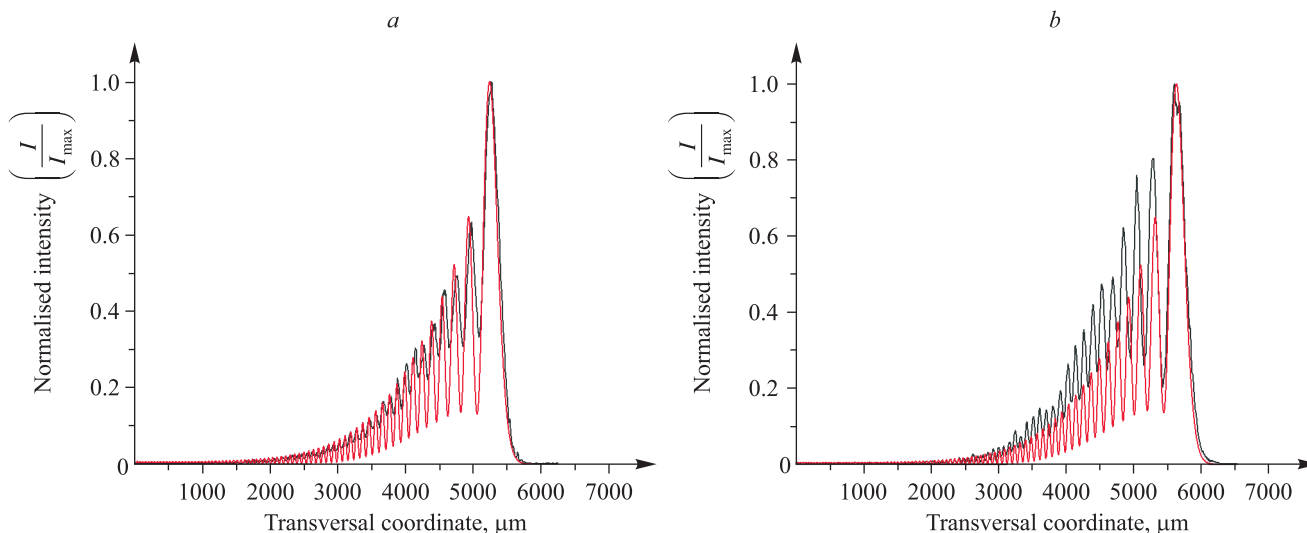


Fig. 6. Experimentally (black curve) and theoretically (red curve) obtained transversal intensity distribution of the field behind the first (a) and the second (b) axicons. Parameters: $\lambda = 532 \text{ nm}$; $R = 1.77 \text{ cm}$; $n = 1.4657$; $\alpha = 2^\circ$; $w_1 = 2 \text{ mm}$; $z = 45 \text{ cm}$

Conclusions

We considered an imperfect axicon with rounded tip approximated by a hyperboloid, and we analysed theoretically and experimentally the properties of the beam generated in far field behind such an axicon. In the course of the study we have established that if the axicon tip deviates in its apex from the ideal sharp tip in the range of tens of micrometers, the transversal intensity distribution is strongly oscillatory. Meanwhile, the ring width d (area, within which normalised intensity does not exceed 0.5) is smaller than in the case of ideal axicon. These oscillations are caused by the interference of parts of the incoming beam propagating through the round, lens-like axicon tip and the conical surface surrounding the tip. It is shown that the periodicity of oscillation depends on the parameter of bluntness a : when it increases the periodicity of oscillations (as well as the radius of the center of the light ring) decreases, and their amplitude increases. We have proposed and tested the method for determining the bluntness of real axicon.

Obtained results can be used to correcting the characteristics of conventional axicons.

References

1. McLeod JH. The axicon: a new type of optical element. *Journal of the Optical Society of America*. 1954;44(8):592–597. DOI: 10.1364/JOSA.44.000592.
2. Khonina SN, Kazanskiy NL, Khorin PA, Butt MA. Modern types of axicons: new functions and applications. *Sensors*. 2021; 21(19):6690. DOI: 10.3390/s21196690.
3. Fan Y, Cluzel B, Petit M, Le Roux X, Lupu A, de Lustrac A. 2D waveguided Bessel beam generated using integrated metasurface-based plasmonic axicon. *ACS Applied Materials & Interfaces*. 2020;12(18):21114–21119. DOI: 10.1021/acsami.0c03420.
4. Ding Z, Ren H, Zhao Y, Nelson JS, Chen Z. High-resolution optical coherence tomography over a large depth range with an axicon lens. *Optics Letters*. 2002;27(4):243–245. DOI: 10.1364/OL.27.000243.
5. Tsampoula X, Garcés-Chávez V, Comrie M, Stevenson DJ, Agate B, Brown CTA, et al. Femtosecond cellular transfection using a nondiffracting light beam. *Applied Physics Letters*. 2007;91(5):053902. DOI: 10.1063/1.2766835.
6. Dufour P, Piché M, Koninck YD, McCarthy N. Two-photon excitation fluorescence microscopy with a high depth of field using an axicon. *Applied Optics*. 2006;45(36):9246–9252. DOI: 10.1364/AO.45.009246.
7. Čižmár T, Garcés-Chávez V, Dholakia K, Zemánek P. Optical conveyor belt for delivery of submicron objects. *Applied Physics Letters*. 2005;86(17):174101. DOI: 10.1063/1.1915543.
8. Shao B, Esener SC, Nascimento JM, Botvinick EL, Berns MW. Dynamically adjustable annular laser trapping based on axicons. *Applied Optics*. 2006;45(25):6421–6428. DOI: 10.1364/AO.45.006421.
9. Garcés-Chávez V, McGloin D, Melville H, Sibbett W, Dholakia K. Simultaneous micromanipulation in multiple planes using a self-reconstructing light beam. *Nature*. 2002;419(6903):145–147. DOI: 10.1038/nature01007.
10. Polesana P, Dubietis A, Porras MA, Kučinskis E, Faccio D, Couairon A, et al. Near-field dynamics of ultrashort pulsed Bessel beams in media with Kerr nonlinearity. *Physical Review E*. 2006;73(5):056612. DOI: 10.1103/PhysRevE.73.056612.

11. Pyragaitė V, Regelskis K, Smilgevičius V, Stabinis A. Self-action of Bessel light beams in medium with large nonlinearity. *Optics Communications*. 2006;257(1):139–145. DOI: 10.1016/j.optcom.2005.07.012.
12. Arlt J, Dholakia K, Allen L, Padgett MJ. Efficiency of second-harmonic generation with Bessel beams. *Physical Review A*. 1999;60(3):2438. DOI: 10.1103/PhysRevA.60.2438.
13. Polesana P, Franco M, Couairon A, Faccio D, Di Trapani P. Filamentation in Kerr media from pulsed Bessel beams. *Physical Review A*. 2008;77(4):043814. DOI: 10.1103/PhysRevA.77.043814.
14. Dubietis A, Polesana P, Valiulis G, Stabinis A, Di Trapani P, Piskarskas A. Axial emission and spectral broadening in self-focusing of femtosecond Bessel beams. *Optics Express*. 2007;15(7):4168–4175. DOI: 10.1364/OE.15.004168.
15. Polesana P, Couairon A, Faccio D, Parola A, Porras MA, Dubietis A, et al. Observation of conical waves in focusing, dispersive, and dissipative Kerr media. *Physical Review Letters*. 2007;99(22):223902. DOI: 10.1103/PhysRevLett.99.223902.
16. Durfee CG, Milchberg HM. Light pipe for high intensity laser pulses. *Physical Review Letters*. 1993;71(15):2409. DOI: 10.1103/PhysRevLett.71.2409.
17. Akturk S, Zhou B, Franco M, Couairon A, Mysyrowicz A. Generation of long plasma channels in air by focusing ultrashort laser pulses with an axicon. *Optics Communications*. 2009;282(1):129–134. DOI: 10.1016/j.optcom.2008.09.048.
18. Polynkin P, Kolesik M, Roberts A, Faccio D, Di Trapani P, Moloney J. Generation of extended plasma channels in air using femtosecond Bessel beams. *Optics Express*. 2008;16(20):15733–15740. DOI: 10.1364/OE.16.015733.
19. Roy G, Blanchard M, Tremblay R. High-pressure amplified stimulated emission effect in a N₂ laser produced plasma with axicon lenses. *Optics Communications*. 1980;33(1):65–68. DOI: 10.1016/0030-4018(80)90094-2.
20. Sochacki J, Kołodziejczyk A, Jaroszewicz Z, Bará S. Nonparaxial design of generalized axicons. *Applied Optics*. 1992;31(25):5326–5330. DOI: 10.1364/AO.31.005326.
21. Soroko LM. Axicons and meso-optical imaging devices. In: Patorski K, Soroko LM, Bassett IM, Welford WT, Winston R, Mihalache D, et al. *Progress in optics. Volume 27*. Wolf E, editor. Amsterdam: North-Holland; 1989. p. 109–160. DOI: 10.1016/S0079-6638(08)70085-4.
22. Dutta R, Saastamoinen K, Turunen J, Friberg AT. Broadband spatiotemporal axicon fields. *Optics Express*. 2014;22(21):25015–25026. DOI: 10.1364/OE.22.025015.
23. Jaroszewicz Z, Burvall A, Friberg AT. Axicon – the most important optical element. *Optics & Photonics News*. 2005;16(4):34–39.
24. Wang Y, Yan S, Friberg AT, Kuebel D, Visser TD. Electromagnetic diffraction theory of refractive axicon lenses. *Journal of the Optical Society of America A*. 2017;34(7):1201–1211. DOI: 10.1364/JOSAA.34.001201.
25. Ren O, Birngruber R. Axicon: a new laser beam delivery system for corneal surgery. *IEEE Journal of Quantum Electronics*. 1990;26(12):2305–2308. DOI: 10.1109/3.64369.
26. Liu Xiaoqing, Xue Changxi. Intensity distribution of diffractive axicon with the optical angular spectrum theory. *Optik*. 2018; 163:91–98. DOI: 10.1016/j.ijleo.2018.02.089.
27. Durnin J, Miceli JJ Jr, Eberly JH. Diffraction-free beams. *Physical Review Letters*. 1987;58(15):1499. DOI: 10.1103/PhysRevLett.58.1499.
28. Schwarz S, Rung S, Esen C, Hellmann R. Fabrication of a high-quality axicon by femtosecond laser ablation and CO₂ laser polishing for quasi-Bessel beam generation. *Optics Express*. 2018;26(18):23287–23294. DOI: 10.1364/OE.26.023287.
29. Horváth ZL, Bor Z. Diffraction of short pulses with boundary diffraction wave theory. *Physical Review E*. 2001;63(2):026601. DOI: 10.1103/PhysRevE.63.026601.
30. Akturk S, Zhou B, Pasquiou B, Franco M, Mysyrowicz A. Intensity distribution around the focal regions of real axicons. *Optics Communications*. 2008;281(17):4240–4244. DOI: 10.1016/j.optcom.2008.05.027.
31. Dépret B, Verkerk P, Hennequin D. Characterization and modelling of the hollow beam produced by a real conical lens. *Optics Communications*. 2002;211(1–6):31–38. DOI: 10.1016/S0030-4018(02)01900-4.
32. Brzobohatý O, Čížmár T, Zemánek P. High quality quasi-Bessel beam generated by round-tip axicon. *Optics Express*. 2008; 16(17):12688–12700. DOI: 10.1364/OE.16.012688.

Received 09.03.2023 / revised 12.04.2023 / accepted 13.04.2023.