ВОПРОСЫ ПРЕПОДАВАНИЯ И ОБУЧЕНИЯ ФИЗИКЕ ПОЛУПРОВОДНИКОВ И НАНОЭЛЕКТРОНИКЕ. СОЦИАЛЬНО-ЭКОЛОГИЧЕСКИЕ ВОПРОСЫ СОВРЕМЕННОЙ ЭЛЕКТРОНИКИ

REFRACTION OF ELECTROMAGNETIC WAVES AT THE INTERFACE BETWEEN ISOTROPIC AND ANISOTROPIC MEDIA

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One particular case of transverse electromagnetic wave refraction at an isotropic/anisotropic medium interface is considered in the dual-type anisotropy approximation, when both electrical and magnetic ordering affect the nature of refraction. When a transverse electromagnetic wave is refracted at the boundary with an anisotropic medium, in the general case, the birefringence takes place, which means that there are two directions along which energy can propagate, depending on the state of polarization. Nevertheless, the wave normal vector remains general and satisfies the traditional Snell's law of refraction, when the product of the refractive indices of the first/second medium and the sine of the angle of incidence/refraction remains constant. It is customary to describe media that are anisotropic in terms of optical properties by tensor material constants, which, if they remain symmetrical, can be reduced to a diagonal form. The report presents a procedure for describing the process of refraction of a transverse electromagnetic wave in the wave vector formalism when the mentioned wave enters from an isotropic medium into an anisotropic optical medium, which is ordered both by electric and magnetic degrees of freedom.

Key words: electromagnetic wave; refraction; isotropic/anisotropic medium interface; birefringence.

ASSUMPTIONS FOR RESEARCH AND ANALYSIS. PROCEDURE FOR SOLVING. DISCUSSION

Here two factors of anisotropy are activated, as equal in importance to the impact on the formation of the response of the anisotropic environment [1–5]. This approach makes sense in view of the development of physical materials science and the emergence of complex in composition and structure not only electronically and magnetically ordered media on the example of transparent ferromagnets in thin layers, magnetic semiconductors-ferrites, material of the so-called ferroelectric segment. Ferroelectrics are known to combine two interrelated order parameters, including electric and magnetic moments, each of which is capable of affecting the properties of the electromagnetic field in a medium. The aforementioned substantiates the fundamental possibility of introducing two tensor coefficients in the description of similar media.

In this case, a medium with double anisotropy is considered by the example of a model optical ferroelectric, in which the dielectric and magnetic properties are characterized by the general permittivity and permeability tensors:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \quad \hat{\mu} = \begin{pmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{pmatrix} \tag{1}$$

To reveal the regularities of refraction at the interface, it is necessary, as in the isotropic case, to formulate the dispersion law for two characteristic types of polarization of the refracted wave.

To obtain the dispersion ratios, the well-known material equations of the connection between the components of the vectors of electric D and magnetic B induction and a strength E and H of the wave field were used, which in the SI unit system look like

$$\overline{H} = \mu_0^{-1}\hat{\mu}^{-1}\overline{B} \tag{2}$$

$$\overline{E} = \varepsilon_0^{-1}\hat{\varepsilon}^{-1}\overline{D} \tag{3}$$

The geometry of incidence and refraction of the wave being plane polarized with respect to the vector of the electric induction (strength) is written in the tensor coordinate system and corresponds to Figure 1. Here the connection between the angles of incidence φ and refraction ψ is found on the basis of the continuity relations and using the form of the wave vector of the wave in the medium, depending on its polarization. Here the *oy* axis is the outward normal to the interface coinciding with the *xz* plane. In accordance with the Fresnel scheme (Fig. 1), the wave vectors of the incident, reflected and refracted waves (k) lie in the *xy* plane.

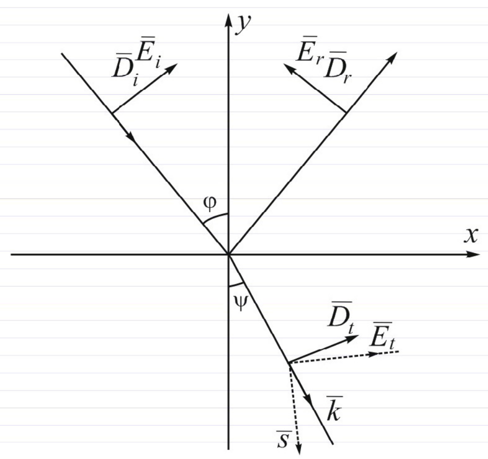

Figure 1. Scheme of refraction of a wave being linearly polarized for *D* in the plane of incidence (angle of incidence - φ, angle of refraction - ψ) on the surface (plane *xz*) of an optical anisotropic, *k* is the wave vector in an anisotropic medium, *s* - ray vector for one of the special case

To formulate the dispersion law for the refracted wave the base Maxwell's equations have been applied:

$$-\nabla \times \hat{\mu}^{-1}\nabla \times \hat{\varepsilon}^{-1}\overline{D} = \varepsilon_0\mu_0\ddot{\overline{D}} \tag{4}$$

Using the unit vectors of the orthogonal Cartesian coordinate system i, j, k the Eq. (4) is applied to the components of the vector D in the most general form of the orientation D and k.

Following well-known expressions like

$$\overline{k}\overline{r} = k_x x + k_y y + k_z z \tag{5}$$

$$\frac{\partial D_x}{\partial x} = -\mathrm{i}k_x D_x e^{i\left(\omega t - \bar{k}\bar{r}\right)} \tag{6}$$

$$\frac{\partial D_x}{\partial y} = -ik_y D_x e^{i\left(\omega t - \omega t - \bar{k}\bar{r}\right)} \tag{7}$$

and so on (here i is the imaginary unit) the relation (4) can be reduced to the form corresponding to the type of the chosen anisotropy with respect to $\hat{\varepsilon}$ and $\hat{\mu}$ (8)

$$-\nabla \times (\hat{\mu}^{-1}\nabla \times \left[i\left(\varepsilon_{xl}{}^{-1}D_l\right) + j\left(\varepsilon_{yl}{}^{-1}D_l\right) + k\left(\varepsilon_{zl}{}^{-1}D_l\right)\right] = i^{-1}\varepsilon_0\mu_0\ddot{D} \tag{8}$$

Here, over the coinciding indices, it is assumed, as usual, the summation. In the coordinate representation, equation (8) after expanding the inner vector product is transformed to the (9)

$$\nabla\times\{i[\mu_{xx}{}^{-1}(\varepsilon_{zl}{}^{-1}D_l k_y - \varepsilon_{yl}{}^{-1}D_l k_z) + \mu_{xy}{}^{-1}(\varepsilon_{xl}{}^{-1}D_l k_z - \varepsilon_{zl}{}^{-1}D_l k_x) + \mu_{xz}{}^{-1}(\varepsilon_{yl}{}^{-1}D_l k_x - \varepsilon_{xl}{}^{-1}D_l k_y)] + j[\mu_{yx}{}^{-1}(\varepsilon_{zl}{}^{-1}D_l k_y - \varepsilon_{yl}{}^{-1}D_l k_z) + \mu_{yy}{}^{-1}(\varepsilon_{xl}{}^{-1}D_l k_z - \varepsilon_{zl}{}^{-1}D_l k_x) + \mu_{yz}{}^{-1}(\varepsilon_{yl}{}^{-1}D_l k_x - \varepsilon_{xl}{}^{-1}D_l k_y)] + k[\mu_{zx}{}^{-1}(\varepsilon_{zl}{}^{-1}D_l k_y - \varepsilon_{yl}{}^{-1}D_l k_z) + \mu_{zy}{}^{-1}(\varepsilon_{xl}{}^{-1}D_l k_z - \varepsilon_{zl}{}^{-1}D_l k_x) + \mu_{zz}{}^{-1}(\varepsilon_{yl}{}^{-1}D_l k_x - \varepsilon_{xl}{}^{-1}D_l k_y)]\} = \mathrm{i}^{-1}\varepsilon_0\mu_0\ddot{D} \tag{9}$$

Here it is necessary to distinguish k as the unit vector of a rectangular coordinate system and the components of the wave vector k_l. Also, as previously mentioned, i is the imaginary unit.

Vector equation (9) allows to write a system of equations for, as mentioned earlier, an arbitrary orientation of the vectors D and k in the coordinate system of the presented tensors. In fact, the component system of equations (9) contains three lines. In general, this system of equations (9) is somewhat redundant in terms of the number of elements, but it still attractive in the sense that it allows one to obtain the dispersion law and the angle of refraction for any particular case in terms of the state of polarization, the type of anisotropy, and the direction of the wave vector.

For example, in the simplest approximation - the incidence and refraction (in the xy plane) of a polarized wave on the surface of a medium being isotropic in optical properties, system (9) is

$$\begin{gathered}\left(\mu_{zz}{}^{-1}\varepsilon_{xx}{}^{-1}k_y{}^2 - \varepsilon_0\mu_0\omega^2\right)D_x - \mu_{zz}{}^{-1}\varepsilon_{yy}{}^{-1}k_x k_y D_y = 0\\ -\mu_{zz}{}^{-1}\varepsilon_{xx}{}^{-1}k_x k_y D_x + \left(\mu_{zz}{}^{-1}\varepsilon_{yy}{}^{-1}k_x{}^2 - \varepsilon_0\mu_0\omega^2\right)D_y = 0\\ \left(\mu_{yy}{}^{-1}\varepsilon_{zz}{}^{-1}k_x{}^2 + \mu_{xx}{}^{-1}\varepsilon_{zz}{}^{-1}k_y{}^2 - \varepsilon_0\mu_0\omega^2\right)D_z = 0 = 0\end{gathered} \tag{10}$$

In (9) there are only diagonal equal components of tensors.

As is known, for given components D determinant of (10) allows us to find the dispersion law, i.e., the wave vector, refractive index as a function of frequency, angle of refraction, and parameters of the medium. Note that in this particular case of wave propagation in the accepted geometry (plane of incidence is xy) determinant of (10) leads to the dispersion equation for an isotropic medium for both types of polarization (11)

$$k_x{}^2 + k_y{}^2 = \frac{\varepsilon_0\mu_0\omega^2}{\varepsilon_{zz}{}^{-1}\mu_{zz}{}^{-1}}. \tag{11}$$

The performed procedure of limiting verification of system (9) gives grounds to consider the anisotropic interaction. In accordance with fig. 1 for a wave being polarized with respect to the vector D in the direction of the z axis, the dispersion law corresponds to the expression (12)

$$k^{-2}\varepsilon_0\mu_0\omega^2 = \varepsilon_{zz}^{-1}(\mu_{yy}^{-1}\sin^2\psi + \mu_{xx}^{-1}\cos^2\psi) + \varepsilon_{zz}^{-1}(\mu_{yx}^{-1} + \mu_{xy}^{-1})\sin\psi\cos\psi - \\ - (\mu_{xz}^{-1}\varepsilon_{xz}^{-1}\cos^2\psi + \mu_{yz}^{-1}\varepsilon_{yz}^{-1}\sin^2\psi) - (\mu_{yz}^{-1}\varepsilon_{xz}^{-1} + \mu_{xz}^{-1}\varepsilon_{yz}^{-1})\sin\psi\cos\psi. \quad (12)$$

For the polarization of the vector D in the xy plane the dispersion law is (13)

$$k^{-2}\varepsilon_0\mu_0\omega^2 = \mu_{zz}^{-1}(\varepsilon_{xx}^{-1}\cos^2\psi + \varepsilon_{yy}^{-1}\sin^2\psi) + \mu_{zz}^{-1}(\varepsilon_{yx}^{-1} + \varepsilon_{xy}^{-1})\cos\psi\sin\psi - \\ - (\mu_{zx}^{-1}\varepsilon_{zx}^{-1}\cos^2\psi + \mu_{zy}^{-1}\varepsilon_{zy}^{-1}\sin^2\psi) - (\mu_{zy}^{-1}\varepsilon_{zx}^{-1} + \mu_{zx}^{-1}\varepsilon_{zy}^{-1})\cos\psi\sin\psi. \quad (13)$$

In both cases, the refractive index depends on the angle ψ, which corresponds to the condition of an extraordinary wave propagating in the xy plane with the dependence of the wave vector on the angle of refraction.

Respectively, for a example, for a wave being polarized with the electric induction vector in the plane of incidence (xy), the ratio between the angles of incidence and refraction has the form (14)

$$\varphi(\psi) = \arcsin\{[\mu_{zz}^{-1}(\varepsilon_{xx}^{-1}\cos^2\psi + \varepsilon_{yy}^{-1}\sin^2\psi) - \mu_{zx}^{-1}\varepsilon_{zx}^{-1}\cos^2\psi - \mu_{zy}^{-1}\varepsilon_{zy}^{-1}\sin^2\psi - \\ - ((\mu_{zy}^{-1}\varepsilon_{zx}^{-1} + \mu_{zx}^{-1}\varepsilon_{zy}^{-1} - \mu_{zz}^{-1}(\varepsilon_{xy}^{-1} + \varepsilon_{yx}^{-1}))0.5\sin(2\psi)]^{-0.5}\varepsilon_1^{-0.5}\mu_1^{-0.5}\sin\psi\}. \quad (14)$$

As mentioned, for double anisotropy, both waves are extraordinary and the energy flux vector will be out of the plane of incidence (xy).

It is rather difficult to analyze the situation in the approximation of the general form of permeability tensors and arbitrary geometry of wave incidence and refraction. We choose the geometry of the external problem, that is, the plane of incidence so that, without losing the generality of properties regarding anisotropy and the presence of optical axes, the tensors can be conveniently reduced to the main axes.

In this special case, when only the Hall components of the dielectric permittivity tensor are nonzero, and for the magnetic permeability tensor they are equal to zero, the normal polarization wave for the electric induction vector, that is, along the z axis becomes ordinary. The wave polarized in the plane of incidence will correspond to the extraordinary dispersion law waves

Indeed, for a medium with mentioned type of anisotropy of the form (15)

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \quad \hat{\mu} = \begin{pmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{pmatrix} \quad (15)$$

for a wave polarized with respect to the vector D in the direction of the z axis, the dispersion law corresponds to the expression (16)

$$k^{-2}\varepsilon_0\mu_0\omega^2 = \varepsilon_{zz}^{-1}(\mu_{yy}^{-1}\sin^2\psi + \mu_{xx}^{-1}\cos^2\psi). \quad (16)$$

For the polarization of the vector D in the xy plane the dispersion law corresponds to the expression (17)

$$k^{-2}\varepsilon_0\mu_0\omega^2 = \mu_{zz}^{-1}(\varepsilon_{xx}^{-1}\cos^2\psi + \varepsilon_{yy}^{-1}\sin^2\psi) + \mu_{zz}^{-1}(\varepsilon_{yx}^{-1} + \varepsilon_{xy}^{-1})\cos\psi\sin\psi. \quad (17)$$

For the case when the components ε_{xx} and ε_{yy} are equal, and this tensor is symmetric, the dispersion law for the waves under consideration with polarization of the vector D along z axis and in the xy plane has respectively more simple forms (18) and (19)

$$k^{-2}\varepsilon_0\mu_0\omega^2 = \varepsilon_{zz}^{-1}\mu_{xx}^{-1}, \quad (18)$$

$$k^{-2}\varepsilon_0\mu_0\omega^2 = \mu_{zz}^{-1}(\varepsilon_{xx}^{-1} + \varepsilon_{xy}^{-1}\sin 2\psi). \quad (19)$$

The chosen form of the tensor $\hat{\varepsilon}$ nevertheless represents a wide range of properties of anisotropic media. It responds to both uniaxial and biaxial materials. Indeed, after reducing tensor to the principal axes, it will have the form (20)

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} + \varepsilon_{xy} & 0 & 0 \\ 0 & \varepsilon_{yy} - \varepsilon_{xy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \quad (20)$$

Depending on the ratio between the components $\hat{\varepsilon}$, this tensor in new coordinates (x', y', z') will represent a medium with either one or two optical axes. For uniaxiality it is required that when the components ε_{xx} and ε_{yy} are equal, the component ε_{zz} is initially equal to either $\varepsilon_{yy} - \varepsilon_{xy}$ or $\varepsilon_{yy} + \varepsilon_{xy}$. Accordingly, after reduction to the principal axes, tensor (20) will correspond to a uniaxial medium in which the cross section of the wave surface in the $x'y'$ plane corresponds to an ovaloid of revolution that touches the circular cross section for a wave with orthogonal polarization.

If initially, at the equality of the components ε_{xx} and ε_{yy} , the component ε_{zz} is not equal to $\varepsilon_{yy} - \varepsilon_{xy}$ or $\varepsilon_{yy} + \varepsilon_{xy}$, then after reduction to the principal axes tensor (20) corresponds to a biaxial medium with three unequal diagonal elements, in which the section in the form of an ovoloid of revolution intersects with a circular section (circular section – for the polarization of the vector D normal to the plane xy).

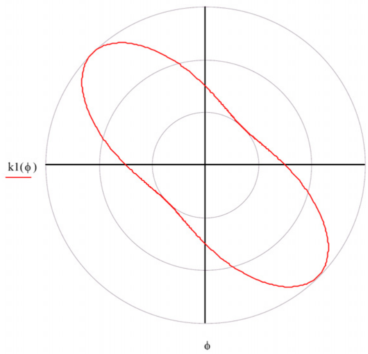

Figure 2. Section of the wave normals surface as an inverse square of the wave phase speed in fractions of square speed of light, $\varepsilon_{xx} = \varepsilon_{yy} = 4$, $\varepsilon_{zz} = 2$, $\varepsilon_{zz} = 4$, $\varepsilon_{zz} = 6\varepsilon_{xy} = \varepsilon_{yx} = 2$, $\mu_{zz} = 1$. For the case of $\varepsilon_{zz} = 2$, the intersection of the ovaloid of revolution and the circular section takes place, determining two special directions along which the phase velocities for the ordinary and extraordinary waves coincide, here the ray vectors are not equal in magnitude and direction

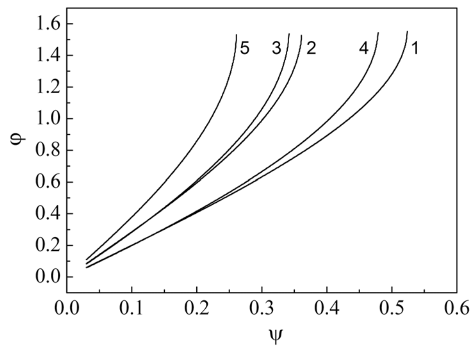

Figure 3. An example of the relationship between the angles of incidence and refraction for a model bianisotropic medium with anisotropy parameters:

***1*) $\varepsilon_{ii} = 4$: $\mu_{ii} = 1$; $\varepsilon_{ik} = 0$: $\mu_{ik} = 0$ – electrically isotropic, non-magnetic,**

***2*) $\varepsilon_{ii} = 4$: $\mu_{ii} = 2$; $\varepsilon_{ik} = 0$: $\mu_{ik} = 0$ – electrically isotropic, magnetically isotropic,**

***3*) $\varepsilon_{ii} = 4$; $\varepsilon_{xy} = \varepsilon_{yx} = 2$, $\varepsilon_{iz} = 0$; $\mu_{ii} = 2$; $\mu_{ik} = 0$ – electrically - anisotropic (biaxial), magnetically – isotropic,**

***4*) $\varepsilon_{ii} = 4$; $\varepsilon_{xy} = \varepsilon_{yx} = 2$, $\varepsilon_{iz} = 0$; $\mu_{ii} = 2$; $\mu_{xy} = \mu_{yx} = 0$. $\mu_{zi} = 1$ – electrically - anisotropic (biaxial), magnetically – anisotropic,**

***5*) $\varepsilon_{ii} = 4$; $\varepsilon_{xy} = \varepsilon_{yx} = 2$, $\varepsilon_{iz} = 0$; $\mu_{ii} = 4$; $\mu_{xy} = \mu_{yx} = 0$. $\mu_{zi} = 1$ – electrically - anisotropic (biaxial), magnetically - anisotropic**

These considerations can be shown in the following diagram. Let the original tensor, after reduction to the main axes, be transformed to the form (21)

$$\hat{\varepsilon} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \quad \text{to} \quad \hat{\varepsilon} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \qquad (21)$$

In this case, the direction cosine between the z and $z^{/}$ axes is equal to one. It is essential that following expression (20) the effective dielectric constant value changes from 2 to 6 as the angle of refraction ψ changes (μ_{zz} is accepted as unit) (Fig. 2).

In conclusion, we display the law of refraction of a wave linearly polarized with electric induction vector in the plane of incidence based on expression (15) for some types of anisotropy (Fig. 3).

The indicated directions of the wave vector of the refracted wave being linearly polarized along the electric induction vector correspond to the direction of phase propagation. Accordingly, the direction of propagation of light energy corresponds to the normal to this section at each point. It can be seen that at normal incidence of light on the surface, the wave with oscillations of the electric induction vector in the xy plane will deflect to the left. In conclusion note that Figure 3 presents the Snell relation for the angles of incidence and refraction for the particular case of the dispersion law based on (13), (14) in accordance with geometry of simplified refraction when the plane of incidence of the wave coincides with the plane of the coordinate system of the dielectric and magnetic permeability tensors.

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ПОДГОТОВКА СПЕЦИАЛИСТОВ НА ФИЗИЧЕСКОМ ФАКУЛЬТЕТЕ БГУ ПО ОБРАЗОВАТЕЛЬНЫМ СТАНДАРТАМ ПОКОЛЕНИЯ 3+

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Изложены основные подходы к разработке образовательных стандартов поколения 3+ и организации образовательного процесса по специальностям ОКРБ 011-2022 с учетом введения в действие Новой редакции Кодекса Республики Беларусь об образовании. Представлены результаты анкетирования абитуриентов 2022 г., подтвер-