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Fracture diagrams of prismatic specimens with an improved contact stress distribution law

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Abstract. The authors have previously improved the principles of distribution of contact normal and tangential stresses at the moment of destruction of the specimen on the basis of L. Prandtl's method as applied to rocks. The article presents further development of the method for constructing out-of-limit curves of "stress - longitudinal deformation" diagrams of rocks. A comparative assessment of the proposed method for calculating diagrams in comparison with diagrams has been constructed by E.P.Unksov method. It has been found that the parameters of the diagrams differ in numerical values in direction of increasing the reliability. Comparison of the calculated diagrams "stress - ordinate of the crack tip" and "stress - deformation" according to the improved method and the method of E.P. Unksov testifies that the level of the current strength values decreases with the development of two cracks at small angles of internal friction. The proposed method allows to determine the ultimate strength and residual strength of rock samples using their shear strength, internal, and contact friction coefficients, elastic modulus, which by simple methods can be established experimentally in laboratories of mining enterprises. The results can be used to control the state of the rock mass and effective destruction during disintegration.

1. Introduction

Important characteristics, which are necessary for the control of stress-strain state of rock mass and their effective destruction during disintegration, are ultimate strength and residual strength of the specimens that are being determined from the "longitudinal stress - longitudinal deformation" diagrams of out-of-limit destruction of rock mass [1-5]. For experimental construction of stress-strain diagrams, prismatic or cylindrical specimens are used.

Since 1960-s, these characteristics have been recorded by special presses that are available at some research institutes. Usually these presses and personnel that can operate it are located far from mining enterprises that need this information on a daily basis. Therefore, there is a clear need for an analytical method for calculating the ultimate and residual strength of rock specimens that could be carried out by using simple methods and at the location of a mining facility [6, 7].

Earlier, attempts were made to mathematical modeling of the processes of destruction of samples [8, 9]. But these models have not been brought to the level of completed analytical methods for calculating the parameters of the diagrams "normal stress - longitudinal deformation" of the out-of-limit destruction of rock specimens. In [10], the author estimates the impact of contact conditions on the values of ultimate parameters, which can be used in the development of new analytical methods for calculating the parameters of diagrams. The article provides a method for calculation of the ultimate state of a specimen under uniaxial compression. According to the design scheme, the authors apply the Coulomb strength criterion in the presence of contact friction. But the lateral sides of the specimen are thought to be rectilinear. In our opinion, in this case, the rule of tangential stresses in the corner regions is inapplicable. The article [11] provides the rationale for the formation of a barrel-shaped form due to the inhibition of transverse deformation from contact friction between the press plate and the specimen. Moreover, the authors come to the conclusion that tensile stresses do not appear at any point in the specimen; the specimen is compressed in all directions. One should agree with this conclusion.

It follows from this concept that in the presence of contact friction, the mathematical model should take into consideration the barrel distortion of the lateral surfaces of the specimen. The barrel distortion would ensure the application of the rule of pairing of tangential stresses in the corner regions during its deformation and destruction.

When solving stress-strain state problems for a specimen with a plane-strain deformation at any point, the Mechanics of Deformable Solids recommends [12] that the conditions of the joint solution of two following partial differential equations must be satisfied

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0\,; \tag{1}$$

$$\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=0 \tag{2}$$

along with the following algebraic equilibrium equation, in this case derived by the authors of [13] and used for rock masses

$$\sigma_x=\frac{2\left(k_n+\mu\sigma_y\right)}{\cos\rho}\left(\sin\rho-\sqrt{1-b^2}\right)+\sigma_y\,, \tag{3}$$

where σ_x and σ_y are horizontal and vertical normal stresses, Pa; τ_{xy} - horizontal tangential stresses inside the material, Pa; x and y - abscissa and ordinate of the examined point, m; k_n is the ultimate shear stress of the material, Pa; ρ is the angle of internal friction (rad); μ = arctan ρ is the coefficient of internal friction; $b=\tau_{xy}/\left(k_n+\mu\sigma_y\right)$ - the ratio of shear stresses from contact friction to shear resistance of the material, taking into account internal friction.

Calculation of ultimate stresses during deformation of bodies is carried out on the basis of the laws of the distribution of contact stresses. To describe patterns of distribution of contact normal and tangential stresses, E.P. Unksov [10] developed a method that based on solving the first ordinary equation instead of two differential equations and on an insufficiently substantiated relation of

$$\frac{\partial\sigma_y}{\partial x}=\frac{\partial\sigma_x}{\partial x} \tag{4}$$

and one algebraic equilibrium equation.

According to E.P. Unksov distribution of normal compressors (σ_{y_i}, Pa) along the unit length of the specimen on the contact plane is described by exponential curves

$$\sigma_{y_i}=\sigma_{y0}\exp\left(\frac{2f_c\cdot x}{h_1}\right) \tag{5}$$

where σ_{y_0} is the normal stress at the corner point, Pa; f_c is the contact friction coefficient; h_1 - sample height, m.

According to L. Prandtl [12], under the bounded condition, $\sigma_y = \sigma_x$ gave a solution to two partial differential equations. But, as a rule, in real circumstances this condition is not met. L. Prandtl's solution with our algebraic equilibrium formula results into the following system of

$$\begin{cases} \sigma_{x_i} = \dfrac{2\tau_c}{h_1}x + \dfrac{2\left(k_n + \mu\sigma_{y_i}\right)}{\cos\rho}\left(\sin\rho - \sqrt{1-b^2}\right) + C_a; \\ \sigma_{y_i} = \dfrac{2\tau_c}{h_1}x + C_a; \\ \tau_{xy} = \tau_c\left(1 - \dfrac{2y}{h_1}\right), \end{cases} \tag{6}$$

where σ_{x_i} is the current value of horizontal stresses, Pa; τ_c - contact shear stress, Pa; C_a - constant of integration, Pa.

L. Prandtl proved that the tangential stresses on the contact plane are constant and do not depend on the abscissa x

$$\tau_c = f_c \cdot \sigma_{y0}, \tag{7}$$

and the vertical contact normal stresses are linearly distributed

$$\sigma_{y_i} = \sigma_{y0}\left(1 + \frac{2f_c \cdot x}{h_1}\right). \tag{8}$$

In the article [14], we have combined the solutions of E.P. Unksov and L. Prandtl to satisfy mentioned requirements by determining the relations between derivatives according differentiating the first equation of the system (6)

$$\frac{\partial\sigma_x}{\partial x} = \left[\frac{2f_c\sigma_y}{h} + 2\mu^2 + \left(\frac{2\left(f_c b - \mu\right)}{\cos\rho\sqrt{1-b^2}}\right)\right]\frac{\partial\sigma_y}{\partial x}.$$

By designating

$$u = \left[\frac{2f_c\sigma_y}{h} + 2\mu^2 + \left(\frac{2\left(f_c b - \mu\right)}{\cos\rho\sqrt{1-b^2}}\right)\right],$$

get

$$\frac{\partial\sigma_x}{\partial x} = u \cdot \frac{\partial\sigma_y}{\partial x}. \tag{9}$$

Using differential equation (1) on the basis of (9) we have

$$\frac{\partial\sigma_x}{\partial x} = u\frac{\partial\sigma_y}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} = 0. \tag{10}$$

The tangential stresses τ_{xy} move away from each of the contact surfaces in absolute value and at $y = 0.5$ become zero, as well as on the axis of symmetry. Therefore, from third equation of system (6) we have

$$\frac{\partial \tau_{yx}}{\partial y} = -\frac{2\tau_c}{h_1} = -\frac{2 f_c \sigma_y}{h_1}. \tag{11}$$

After substituting expression (11) into equation (10), we have

$$u\frac{\partial \sigma_y}{\partial x} - \frac{2 f_c \sigma_y}{h_1} = 0.$$

After integration of the previous expression with regard to the boundary conditions we obtained that the distribution of the current values of normal contact stresses can be described by the following formula

$$\sigma_{y_i} = \sigma_y \cdot e^{\frac{2 f_c \cdot x}{u \cdot h_1}}. \tag{12}$$

In the article [14], a comparison calculation of the ultimate strength of the specimen that has destruction in a shape of a truncated wedge with the use of new, in our opinion, a more accurate distribution of contact normal and tangential stresses as compared with a known exponential distribution E.P Unksov. It was described that the relative error of the estimated ultimate strength with the experimental data is 0.135, which is significantly below the relative error in the calculation of 0.335, obtained by the authors [14] by E.P. Unksov with identical initial data.

The purpose of this article is to develop a method for calculating the diagram parameters of "longitudinal stress - longitudinal deformation" for improved distribution of the contact normal stresses.

2. Methods

Now it is necessary to construct using the new distribution of contact normal and tangential stresses diagrams "stress - deformation" of prismatic rock samples necessary to control the stress-strain state of the rock mass and effective destruction of rocks during disintegration. As an initial object, we take a sample convex due to deformation (figure 1), which satisfies the conditions of pairing of tangential stresses in the corner regions.

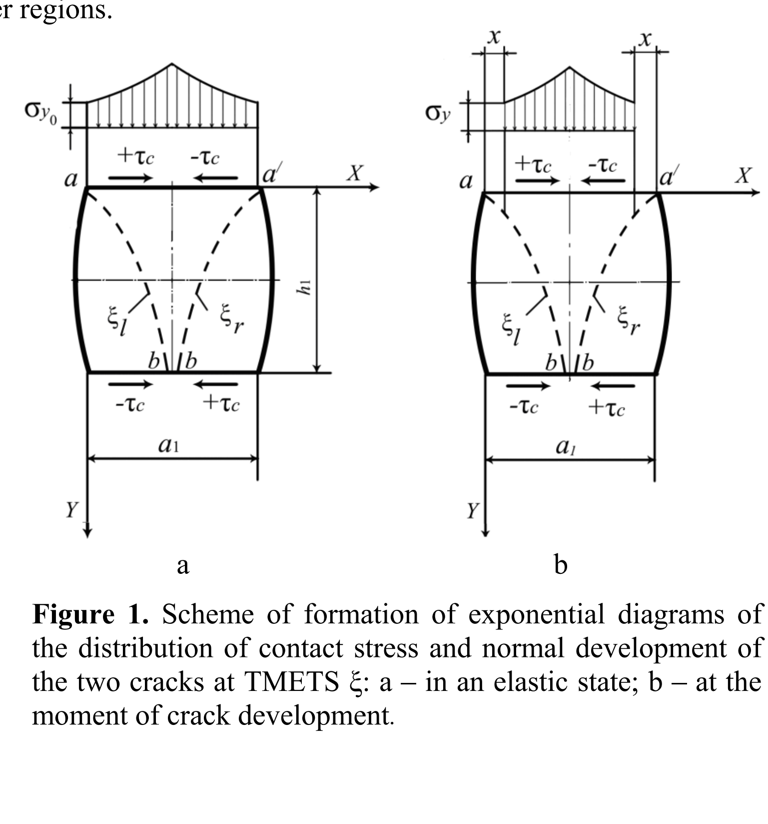

Figure 1. Scheme of formation of exponential diagrams of the distribution of contact stress and normal development of the two cracks at TMETS ξ: a – in an elastic state; b – at the moment of crack development.

The sign of shear stresses according to the well-known rule of signs of shear stresses on the left upper vertical quarter of the specimen to the horizontal axis of symmetry is assumed to be positive, on the right - negative. On the lower half, the signs are symmetrically reversed. Let us present the scheme of crack propagation along the trajectories of maximum effective tangential stresses (TMETS) for the truncated-wedge fracture of prismatic specimens (figure 1). Tangential τ_c and normal angular and current stresses arising from contact friction are shown on the contact surfaces, inside - TMETS (left ξ_l and right ξ_r), formed in the material according to the Coulomb strength criterion. In figure 1, a) shows a sample in an elastic state, figure 1, b) shows a sample at the moment of crack development. On the contact planes there are the normal stress σ_y and the abscissa x of the point at its vertex.

Methods of analytical construction of diagrams for prismatic samples according to E.P Unksov for exponential distribution are detailed in the book [13]. Let's use a ready-made solution. To build the diagrams, first of all, two parameters must be determined: the current value of the strength on the bearing area of the specimen (on the area that did not come out of the load during the development of two cracks from the upper corners) and the deformation of the specimen.

To determine these parameters, the current values of the specific force (p, Pa) are required on the sample area, which does not go beyond the load, which, according to the method of E.P. Unksov are determined by the formula

$$p = \sigma_{y\xi} \frac{h_1}{f_c \cdot (a_1 - 2x_\xi)} \left(\exp\left(\frac{f_c \cdot (a_1 - 2x_\xi\ )}{h_1} \right) - 1 \right), \tag{13}$$

and using expression (12) – according to the formula

$$p = \sigma_{y\xi} \frac{uh_1}{f_c \cdot (a_1 - 2x_\xi)} \left(\exp\left(\frac{f_c \cdot (a_1 - 2x_\xi\ )}{uh_1} \right) - 1 \right), \tag{14}$$

where σ_{y_ξ} – is the normal stress at the crack tip, Pa; a_1 – sample length, m; x_ξ is the abscissa of the crack tip, m.

The current value of the strength σ_c (Pa) at the TMETS ξ is determined by the formula

$$\sigma_c = p \cdot S_\xi, \tag{15}$$

where S_ξ is the ratio of the current bearing area to the initial site.

This ratio is determined by the formula

$$S_\xi = \frac{a_1 - 2x_\xi}{a_1}. \tag{16}$$

The deformation value is determined by Hooke's law

$$\varepsilon = \frac{p}{E}, \tag{17}$$

where E is the modulus of elasticity of the rock, Pa.

The book [13] gives a method for determining the normal stress at the crack tip

$$\sigma_{y_\xi} = \frac{1}{\mu} \left(\frac{k_n \left(1 + \sin\rho\sqrt{1 - b_\xi^2} \right) \cdot \exp\left(2\mu\left(\beta_\xi + \beta_b \right) \right)}{1 - \sin\rho\sqrt{1 - b^2}} - k_b \right), \tag{18}$$

where

$$k_b = \frac{\left(k_n + \mu\sigma_{y\xi}\right)\left(1 - \sin\rho\sqrt{1-b^2}\right)}{\left(1 + \sin\rho\sqrt{1-b^2}\right)\exp\left(-4\mu\beta_b\right)};$$

$$b_\xi = \frac{f_c\left(1 - \frac{2y}{h_1}\right)\cdot\sigma_{y\xi}\cdot\exp\left(\frac{2f_c\cdot x_\xi}{uh_1}\right)}{k_n + \mu\sigma_{y\xi}\cdot\exp\left(\frac{2f_c\cdot x_\xi}{uh_1}\right)}; \qquad b_b = -\frac{f_c\cdot\sigma_{y\xi}\cdot\exp\left(\frac{2f_c\cdot x_b}{uh_1}\right)}{k_b + \mu\sigma_{y\xi}\cdot\exp\left(\frac{2f_c\cdot x_b}{uh_1}\right)};$$

$$\beta_\xi = \frac{1}{2}\mathrm{arctg}\frac{b_\xi\cos\rho}{\sin\rho - \sqrt{1-b^2}}; \qquad \beta_b = \frac{1}{2}\mathrm{arctg}\frac{b_b\cos\rho}{\sin\rho - \sqrt{1-b^2}},$$

k_n – shear strength, Pa; k_b – effective shear stress at point b, Pa; b_ξ – the parameter of contact friction on the TMETS ξ; b_b – the contact friction parameter at point b; β_ξ – angle of rotation of TMETS ξ at the crack tip, rad; β_b – the angle of rotation of TMETS ξ at point b, rad.

3. Results and their discussion

Thus, we have developed a set of systems of equations for constructing "stress - strain" diagrams and an intermediate diagram "stress - ordinate of the crack tip". As you can see, the set of systems of equations in an explicit form is undecidable, since some of the formulas are transcendental. The determination of the parameters of the diagrams of the samples of the truncated-wedge shape of fracture using these systems is carried out on a computer by the iteration method based on the geometric properties of the TMETS, i.e. Instead of solving in an explicit form, we will determine the sought stresses in a finite number of nodal points of the TMETS grid, as is customary in approximate graphical methods for solving problems of deformation of solids. To do this, we divide the sample into several layers n, for example, 18-20.

The angle of inclination of the TMETS layer (α_ξ, rad) relative to the abscissa x is determined by its upper boundary according to the formula derived by the authors

$$\alpha_\xi = \frac{\pi}{4} + \frac{\rho}{2} - \beta_\xi \tag{19}$$

The layer is assumed to be constant in thickness. The values of the coordinates are determined by adding the corresponding elements Δy and Δx, while Δx is defined as $\Delta x = \Delta y \cdot \mathrm{ctg}\alpha$. The set and calculated parameters of all 18-20 layers are displayed on the screen. The next operation displays on the screen the images of the TMETS emerging from the corners of the sample. The solution allows you to determine the limit values of vertical and horizontal normal stresses, deformation and a number of values of intermediate parameters at the tops of cracks as they develop. In figure 2 shows for comparison the calculated diagrams "stress - ordinate of the crack tip", and in figure 3 - design diagrams "stress - strain", the parameters of which are determined by formulas (14) and (18) with exponential laws of distribution of contact stresses by the improved method and by the method of E.P. Unksov.

The figures show that the level of the current strength values decreases with the development of two cracks at small angles of internal friction, while the so-called decay modulus with an improved distribution law is less value than with the law of E.P. Unksov, except for points b and b' of the exit of two TMETS to the lower contact surface (figure 1), in which at ρ = 45° and f_c = 0.25 the residual strength (σ_{res}, MPa) has the same values. At high values of the angles of internal friction, the character of the decay modulus changes. It takes the form of a concave quadratic function, while the equality of the residual strength is not preserved, but the current value of strength with the improved law of stress distribution is always less than that under the law of E.P. Unksov. It follows from this conclusion that the decay modulus depends on the parameters of the physical and mechanical properties of rocks and is not a constant value, although some authors think so. Figure 3 shows line 5 of the true diagram "stress - longitudinal deformation", which is determined by formulas (13) and (15) without taking into account the ratio of the current bearing area to the initial site. This line has the form of a straight line, although the TMETS and the transcendental branches of the conditional diagrams are depicted by curved lines.

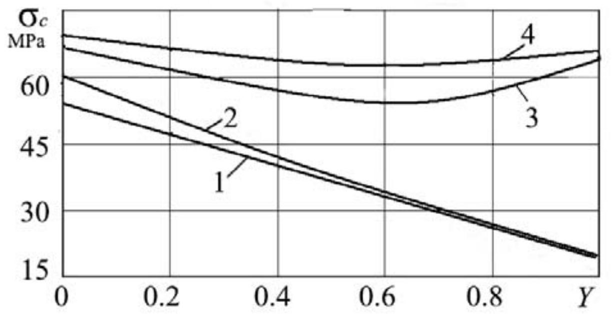

Figure 2. Design diagrams "stress - ordinate of the crack tip" according to the improved (1, 3) and according to E.P. Unksov (2, 4) distributions for two TMETS ξ at k_n = 10.0 MPa: 1, 2 – ρ=45°, f_c=0,25; 3, 4 – ρ=55°, f_c=0,25.

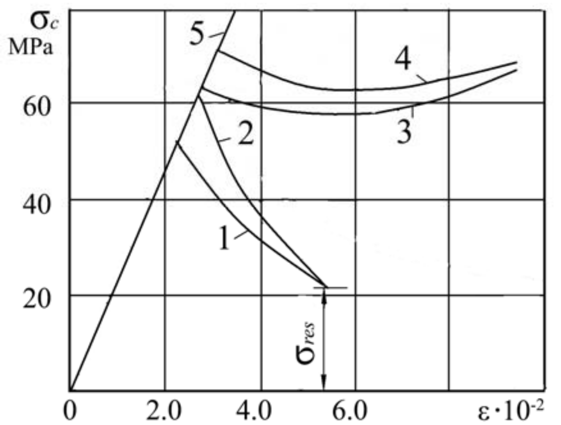

Figure 3. Design conditional diagrams "stress - ordinate of the crack tip" according to the improved (1, 3) and according to E. P. Unksov (2, 4) distributions for two TMETS ξ at k_n = 10.0 MPa: 1, 2 – ρ =45°, f_c =0,25; 3, 4 – ρ =55°, f_c =0,25; 5 – true chart line.

4. Conclusions

1. A method has been developed for calculating the parameters of the "stress – ordinate of the crack tip" and "stress – deformation" diagrams with an improved distribution of normal contact stresses.

2. Comparison of the calculated diagrams "stress – ordinate of the crack tip" and "stress – deformation" with exponential laws of distribution of contact stresses according to the improved method and according to E.P. Unksov shows that the level of the current strength values on the branches of the diagrams decreases with the development of two cracks at small angles of internal friction, while the so-called decay modulus with an improved distribution law is less value than with the law of E.P. Unksov, except for points *b* and *b′* of the exit of two TMETS to the lower contact surface, in which at ρ = 45° and f_c = 0.25 the residual strength has the same values.

3. At high values of the angles of internal friction, the character of the decay modulus changes. It takes the form of a concave quadratic function, while the equality of the residual strength is not preserved, but the current value of strength with the improved law of stress distribution is always less than that under the law of E. P. Unksov.

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