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MINIMAL PROJECTIVE MODULES AND STEINBERG-LIKE CHARACTERS OF FINITE SIMPLE GROUPS OF LIE TYPE

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Let G be a finite group and p a prime dividing the order $|G|$ of G. Let F be an algebraically closed field of characteristic p.

It is well known that the dimension of a projective FG-module Φ is a multiple of $|G|_p$, the p-part of $|G|$. We call Φ minimal if $\dim\Phi = |G|_p$. Surprisingly little is known about these modules, it is not even clear for which groups and primes p minimal projective modules exist. By a result of Fong (1962), they exist for p-solvable groups, and Steinberg (1957) proved that every simple group G of Lie type with defining characteristic p has a unique irreducible projective FG-module St, which is minimal. One can extract from known results a full list of irreducible minimal projective modules for all simple groups and all primes p dividing $|G|$. So the problem is to determine reducible minimal projective modules. Malle and Weigel (2008) determined them under certain rather heavy restrictions.

Theorem 1. *Let G be a simple group of Lie type with defining characteristic p. Then every reducible minimal projective FG-modules belongs to the list of Malle and Weigel.*

To every projective FG-module one corresponds a representation over the complex numbers, which character vanishes at the p-singular elements. We call characters with this property p-vanishing, and those of degree $|G|_p$ are called Steinberg-like. So minimal projective FG-modules yield Steinberg-like characters, the converse is true only for irreducible characters. Jointly with M. Pellegrini (Univ. of Brasilia) we study Steinberg-like characters for a finite simple group of Lie type G in defining characteristic p. These groups have a BN-pair structure, which involves a parameter called the rank of G. For instance, the rank of $PSL(n,q)$ is $n-1$.

Theorem 2. *If the rank of G is greater than 5, then G has no reducible Steinberg-like character.*

Many groups of smaller rank have Steinberg-like characters, in particular, they always exist for the groups of rank 1.

Some results for other simple groups will be reported in the talk.

ABELIAN-BY-CYCLIC MOUFANG LOOPS

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A group G possessing automorphisms ρ and σ that satisfy $\rho^3 = \sigma^2 = (\rho\sigma)^2 = 1$ is called a *group with a triality* $\langle\rho,\sigma\rangle$ if

$$(x^{-1}x^{\sigma})(x^{-1}x^{\sigma})^{\rho}(x^{-1}x^{\sigma})^{\rho^2} = 1$$

for every x in G. A loop M is called a *Moufang loop* if

$$xy \cdot zx = (x \cdot yz)x$$

for all $x, y, z \in M$. When the connections between groups with a triality and Moufang loops were discovered [1], new ways of constructing Moufang loops using groups with a triality appeared.

We construct a new series of groups with a triality and then derive an explicit multiplication formula for the corresponding Moufang loops. In particular, we obtain a series of *abelian-by-cyclic* Moufang loops (i. e. an upward extension of an abelian group by a cyclic group). To be more precise, we prove the following

Theorem. *Let R be an arbitrary associative commutative unital ring, and let R_0 be a cyclic group of invertible elements of R. Then the set of tuples (r, x, y, z), where $r \in R_0$, $x, y, z \in R$, with the multiplication*

$$(r_1, x_1, y_1, z_1)(r_2, x_2, y_2, z_2) = (r_1r_2, x_1 + r_1x_2, y_1 + r_1y_2, r_2z_1 + z_2 + (1 - 2r_1^{-1}r_2)x_1y_2 - x_2y_1) \quad (1)$$

is an abelian-by-cyclic nonassociative Moufang loop of the form $R_0.(R + R + R)$ provided that either the order of R_0 is 3 or the characteristic of R is 2.

The minimal finite loops of this type clearly arise if R_0 has a prime order p and R is a minimal finite field with an element of multiplicative order p. For example, this gives abelian-by-cyclic proper Moufang loops of orders 3.2^6, 7.2^9, 3.5^6, 3.7^3, etc.

The abelian-by-cyclic Moufang loops are of interest in the connection with the following problem proposed by M. Kinyon and based on a paper by O. Chein:

Problem. Let M be a Moufang loop with a normal abelian subgroup (i. e. an associative subloop) N of odd order such that M/N is a cyclic group of order bigger than 3. If the orders of N and M/N are coprime, is M a group?

Although the finite loops of the form (1) are not counterexamples to this problem, there are reasons to believe that they are essentially the only types of abelian-by-cyclic Moufang loops such that the orders of N and M/N are coprime and thus the answer to the question is affirmative.

The loops (1) are constructed as particular cases of a wider class of Moufang loops $M_{a,b}$, $a, b \in R$, not all of which are abelian-by-cyclic, but all have the general structure $R_0.(R + R).R$ for a given subgroup $R_0 \leqslant R^\times$. We show that some members of this series can be embedded into the Cayley algebra $\mathbb{O}(R)$ and prove a result connected with the isomorphism problem for the loops $M_{a,b}$.

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