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MINIMAL PROJECTIVE MODULES AND STEINBERG-LIKE CHARACTERS OF FINITE SIMPLE GROUPS OF LIE TYPE

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Let $G$ be a finite group and $p$ a prime dividing the order $|G|$ of $G$. Let $F$ be an algebraically closed field of characteristic $p$.

It is well known that the dimension of a projective $FG$-module $\Phi$ is a multiple of $|G|_p$, the $p$-part of $|G|$. We call $\Phi$ minimal if $\dim \Phi = |G|_p$. Surprisingly little is known about these modules, it is not even clear for which groups and primes $p$ minimal projective modules exist. By a result of Fong (1962), they exist for $p$-solvable groups, and Steinberg (1957) proved that every simple group $G$ of Lie type with defining characteristic $p$ has a unique irreducible projective $FG$-module $St$, which is minimal. One can extract from known results a full list of irreducible minimal projective modules for all simple groups and all primes $p$ dividing $|G|$. So the problem is to determine reducible minimal projective modules. Malle and Weigel (2008) determined them under certain rather heavy restrictions.

Theorem 1. Let $G$ be a simple group of Lie type with defining characteristic $p$. Then every reducible minimal projective $FG$-modules belongs to the list of Malle and Weigel.

To every projective $FG$-module one corresponds a representation over the complex numbers, which character vanishes at the $p$-singular elements. We call characters with this property $p$-vanishing, and those of degree $|G|_p$ are called Steinberg-like. So minimal projective $FG$-modules yield Steinberg-like characters, the converse is true only for irreducible characters. Jointly with M. Pellegrini (Univ. of Brasilia) we study Steinberg-like characters for a finite simple group of Lie type $G$ in defining characteristic $p$. These groups have a BN-pair structure, which involves a parameter called the rank of $G$. For instance, the rank of $PSL(n, q)$ is $n - 1$.

Theorem 2. If the rank of $G$ is greater than 5, then $G$ has no reducible Steinberg-like character.

Many groups of smaller rank have Steinberg-like characters, in particular, they always exist for the groups of rank 1.

Some results for other simple groups will be reported in the talk.

ABELIAN-BY-CYCLIC MOUFCANG LOOPS

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A group $G$ possessing automorphisms $\rho$ and $\sigma$ that satisfy $\rho^3 = \sigma^2 = (\rho \sigma)^2 = 1$ is called a group with a triality $\langle \rho, \sigma \rangle$ if

$$(x^{-1}x^\sigma)(x^{-1}x^\sigma)^\rho(x^{-1}x^\sigma)^{\rho^2} = 1$$
for every $x$ in $G$. A loop $M$ is called a Moufang loop if

$$xy \cdot zx = (x \cdot yz)x$$

for all $x, y, z \in M$. When the connections between groups with a triality and Moufang loops were discovered [1], new ways of constructing Moufang loops using groups with a triality appeared.

We construct a new series of groups with a triality and then derive an explicit multiplication formula for the corresponding Moufang loops. In particular, we obtain a series of abelian-by-cyclic Moufang loops (i.e. an upward extension of an abelian group by a cyclic group). To be more precise, we prove the following

**Theorem.** Let $R$ be an arbitrary associative commutative unital ring, and let $R_0$ be a cyclic group of invertible elements of $R$. Then the set of tuples $(r, x, y, z)$, where $r \in R_0$, $x, y, z \in R$, with the multiplication

$$(r_1, x_1, y_1, z_1)(r_2, x_2, y_2, z_2) = (r_1r_2, x_1 + r_1x_2, y_1 + r_1y_2, r_2z_1 + z_2 + (1 - 2r_1^{-1}r_2)x_1y_2 - x_2y_1)$$  \(1\)

is an abelian-by-cyclic nonassociative Moufang loop of the form $R_0.(R + R + R)$ provided that either the order of $R_0$ is 3 or the characteristic of $R$ is 2.

The minimal finite loops of this type clearly arise if $R_0$ has a prime order $p$ and $R$ is a minimal finite field with an element of multiplicative order $p$. For example, this gives abelian-by-cyclic proper Moufang loops of orders $3.2^6$, $7.2^9$, $3.5^6$, $3.7^3$, etc.

The abelian-by-cyclic Moufang loops are of interest in the connection with the following problem proposed by M. Kinyon and based on a paper by O. Chein:

**Problem.** Let $M$ be a Moufang loop with a normal abelian subgroup (i.e. an associative subloop) $N$ of odd order such that $M/N$ is a cyclic group of order bigger than 3. If the orders of $N$ and $M/N$ are coprime, is $M$ a group?

Although the finite loops of the form (1) are not counterexamples to this problem, there are reasons to believe that they are essentially the only types of abelian-by-cyclic Moufang loops such that the orders of $N$ and $M/N$ are coprime and thus the answer to the question is affirmative.

The loops (1) are constructed as particular cases of a wider class of Moufang loops $M_{a,b}$, $a, b \in R$, not all of which are abelian-by-cyclic, but all have the general structure $R_0.(R + R).R$ for a given subgroup $R_0 \leq R^\times$. We show that some members of this series can be embedded into the Cayley algebra $O(R)$ and prove a result connected with the isomorphism problem for the loops $M_{a,b}$.

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