ON LOCAL NORMALITY OF MAXIMAL FITTING CLASSES

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All groups considered are finite. We use standard definitions and notation taken from [1].

A normally hereditary class of groups $\mathfrak{F}$ is called a Fitting class if it is closed under the products of normal $\mathfrak{F}$-subgroups. A Fitting class $\mathfrak{F}$ is called a maximal (by inclusion) subclass of a Fitting class $\mathfrak{H}$ (this is denoted by $\mathfrak{F} < \cdot\mathfrak{H}$), if $\mathfrak{F} \subset \mathfrak{H}$ and the condition $\mathfrak{F} \subseteq \mathfrak{M} \subseteq \mathfrak{H}$, where $\mathfrak{M}$ is a Fitting class, always implies $\mathfrak{M} \in \{\mathfrak{F}, \mathfrak{H}\}$. A non-empty Fitting class $\mathfrak{F}$ is called locally normal (or normal in a Fitting class $\mathfrak{H}$, or $\mathfrak{H}$-normal, this is denoted by $\mathfrak{F} \trianglelefteq \mathfrak{H}$) if $\mathfrak{F} \subseteq \mathfrak{H}$ and for every $\mathfrak{H}$-group G its $\mathfrak{F}$-radical $G_\mathfrak{F}$ is an $\mathfrak{F}$-maximal subgroup of G. Recall that a unique maximal normal $\mathfrak{F}$-subgroup of an arbitrary group G is called its $\mathfrak{F}$-radical and denoted by $G_\mathfrak{F}$.

Since 1970s maximal and normal Fitting classes have been deeply investigated in the class $\mathfrak{S}$ of all soluble groups. In particular, it is proved [2] that every maximal Fitting subclass of $\mathfrak{S}$ is $\mathfrak{S}$-normal. This result was extended [3] for the class $\mathfrak{S}_\pi$ of all soluble π-groups (π is a non-empty set of prime numbers). In the class $\mathfrak{E}$ of all groups it is also proved [4] that every maximal Fitting subclass of the class $\mathfrak{E}$ is normal in $\mathfrak{E}$.

In this paper the property of local normality of maximal Fitting classes of π-groups is established.

Let $\mathbb{P}$ be a set of all primes and $\varnothing \neq \pi \subseteq \mathbb{P}$. The symbol $\mathfrak{E}_\pi$ denotes the Fitting class of all π-groups.

Theorem. *Let $\mathfrak{F}$ and $\mathfrak{H}$ be Fitting classes such as $\mathfrak{F} < \cdot\mathfrak{H} \subseteq \mathfrak{E}_\pi$, where π denotes a non-empty set of prime numbers. Then $\mathfrak{F}$ is $\mathfrak{H}$-normal.*

This theorem implies the following four corollaries:

Corollary 1 ($\mathfrak{H} = \mathfrak{E}_\pi$, $\emptyset \neq \pi \subseteq \mathbb{P}$). *If $\mathfrak{F}$ is a Fitting class such as $\mathfrak{F} < \cdot\mathfrak{E}_\pi$ then $\mathfrak{F} \lhd \mathfrak{E}_\pi$.*

Corollary 2 [4] ($\mathfrak{H} = \mathfrak{E}_\pi$, $\pi = \mathbb{P}$). *If $\mathfrak{F}$ is a Fitting class such as $\mathfrak{F} < \cdot\mathfrak{E}$ then $\mathfrak{F} \lhd \mathfrak{E}$.*

Corollary 3 [3] ($\mathfrak{H} = \mathfrak{S}_\pi$, $\emptyset \neq \pi \subseteq \mathbb{P}$). *If $\mathfrak{F}$ is a Fitting class such as $\mathfrak{F} < \cdot\mathfrak{S}_\pi$ then $\mathfrak{F} \lhd \mathfrak{S}_\pi$.*

Corollary 4 [2] ($\mathfrak{H} = \mathfrak{S}_\pi$, $\pi = \mathbb{P}$). *If $\mathfrak{F}$ is a Fitting class such as $\mathfrak{F} < \cdot\mathfrak{S}$ then $\mathfrak{F} \lhd \mathfrak{S}$.*

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A METHOD OF TEACHING DETERMINANTS

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We adopt the notion of diagonal of a matrix from combinatorics: maximal collection of matrix places being in its different rows and in different columns is called a *diagonal of this matrix.* This notion is the base of the notion of determinant.

A collection of cells of an n order square matrix such that both their sets of rows and their sets of columns are partitions of $\{1, \ldots, n\}$ is said to be a *cell diagonal* of the matrix. A *cell* *summand* of a determinant $|A|$ being defined by a cell diagonal $\widehat{\beta} := \{B_1, B_2, \ldots, B_s\}$ is said to be a number $d_{\widehat{\beta}}$, which is defined by

$$d_{\widehat{\beta}} := (-1)^{\mathrm{inv}\widehat{\beta}} \cdot |B_1| \cdot |B_2| \cdot \ldots \cdot |B_s|,$$

where $\mathrm{inv}\widehat{\beta}$ denotes a number of inversions in $\widehat{\beta}$. Using the determinant definition we find an elementary proof of the following theorem.

Theorem. *Sum of all cell summands of a matrix having the same row partition is equal to the determinant of the matrix.*

The other properties of determinants, including Laplace theorem and Binet — Cauchy formula, follow immediately from this theorem.

ON FINITE GROUPS ISOSPECTRAL TO FINITE SIMPLE EXCEPTIONAL GROUPS OF TYPE E_7

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Denote $\omega(G)$ the *spectrum* of a fixed finite group G, i. e., the set of its element orders. Groups with the same spectra are called *isospectral*. A finite group G is said to be *recognizable by spectrum* (briefly, *recognizable*) if every finite group H isospectral to G is isomorphic to G. Since a finite group with a nontrivial normal soluble subgroup is not recognizable [1], of prime interest is the recognition problem for nonabelian simple groups. In this abstract we consider finite simple exceptional groups of Lie type. At present it is known that the groups ${}^2B_2(q)$ [2], ${}^2G_2(q)$ [3], ${}^2F_4(q)$ [4], $F_4(2^m)$ [5], $E_8(q)$ [6], and $E_7(q)$ with $q = 2, 3$ [7] are recognizable. Moreover, it is still unknown whether there exists a finite simple exceptional group of Lie type which is not recognizable by its spectrum (see question 16.24 in [8]). Here we investigate the composition structure of finite groups isospectral to finite simple exceptional groups of type E_7.

Theorem. *Let L be a finite simple exceptional group $E_7(q)$ with $q > 3$. If G is a finite group with $\omega(G) = \omega(L)$ and K is the maximal normal soluble subgroup of G, then $L \leqslant G/K \leqslant$ $\leqslant \mathrm{Aut}\, L$.*

Together with previous results the theorem yields the following assertion.

Corollary. *Let L be a finite simple exceptional group of Lie type. If G is a finite group with $\omega(G) = \omega(L)$ and K is the maximal normal soluble subgroup of G, then $L \leqslant G/K \leqslant \mathrm{Aut}\, L$.*

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