# Wave optics of quantum gravity for massive particles

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#### S.L. Cherkas<sup>†</sup> and V.L. Kalashnikov<sup>‡</sup>

† Institute for Nuclear Problems, Bobruiskaya 11, Minsk 220006, Belarus
‡ Facoltá di Ingegneria dell'Informazione, Informatica e Statistica, Sapienza Universitá di Roma, Via Eudossiana 18 00189 - Roma, RM, Italia

**Abstract.** Effects of the quantum gravity under Minkowski space-time background are considered. It is shown that despite the absence of the complete theory of quantum gravity, some concrete predictions could be made for the influence of the quantum gravitational fluctuations on the propagation of the massive particles. We demonstrate that although the gravitational potential fluctuations do not produce particle scattering, they cause decoherence of the matter waves due to off-shell effects. For point-like massive particles of the Planck mass order, the effect is considerable. However, this type of decoherence is beyond the measurable possibility for the real particles of the finite size.

# 1. Introduction

It is widely stated that the complete theory of quantum gravity (QG) is not built yet. Indeed, it is true. At the same time, it is usually implied that the quantum gravitational fluctuations of space-time should be small. However, within the theory of general relativity (GR), one could hardly state that the quantum gravitational fluctuations are small because the coordinates' transformation to the reference frame where an observer has a highly oscillating position would result in substantial quantum gravitational fluctuations. Moreover, a number of real particles will be created from a vacuum in such a reference frame [1].

The situation changes cardinally when some preferred system of reference exists. For instance, the cosmic microwave background (CMB) defines the reference frame where CMB dipole anisotropy is absent [2]. That suggests considering all the phenomena in this particular frame. However, the CMB alone is not sufficient for determining the reference frame uniquely.

Another landmark is the vacuum energy problem insisting and specifying a class of permitted metrics [3–5]. As shown, a conformally-unimodular gauge [3] allows extending the GR to some theory admitting a Hamiltonian constraint satisfied up to some constant [3]. That explains why the main part of vacuum energy  $\rho_{vac} \sim M_p^4$  does not contribute to gravity [5], i.e., does not lead to the very fast universe expansion. Observation of the QG effects in table-top, the accelerator experiments or astrophysics is a dream of the several physicist generations [6–13]. Here we will consider the simplest vacuum model as a medium with the stochastic gravitational potential [14] and consider propagating the massive particles through it.

# 2. From GR to the gauge violating theory of gravity

In GR, any spatially uniform energy density (including that of zero-point fluctuations of the quantum fields) causes the expansion of the universe. Using the Planck level of the ultra-violet (UV) cutoff of momentum results in the Planckian vacuum energy density  $\rho_{vac} \sim M_p^4$  [15], which must lead to the universe expansion with the Planckian rate [16]. In this sense, because such a fast expansion is not experimentally visible, the vacuum energy problem is an observational fact [5]. One of the obvious solutions is to build a theory of gravity, allowing an arbitrarily reference level of energy density. One such theory has long been known. That is the unimodular gravity [17–21], which admits an arbitrary cosmological constant. However, under using the UV comoving momentums cutoff, the vacuum energy density scales with time as radiation [5, 22], but not as the cosmological constant.

Another theory [3] could also lead to Friedmann's equation defined up to some arbitrary constant, but this constant corresponds to the invisible radiation and can compensate the vacuum energy. This five-vector theory of gravity (FVT) [3] assumes the gauge invariance violation of GR by constraining the class of all possible metrics in varying the standard Einstein-Hilbert action. One has to vary not over all possible space-time metrics  $g_{\mu\nu}$ , but over some class of conformally-unimodular metrics

$$ds^{2} \equiv g_{\mu\nu} dx^{\mu} dx^{\nu} = a^{2} \left(1 - \partial_{m} P^{m}\right)^{2} d\eta^{2} - \gamma_{ij} (dx^{i} + N^{i} d\eta) (dx^{j} + N^{j} d\eta),$$
(1)

where  $x^{\mu} = \{\eta, \boldsymbol{x}\}, \eta$  is a conformal time,  $\gamma_{ij}$  is a spatial metric,  $a = \gamma^{1/6}$  is a locally defined scale factor, and  $\gamma = \det \gamma_{ij}$ . The spatial part of the interval (1) reads as

$$dl^2 \equiv \gamma_{ij} dx^i dx^j = a^2(\eta, \boldsymbol{x}) \tilde{\gamma}_{ij} dx^i dx^j, \qquad (2)$$

where  $\tilde{\gamma}_{ij} = \gamma_{ij}/a^2$  is a matrix with the unit determinant.

The interval (1) is similar formally to the ADM one [23], but with the lapse function defined as  $N = a(1 - \partial_m P^m)$ , where  $P^m$  is a three-dimensional vector, and  $\partial_m$  is a conventional partial derivative. Finally, restrictions  $\partial_n(\partial_m N^m) = 0$  and  $\partial_n(\partial_m P^m) = 0$ arise on the Lagrange multipliers  $\mathbf{N}$  and  $\mathbf{P}$  in FVT. The Hamiltonian  $\mathcal{H}$  and momentum  $\mathcal{P}_i$  constraints in the particular gauge  $P^i = 0$ ,  $N^i = 0$  obey the constraint evolution equation [3]:

$$\partial_{\eta} \mathcal{H} = \partial_i \left( \tilde{\gamma}^{ij} \mathcal{P}_j \right), \tag{3}$$

$$\partial_{\eta} \mathcal{P}_i = \frac{1}{3} \partial_i \mathcal{H},\tag{4}$$

which admits adding of some constant to  $\mathcal{H}$ . Thus, the constraint  $\mathcal{H}$  is not necessarily to be zero, but  $\mathcal{H} = const$  is also allowed.

## 3. Perturbations under Minkowski background

Here we will consider an empty space-time filled only by vacuum and taking into account its quantum properties. The well-known solution for an empty universe was found by Milne [24]. Although Milne himself does not use GR, from the point of GR his universe represents closed empty universe expanding linearly in cosmic time. Consideration of the mean vacuum energy density and pressure in the framework of FVT gives a a flat universe, which has a Milne's-like expansion stage [4, 25, 26] changed by the accelerated expansion.

Below, the scalar perturbations of the metric will be considered, which look in the conformally-unimodular frame as [28]

$$ds^{2} = a(\eta, \boldsymbol{x})^{2} \left( d\eta^{2} - \left( \left( 1 + \frac{1}{3} \sum_{m=1}^{3} \partial_{m}^{2} F(\eta, \boldsymbol{x}) \right) \delta_{ij} - \partial_{i} \partial_{j} F(\eta, \boldsymbol{x}) \right) dx^{i} dx^{j} \right),$$
(5)

where the perturbations of the locally defined scale factor

$$a(\eta, \boldsymbol{x}) = e^{\alpha(\eta)} (1 + \Phi(\eta, \boldsymbol{x})), \tag{6}$$

are expressed through a gravitational potential  $\Phi$ . A stress-energy tensor could be written in the hydrodynamic approximation [2]

$$T_{\mu\nu} = (p+\rho)u_{\mu}u_{\nu} - p g_{\mu\nu}.$$
(7)

The perturbations of the energy density  $\rho(\eta, \boldsymbol{x}) = \rho_v + \delta \rho(\eta, \boldsymbol{x})$  and pressure  $p(\eta, \boldsymbol{x}) = p_v + \delta p(\eta, \boldsymbol{x})$  will be considered around the vacuum mean values, where the index v will denote an uniform component of the vacuum energy density and pressure.

The zero-order equations for a flat universe take the form [4, 25, 26]

$$M_p^{-2}e^{4\alpha}\rho_v - \frac{1}{2}e^{2\alpha}\alpha'^2 = const,$$
(8)

$$\alpha'' + \alpha'^2 = M_p^{-2} e^{2\alpha} (\rho_v - 3p_v), \tag{9}$$

where  $\alpha(\eta) = \log a(\eta)$ . Here and everywhere further, the system of units  $\hbar = c = 1$  is used as well as the reduced Planck mass  $M_p = \sqrt{\frac{3}{4\pi G}}$  is implied. According to FVT [3], the first Friedmann equation (8) is satisfied up to some constant, and the main parts of the vacuum energy density and pressure

$$\rho_v \approx (N_{boson} - N_{ferm}) \frac{k_{max}^4}{16\pi^2 a^4},\tag{10}$$

$$p_v = \frac{1}{3}\rho_v \tag{11}$$

do not contribute to the universe expansion. In the formula (10), the UV cut-off and the number of bosonic and fermionic degrees of freedom of the quantum fields appear because the zero-point stress-energy tensor is an additive quantity [22]. Here, we do not consider the supersymmetry hypotheses [27] due to the absence of evidence of the supersymmetric particles to date.

Other contributors to the vacuum energy density are the terms depending on the derivatives of the universe expansion rate [4, 26]. They have the right order of

 $\rho_v \sim M_p^2 H^2$ , where H is the Hubble constant, and allow explaining the accelerated expansion of the universe. Then, the energy density and pressure are [4, 26]:

$$\rho_v = \frac{a'^2}{2a^6} M_p^2 (2 + N_{sc}) \mathcal{S}_0, \quad p_v = \frac{M_p^2 (2 + N_{sc}) \mathcal{S}_0}{a^6} \left(\frac{1}{2} a'^2 - \frac{1}{3} a'' a\right), \quad (12)$$

where,  $S_0 = \frac{k_{max}^2}{8\pi^2 M_p^2}$ . Eqs. (12) include the number of minimally coupled scalar fields  $N_{sc}$  plus two, because the gravitational waves give two additional degrees of freedom [26], whereas massless fermions and photons do not contribute to (12) [26].

The residual vacuum energy density and pressure (12) lead to the accelerated universe expansion, which allows finding a momentum UV cut off

$$k_{max} \approx \frac{12M_p}{\sqrt{2+N_{sc}}}.$$
(13)

from the experimental value of the universe decceleration parameter [4, 5, 26].

In this paper, we are interested in the local properties of a vacuum. Without including a real matter, if the constant in Eq. (8) compensates vacuum energy (10) exactly, one comes to the static Minkowski space-time. Further, we will consider the perturbations [28] under this background and set  $\alpha(\eta) = 0$  in (6).

Generally, a vacuum can be considered as some fluid, i.e., "ether" [4], but with some stochastic properties among the elastic ones. Let us return to the stress-energy tensor (7) and introduce other variables

$$\wp(\eta, \boldsymbol{x}) = a^4(\eta, \boldsymbol{x})\rho(\eta, \boldsymbol{x}),\tag{14}$$

$$\Pi(\eta, \boldsymbol{x}) = a^4(\eta, \boldsymbol{x})p(\eta, \boldsymbol{x})$$
(15)

for the reasons which will be explained below. The perturbations around the uniform values can be written now as  $\wp(\eta, \mathbf{x}) = \rho_v + \delta \wp(\eta, \mathbf{x})$ ,  $\Pi(\eta, \mathbf{x}) = p_v + \delta \Pi(\eta, \mathbf{x})$ . The vacuum-ether 4-velocity u is represented in the form of

$$u^{\mu} = \{ (1 - \Phi(\eta, \boldsymbol{x})), \boldsymbol{\nabla} \frac{v(\eta, \boldsymbol{x})}{\wp(\eta, \boldsymbol{x}) + \Pi(\eta, \boldsymbol{x})} \} \approx \{ (1 - \Phi(\eta, \boldsymbol{x})), \boldsymbol{\nabla} \frac{v(\eta, \boldsymbol{x})}{\rho_v + p_v} \},$$
(16)

where  $v(\eta, \boldsymbol{x})$  is a scalar function. Expanding all perturbations into the Fourier series  $\delta \wp(\eta, \boldsymbol{x}) = \sum_{\boldsymbol{k}} \delta \wp_{\boldsymbol{k}}(\eta) e^{i\boldsymbol{k}\boldsymbol{x}}$ ... etc. results in the equations for the perturbations:

$$-6\hat{\Phi}'_{k} + k^{2}\hat{F}'_{k} + \frac{18}{M_{p}^{2}}\hat{v}_{k} = 0, \qquad (17)$$

$$-6k^{2}\hat{\Phi}_{k} + k^{4}\hat{F}_{k} + \frac{18}{M_{p}^{2}}\delta\hat{\wp}_{k} = 0, \qquad (18)$$

$$-12\hat{\Phi}_{k} - 3\hat{F}_{k}'' + k^{2}\hat{F}_{k} = 0,$$
(19)

$$-9\hat{\Phi}_{k}'' - 9k^{2}\hat{\Phi}_{k} + k^{4}\hat{F}_{k} - \frac{9}{M_{p}^{2}}\left(3\delta\hat{\Pi}_{k} - \delta\hat{\wp}_{k}\right) = 0,$$
(20)

$$-\delta\hat{\varphi}'_{\boldsymbol{k}} + k^2 \hat{v}_{\boldsymbol{k}} = 0, \tag{21}$$

$$\delta \hat{\Pi}_{k} + \hat{v}'_{k} = 0. \tag{22}$$

It is remarkable that the choice of the variables (14), (15), (16) means that the values  $\rho_v$ and  $p_v$  do not appear in the system (17)-(22). The second point is that the continuity and the Newton second law equations (21), (22) do not contain metric perturbation. From now we will begin to consider the perturbation in Eqs. (17)-(22) as operators by writing a "hat" under every quantity. Here, we do not suppose the strong nonlinearity [29] and assume a smallness of the quantum fluctuations of space-time in this particular conformally unimodular metric. Let us emphasize that the system (17)-(22) for a perturbation evolution is exact in the first order on perturbations. However, it is not closed. To obtain a closed system, one needs, for instance, to specify the sound speed for a perturbation of pressure. Still, alternatively, as an approximation, we could strictly calculate pressure and energy density by using the field theory under unperturbed Minkowski space-time. Expressing  $F_k$  from Eq. (18) and substituting it into Eq. (20) leads to

$$\hat{\Phi}_{\boldsymbol{k}}^{\prime\prime} + \frac{1}{3}k^2\hat{\Phi}_{\boldsymbol{k}} + \frac{1}{M_p^2}\left(3\delta\hat{\Pi}_{\boldsymbol{k}} + \delta\hat{\varphi}_{\boldsymbol{k}}\right) = 0.$$
(23)

Although, generally, a gravity causing an arbitrary curved space-time background does not allow a well-defined and covariant vacuum state [1], we will approximately consider an operator  $3\delta \hat{\Pi}_{k} + \delta \hat{\wp}_{k}$  by using the creation and annihilation operators under the Minkowski space-time background. Such an approximation allows closing the system (17)-(22). Nevertheless, let us point out the difference between the quantum field theory (QFT) and QG. As is shown in Fig.1, a test particle moves straightforwardly in QFT. In a framework of the QG [30], the particle has to undergo interaction with ether.

# 3.1. Quantum fields as a source for energy density and pressure perturbations

Let us consider a single scalar field as an example of a quantum field. Energy density and pressure of the scalar field in the pure Minkowski space-time (without metric perturbation) have the form [22]

$$\hat{p}(\eta, \boldsymbol{x}) = \frac{\hat{\varphi}^{\prime 2}}{2} - \frac{(\boldsymbol{\nabla}\hat{\varphi})^2}{6},\tag{24}$$

$$\hat{\rho}(\eta, \boldsymbol{x}) = \frac{\hat{\varphi}^{\prime 2}}{2} + \frac{(\boldsymbol{\nabla}\hat{\varphi})^2}{2}.$$
(25)

All the quantities may be expanded into the Fourier series  $\hat{\varphi}(\eta, \boldsymbol{x}) = \sum_{\boldsymbol{k}} \hat{\phi}_{\boldsymbol{k}}(\eta) e^{i\boldsymbol{k}\boldsymbol{x}}$ ,  $\hat{p}(\eta, \boldsymbol{x}) = \sum_{\boldsymbol{k}} \hat{p}_{\boldsymbol{k}}(\eta) e^{i\boldsymbol{k}\boldsymbol{x}}$ , where  $\hat{p}_{\boldsymbol{k}}(\eta) = \int \hat{p}(\eta, \boldsymbol{x}) e^{-i\boldsymbol{k}\boldsymbol{x}} d\boldsymbol{x}$ , etc. For  $\boldsymbol{k} \neq 0$ , the approximate identifying  $\delta \hat{\Pi}_{\boldsymbol{k}} = \hat{p}_{\boldsymbol{k}}$  and  $\delta \hat{\varphi}_{\boldsymbol{k}} = \hat{\rho}_{\boldsymbol{k}}$  results in

$$\delta \hat{\Pi}_{\boldsymbol{k}} = \sum_{\boldsymbol{q}} \frac{1}{2} \hat{\phi}_{\boldsymbol{q}}^{\prime\prime} \hat{\phi}_{\boldsymbol{q}+\boldsymbol{k}}^{\prime} - \frac{1}{6} (\boldsymbol{q}+\boldsymbol{k}) \boldsymbol{q} \, \hat{\phi}_{\boldsymbol{q}}^{+} \hat{\phi}_{\boldsymbol{q}+\boldsymbol{k}}, \tag{26}$$

$$\delta\hat{\varphi}_{\boldsymbol{k}} = \sum_{\boldsymbol{q}} \frac{1}{2} \hat{\phi}_{\boldsymbol{q}}^{+\prime} \hat{\phi}_{\boldsymbol{q}+\boldsymbol{k}}^{\prime} + \frac{1}{2} (\boldsymbol{q}+\boldsymbol{k}) \boldsymbol{q} \, \hat{\phi}_{\boldsymbol{q}}^{+} \hat{\phi}_{\boldsymbol{q}+\boldsymbol{k}}, \tag{27}$$

so that the quantity  $3\delta \hat{\Pi}_{k} + \delta \hat{\wp}_{k}$  from Eq. (23) is reduced to

$$3\delta\hat{\Pi}_{\boldsymbol{k}} + \delta\hat{\wp}_{\boldsymbol{k}} = 2\sum_{\boldsymbol{q}} \hat{\phi}_{\boldsymbol{q}}^{+\prime} \hat{\phi}_{\boldsymbol{q}+\boldsymbol{k}}^{\prime}.$$
(28)



Figure 1. Illustration of vacuum influence to the particle propagation a) in the QFT, where the vacuum loops renormalize mass and charge of a particle, but do not prevent its free motion b) and in the QG, where the space is filled by ether due to the absence of a vacuum state.

Writing quantized field explicitly with creation and annihilation operators [1]

$$\hat{\phi}_{\boldsymbol{k}}(\eta) = \frac{1}{\sqrt{2\omega_k}} \left( \hat{a}^+_{-\boldsymbol{k}} e^{i\omega_k \eta} + \hat{a}_{\boldsymbol{k}} e^{-i\omega_k \eta} \right), \tag{29}$$

anows obtaining from the Eqs. (28) and (29)  

$$3\,\delta\hat{\Pi}_{\boldsymbol{k}} + \delta\hat{\wp}_{\boldsymbol{k}} = \sum_{\boldsymbol{q}} \sqrt{\omega_{\boldsymbol{q}}\omega_{|\boldsymbol{q}+\boldsymbol{k}|}} \Big( \hat{a}_{-\boldsymbol{q}}\hat{a}^{+}_{-\boldsymbol{q}-\boldsymbol{k}} e^{i(\omega_{|\boldsymbol{q}+\boldsymbol{k}|}-\omega_{\boldsymbol{q}})\eta} + \hat{a}^{+}_{\boldsymbol{q}}\hat{a}_{\boldsymbol{q}+\boldsymbol{k}} e^{i(\omega_{q}-\omega_{|\boldsymbol{q}+\boldsymbol{k}|})\eta} - \hat{a}_{-\boldsymbol{q}}\hat{a}_{\boldsymbol{q}+\boldsymbol{k}} e^{i(\omega_{|\boldsymbol{q}+\boldsymbol{k}|}+\omega_{\boldsymbol{q}})\eta} - \hat{a}^{+}_{\boldsymbol{q}}\hat{a}^{+}_{-\boldsymbol{q}-\boldsymbol{k}} e^{i(\omega_{|\boldsymbol{q}+\boldsymbol{k}|}+\omega_{\boldsymbol{q}})\eta} \Big), \quad (30)$$

where for a massless scalar field  $\omega_{\mathbf{k}} = |\mathbf{k}|$ . As is seen from Eq. (30), the perturbations have the general form:

$$3\,\delta\hat{\Pi}_{\boldsymbol{k}} + \delta\hat{\wp}_{\boldsymbol{k}} = \sum_{m} \hat{\mathcal{P}}_{m\boldsymbol{k}} e^{i\Omega_{m\boldsymbol{k}}\eta},\tag{31}$$

where the frequencies  $\Omega_{m\mathbf{k}}$  take the values of  $\omega_q - \omega_{|\mathbf{q}+\mathbf{k}|}$ ,  $-\omega_q + \omega_{|\mathbf{q}+\mathbf{k}|}$ ,  $\omega_q + \omega_{|\mathbf{q}+\mathbf{k}|}$  and  $-\omega_q - \omega_{|\mathbf{q}+\mathbf{k}|}$ . That allows finding the solution of Eq. (23) as

$$\hat{\Phi}_{\boldsymbol{k}}(\eta) = -\frac{1}{M_p^2} \sum_m \frac{\hat{\mathcal{P}}_{m\boldsymbol{k}} e^{i\Omega_{m\boldsymbol{k}}\eta}}{\Omega_{m\boldsymbol{k}}^2 - k^2/3}.$$
(32)

Using Eqs. (30) and (32), the final expression for the metric perturbation  $\hat{\Phi}_{k}(\eta)$  acquires the form

$$\hat{\Phi}_{\boldsymbol{k}}(\eta) = \frac{1}{M_p^2} \sum_{\boldsymbol{q}} \sqrt{\omega_q \omega_{|\boldsymbol{q}+\boldsymbol{k}|}} \left( \frac{1}{(\omega_{|\boldsymbol{q}+\boldsymbol{k}|} + \omega_q)^2 - k^2/3} \left( \hat{\mathbf{a}}_{-\boldsymbol{q}} \hat{\mathbf{a}}_{\boldsymbol{q}+\boldsymbol{k}} e^{-i(\omega_{|\boldsymbol{q}+\boldsymbol{k}|} + \omega_q)\eta} + \frac{1}{(\omega_{|\boldsymbol{q}+\boldsymbol{k}|} + \omega_q)^2 - k^2/3} \right) \right)$$

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$$\hat{\mathbf{a}}_{\boldsymbol{q}}^{+}\hat{\mathbf{a}}_{-\boldsymbol{q}-\boldsymbol{k}}^{+}e^{i(\omega_{|\boldsymbol{q}+\boldsymbol{k}|}+\omega_{q})\eta}\Big) - \frac{1}{(\omega_{|\boldsymbol{q}+\boldsymbol{k}|}-\omega_{q})^{2}-k^{2}/3}\Big(\hat{\mathbf{a}}_{-\boldsymbol{q}}\hat{\mathbf{a}}_{-\boldsymbol{q}-\boldsymbol{k}}^{+}e^{i(\omega_{|\boldsymbol{q}+\boldsymbol{k}|}-\omega_{q})\eta} + \hat{\mathbf{a}}_{\boldsymbol{q}}^{+}\hat{\mathbf{a}}_{\boldsymbol{q}+\boldsymbol{k}}e^{i(\omega_{q}-\omega_{|\boldsymbol{q}+\boldsymbol{k}|})\eta}\Big)\Big).$$

$$(33)$$

The most interesting parameter is a correlator:

$$<0|\hat{\Phi}(\eta,\boldsymbol{x})\hat{\Phi}(\tau,\boldsymbol{x}')|0>=\chi(\tau-\eta,|\boldsymbol{x}-\boldsymbol{x}'|),\qquad\chi(\tau-\eta,x)=\sum_{\boldsymbol{k}}S(\tau-\eta,k)e^{i\boldsymbol{k}\boldsymbol{x}},\quad(34)$$

which determines fluctuations of the gravitational potential  $\hat{\Phi}(\eta, \mathbf{r})$  in a vacuum state defined for the creation and annihilation operators. An explicit formula for  $S(\eta - \tau, k)$  looks as

$$S(\tau - \eta, k) = <0|\hat{\Phi}_{k}^{+}(\eta)\hat{\Phi}_{k}(\tau)|0> = \frac{18}{(2\pi)^{3}M_{p}^{4}}\int \frac{e^{i(\tau - \eta)(\omega_{q} + \omega_{q+k})}\omega_{q}\omega_{k+q}d^{3}q}{(k^{2} - 3(\omega_{q} + \omega_{k+q})^{2})^{2}},$$
(35)

where the summation over  $\boldsymbol{q}$  has been changed by the integration as  $\sum_{\boldsymbol{q}} \rightarrow \frac{1}{(2\pi)^3} \int d^3 \boldsymbol{q}$ . To calculate this integral, the spherical coordinates can be applied, in which  $\omega_{\boldsymbol{k}+\boldsymbol{q}} = \sqrt{k^2 + 2kq\cos\theta + q^2}$ ,  $\boldsymbol{q}(\boldsymbol{q}+\boldsymbol{k}) = q^2 + kq\cos\theta$ ,  $d^3\boldsymbol{q} = 2\pi q^2 dq\sin\theta d\theta$ . It is more convenient to calculate a spectral function  $\tilde{S}(\omega, \boldsymbol{q})$  of the correlator (35)

$$\tilde{S}(\omega,k) = \frac{1}{2\pi} \int S(\eta,k) e^{-i\omega\eta} d\eta = \frac{18}{(2\pi)^3 M_p^4} \int_{q < k_{max}} \frac{\delta(\omega_{\boldsymbol{q}} + \omega_{\boldsymbol{q}+\boldsymbol{k}} - \omega)\omega_{\boldsymbol{q}}\omega_{\boldsymbol{k}+\boldsymbol{q}} d^3 \boldsymbol{q}}{(k^2 - 3(\omega_{\boldsymbol{q}} + \omega_{\boldsymbol{k}+\boldsymbol{q}})^2)^2} = \begin{cases} \frac{1}{160\pi^2 M_p^4} \left(5 + \frac{4k^4}{(k^2 - 3\omega^2)^2}\right), \ q < \omega < 2k_{max} \\ 0, \ \text{otherwise} \end{cases} \approx \begin{cases} \frac{1}{32\pi^2 M_p^4}, \ q < \omega < 2k_{max} \\ 0, \ \text{otherwise.} \end{cases}$$
(36)

Taking into account that the main contribution originates from large q, one could also calculate simultaneous correlator

$$<0|\hat{\Phi}(\eta,\boldsymbol{x})\hat{\Phi}(\eta,\boldsymbol{x}')|0>\approx\frac{k_{max}}{4(2\pi)^2M_p^4}\delta(\boldsymbol{x}-\boldsymbol{x}'),$$
(37)

which corresponds to the contact interaction and was used in [31]. However, more careful analysis based on Appendix B shows that using of (37) is insufficient and the spectral function (36) of the non-simultaneous correlator (34) plays a role.

# 4. Massive particle in a random medium

#### 4.1. Point particles

Let us first consider nonrelativistic point massive particles propagating among the fluctuations of the gravitational potential [32]. The evolution of a system could be described by the Fokker-Plank type equation given in Appendix B

$$\partial_{\eta} f_{\boldsymbol{k}}(\boldsymbol{p}) + i(E_{\boldsymbol{p}+\boldsymbol{k}/2} - E_{\boldsymbol{p}-\boldsymbol{k}/2}) f_{\boldsymbol{k}}(\boldsymbol{p}) = -i K_1 \, \boldsymbol{k} \frac{\partial f_{\boldsymbol{k}}}{\partial \boldsymbol{p}} + 2i K_2 \, \boldsymbol{k} \boldsymbol{p} \, \Delta_{\boldsymbol{p}} f_{\boldsymbol{k}}(\boldsymbol{p}) + 2i K_3 \, p_i k_j \frac{\partial^2 f_{\boldsymbol{k}}}{\partial p_j \partial p_i}, \quad (38)$$

where  $\Delta_{\mathbf{p}}$  is a Laplacian over  $\mathbf{p}$ , the constants  $K_1 = \frac{m^2 N_{all}}{32\pi^2 M_p^4} \tilde{K}_1$ ,  $K_2 = \frac{m^2 N_{all}}{32\pi^2 M_p^4} \tilde{K}_2$ , ..., and  $\tilde{K}_1, \tilde{K}_2, \tilde{K}_3$  are given in Appendix B. In the difference from (10), the quantities  $K_i$  contain sum  $N_{all} = N_{boson} + N_{ferm}$  of the bosonic and fermionic degrees of freedom, because correlator (35) is the second order on gravitational potential  $\Phi$ , whereas  $\Phi$  is proportional to the energy density and pressure according to Eq. (23).

It is suggested that the Fokker-Planck equation is applicable for particles of large mass when the momentum of a particle is larger than the maximal momentum transferred, which is considered to be of the order of  $M_p$  for point-like particle. Since the Migdal equation (B.11) is too complicated for solution, the Fokker-Planck equation could be used to obtain an estimation of decoherence for particles of a smaller mass.

Due to the smallness of the right-hand side of (38), one needs to find a solution only in the first order on the constants  $K_1$ ,  $K_2$ ,  $K_3$ . For this aim, it is sufficient to substitute approximate solution (A.4) into the right-hand side of (38) and then solve it. This gives

$$f_{k}(\boldsymbol{p},\eta) \approx \tilde{f}_{k}(\boldsymbol{p},\eta) \left(1 + K_{1}\left(-\frac{k^{2}\eta^{2}}{2m} + \frac{2i\eta\boldsymbol{k}(\boldsymbol{p}-\boldsymbol{p}_{0})}{\Gamma^{2}}\right) + K_{2}\left(\frac{2\eta\boldsymbol{k}\boldsymbol{p}\left(-i\Gamma^{4}k^{2}\eta^{2} - 6\Gamma^{2}m(\boldsymbol{k}(\boldsymbol{p}-\boldsymbol{p}_{0})\eta + 3im) + 12im^{2}(\boldsymbol{p}-\boldsymbol{p}_{0})^{2}\right)}{3\Gamma^{4}m^{2}}\right) + K_{3}\left(-\frac{2\eta\left(\eta\left(-(\boldsymbol{k}\boldsymbol{p}_{0})(\boldsymbol{k}\boldsymbol{p}) + (\boldsymbol{k}\boldsymbol{p})^{2} + k^{2}(p^{2} - \boldsymbol{p}\boldsymbol{p}_{0})\right) + 2im\boldsymbol{k}\boldsymbol{p}\right)}{\Gamma^{2}m} + \frac{8i\eta(\boldsymbol{k}\boldsymbol{p}-\boldsymbol{k}\boldsymbol{p}_{0})(p^{2} - \boldsymbol{p}_{0}\boldsymbol{p})}{\Gamma^{4}} - \frac{2i(\boldsymbol{k}\boldsymbol{p})k^{2}\eta^{3}}{3m^{2}}\right)\right),$$
(39)

where  $\tilde{f}_{k}(\boldsymbol{p},\eta)$  is given in Appendix A by (A.4). Substituting the solution (39) into (A.5) gives in the first order on the constants  $K_1, K_2, K_3$ 

$$\int f_{\boldsymbol{k}}(\boldsymbol{p},\eta) f_{-\boldsymbol{k}}(\boldsymbol{p},\eta) d^3 \boldsymbol{p} d^3 \boldsymbol{k} \approx 1 - (3K_1 + 3K_2 + 6K_3) \frac{\Gamma^2 \eta^2}{m}.$$
(40)

As one could see, the interaction with vacuum produces decoherence expressed in the decreasing of a "purity" (A.5) of a particle state according to (40). From Eq. (40), the decoherence time is estimated as

$$t_{dec} \approx \frac{1}{\Gamma} \sqrt{\frac{m}{3K_1 + 3K_2 + 6K_3}}.$$
 (41)

It is convenient to measure the decoherence length  $L_{dec} = t_{dec} \mathcal{V}$  in terms of the localization length  $1/\Gamma$  of the wave packet (see Appendix A). Particle velocity is defined as  $\mathcal{V} = p_0/m$ . Dependence of the constant  $\sqrt{\frac{m}{3K_1+3K_2+6K_3}}$  is shown in Fig. 2, where also an approximate expression is shown. Using this approximate expression, one comes to  $L_{dec} \approx \frac{4M_p}{3\sqrt{3N_{all}\pi m}} \frac{\mathcal{V}}{\Gamma}$ . That is, a point-like particle of mass  $m \sim \frac{4M_p\mathcal{V}}{3\sqrt{3N_{all}\pi}}$  loses coherence at a distance equal to the length of the wave packet  $1/\Gamma$ . It should be noted that interaction with the ether does not produce particle scattering because the momentum distribution  $f_0(\mathbf{p})$  does not change, nevertheless the decoherence arises.

### 4.2. Particles of a finite size

A real particle of a large mass has a finite size, which restricts momentums transferred by the form factor. Approximately, momentum transferred q in the Eqs. (B.12), (B.13) should be restricted by q < 1/d, where d is size of a particle. In this case, the calculation



**Figure 2.** A dimensionless quantity determining decoherence time and length by Eq. (41) (solid line), an approximation  $\sqrt{\frac{m}{3K_1+3K_2+6K_3}} \sim \frac{4M_p}{3\sqrt{3N_{all}\pi m}}$  (dashed line). It is taken  $N_{sc} = 4$  and  $N_{all} = 126$  in Eq. (13) for  $k_{max}$ .

of the integrals gives  $K_1 = \frac{m N_{all}}{192\pi^2 (M_p d)^4}$ ,  $K_2 = \frac{N_{all}}{1200\pi^2 M_p^4 d^5}$ ,  $K_3 = \frac{N_{all}}{2400\pi^2 M_p^4 d^5}$ . The main contribution to the decoherence length for large mass particles originates from the constant  $K_1$  and gives

$$L_{dec} \approx \frac{\mathcal{V}}{\Gamma} \sqrt{\frac{m}{3K_1}} \approx \frac{8\pi (M_p d)^2}{\sqrt{N_{all}}} \frac{\mathcal{V}}{\Gamma}.$$
 (42)

This quantity seems very large and unobservable in a matter-wave interferometry [33–38], because increasing of the particle mass does not decrease decoherence length. On the other hand, the large mass particles usually have internal degrees of freedom and another decoherence mechanisms [39–46] related with these internal degrees of freedom works. Also, a particle spin could be considered as an internal degree of freedom and, thereby, produce decoherence [47, 48].

It should also be noted that another branch of combining gravity and quantum mechanics exists, namely, reduction of the wave function due to gravitational interaction [49]. That is beyond an "usual" QG and the content of this paper.

# 5. Discussion and conclusion

The QG must produce a considerable decoherence effect for a pure problem formulated for the point-like massive particles. Interestingly, this effect originates not due to the on-shell multiple scattering [50, 51], forbidden by the energy conservation, but from the off-shell effects.

For the real particles of finite size, the form factor restricts the momentum transferred. That reduces the effect of QG decoherence to an unobservable level. This QG decoherence could not compete with the decoherence arising from the interaction of the internal degrees of freedom of a composite particle with the gravitational field.

A general remark about the decoherence should also be made. From the point of quantum gravity, the universe as a whole exists in a single quantum state [52] and has zero entropy. Consequently, one could not consider a massive particle as completely isolated because it is always embedded into the general quantum state. Thus, any object does not lose its quantum properties but becomes more entangled with the universe's general quantum state.

It is of interest to analyze QG vacuum effects on the propagation of high energy gamma quanta [53, 54] in the universe. We plan to perform this investigation in the nearest future because the preliminary analysis [31] based on a simultaneous correlator of the gravitational potential is insufficient.

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#### Appendix A. Quantum mechanical evolution of the wave packet

A momentum wave packet of a freely moving particle can be written as

$$\psi(p,t) = \psi_0(p)e^{-i\frac{p^2}{2m}t},$$
(A.1)

because it obeys the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = \frac{p^2}{2m}\psi.$$
(A.2)

The function  $f_{\mathbf{k}}(\mathbf{p})$  corresponding to this pure quantum state is

$$f_{\boldsymbol{k}}(\boldsymbol{p}) = \psi(\boldsymbol{p} + \boldsymbol{k}/2)\psi^*(\boldsymbol{p} - \boldsymbol{k}/2). \tag{A.3}$$

For the Gaussian wave packet  $\psi(\mathbf{p},t) = \pi^{-3/4} \Gamma^{-3/2} e^{-\frac{(\mathbf{p}-\mathbf{p}_0)^2}{2\Gamma^2} - i\frac{\mathbf{p}^2}{2m}t}$ , the function  $f_{\mathbf{k}}(\mathbf{p})$  takes the form of

$$f_{k}(\boldsymbol{p}) = \pi^{-3/2} \Gamma^{-3} e^{-\frac{(\boldsymbol{p}-\boldsymbol{p}_{0})^{2}}{\Gamma^{2}} - \frac{k^{2}}{4\Gamma^{2}} - i\frac{\boldsymbol{k}\boldsymbol{p}}{m}t}.$$
(A.4)

For pure states, the density matrix  $\rho_{pp'} = \frac{1}{(2\pi)^3} f_{p-p'} \left(\frac{p'+p}{2}\right)$  satisfies [55]  $\sum_{p'} \rho_{pp'} \rho_{p'p''} = \rho_{pp'}$  or  $\sum_{p,p'} \rho_{pp'} \rho_{pp'} \rho_{p'p} = 1$ . The last equality expressed in terms of the function  $f_k(p)$  as

$$\int f_{\boldsymbol{k}}(\boldsymbol{p}) f_{-\boldsymbol{k}}(\boldsymbol{p}) d^3 \boldsymbol{p} d^3 \boldsymbol{k} = 1$$
(A.5)

could serve as a criterion of "purity" of a system state.

# Appendix B. Wigner function evolution in a random medium

Let us consider an equation for the density matrix  $\wp$ 

$$i\partial_{\eta}\hat{\wp} = [\hat{H}_1 + H_2 + \hat{V}, \hat{\wp}],\tag{B.1}$$

where operators  $\hat{H}_1$ ,  $\hat{H}_2$  describe the test particle and ether-medium, respectively [51, 56, 57]. The operator  $\hat{V}(\boldsymbol{x})$  is an operator acting in a test particle Hilbert space described by  $\boldsymbol{x}$ , and, besides, acting to the vacuum-ether variables. We will omit hats everywhere further in this Appendix. Let us introduce the density matrix of a particle by the averaging  $\rho = Tr_2 \varphi$ , then it satisfies the equation

$$i\partial_{\eta}\rho = [H_1, \rho] + Tr_2[V, \wp]. \tag{B.2}$$

The formal solution of Eq. (B.1) could be written as

$$\wp(\eta) = -i \int_{-\infty}^{\eta} e^{i(H_1 + H_2)(\tau - \eta)} [V, \wp(\tau)] e^{-i(H_1 + H_2)(\tau - \eta)} d\tau.$$
(B.3)

This expression can be substituted into the Eq. (B.2) and one comes to

$$i\partial_{\eta}\rho = [H_1, \rho] - iTr_2 \int_{-\infty}^{\eta} [V, e^{i(H_1 + H_2)(\tau - \eta)} [V, \wp(\tau)] e^{-i(H_1 + H_2)(\tau - \eta)}] d\tau.$$
(B.4)

For further approximation, the density matrix is factorized as  $\wp(\tau) = \rho(\tau)\rho_2(\tau)$ . Then, one has to take into account that the calculations of the correlator of the interaction in the Sec. 3 have been performed in the Heisenberg picture over medium-ether variables. Thus, we have to put interaction into the Heisenberg form using  $V = e^{-iH_2\tau}V(\tau)e^{iH_2\tau}$ , and, respectively bring the density matrix of a vacuum-medium into to the static form by writing  $\rho_2(\tau) = e^{-iH_2\tau}\rho_2 e^{iH_2\tau}$ . This leads to the equation

$$i\partial_{\eta}\rho = [H_1, \rho] - iTr_2 \int_{-\infty}^{\eta} [V(\eta), e^{iH_1(\tau - \eta)} [V(\tau), \rho(\tau)\rho_2] e^{-iH_1(\tau - \eta)}] d\tau.$$
(B.5)

Else, in terms of the matrix elements corresponding to the plane waves [56, 57]:

$$i\partial_{\eta}\rho_{pp'} = (E_{p} - E_{p'})\rho_{pp'} - i\sum_{q,q'} \int_{-\infty}^{\eta} \left( \langle V_{p-q}(\eta)V_{q-q'}(\tau) \rangle \rho_{q'p'}(\tau)e^{i(E_{q} - E_{p'})(\tau-\eta)} - \langle V_{p-q}(\eta)V_{q'-p'}(\eta) \rangle \rho_{qq'}(\tau)e^{i(E_{p} - E_{q'})(\tau-\eta)} + \rho_{pq}(\tau) \langle V_{q-q'}(\tau)V_{q'-p'}(\eta) \rangle e^{i(E_{p} - E_{q'})(\tau-\eta)} + \rho_{pq}(\tau) \langle V_{q-q'}(\tau)V_{q'-p'}(\eta) \rangle e^{i(E_{p} - E_{q'})(\tau-\eta)} \right) d\tau,$$
(B.6)

where an averaging  $\langle \rangle$  implies

$$\langle V_{\boldsymbol{q}}(\eta)V_{\boldsymbol{p}}(\tau)\rangle = Tr_2\Big(V_{\boldsymbol{q}}(\eta)V_{\boldsymbol{p}}(\tau)\rho_2\Big) \equiv \langle 0|V_{\boldsymbol{q}}(\eta)V_{\boldsymbol{p}}(\tau)|0\rangle$$

According to (35), (36), one has:

$$<0|V_{\boldsymbol{q}'}(\eta)V_{\boldsymbol{q}}(\tau)|0>=m^{2}\delta_{-\boldsymbol{q}',\boldsymbol{q}}S(\boldsymbol{q},\tau-\eta)=m^{2}\delta_{-\boldsymbol{q}',\boldsymbol{q}}\int\tilde{S}(\boldsymbol{q},\omega)e^{i\omega(\tau-\eta)}d\omega.$$
(B.7)

After changing the integration variable  $\tau' = \tau - \eta$  in the integral (B.6) and using approximately

$$\rho_{pq}(\eta + \tau') \approx e^{-i(E_p + E_q)\tau'} \rho_{pq}(\eta) \tag{B.8}$$

in the second-order on the interaction expression in the right-hand side of (B.6), we come to the following equation:

$$i\partial_{\eta}\rho_{\boldsymbol{p}\boldsymbol{p}\boldsymbol{p}'} = (E_{\boldsymbol{p}} - E_{\boldsymbol{p}'})\rho_{\boldsymbol{p}\boldsymbol{p}\boldsymbol{p}'} - im^{2}\sum_{\boldsymbol{q}} \int \Big(\tilde{S}(q,\omega)\Big((\Delta(E_{\boldsymbol{p}+\boldsymbol{q}} - E_{\boldsymbol{p}} + \omega) + \Delta(E_{\boldsymbol{p}'} - E_{\boldsymbol{p}'+\boldsymbol{q}} - \omega))\rho_{\boldsymbol{p}\boldsymbol{p}\boldsymbol{p}'}(\eta) - (\Delta(\omega - E_{\boldsymbol{p}'} + E_{\boldsymbol{p}'+\boldsymbol{q}}) + \Delta(-\omega + E_{\boldsymbol{p}} - E_{\boldsymbol{p}+\boldsymbol{q}}))\rho_{\boldsymbol{p}+\boldsymbol{q},\boldsymbol{p}'+\boldsymbol{q}}\Big)d\omega,$$
(B.9)

where

$$\Delta(\omega) = \int_{-\infty}^{0} e^{i\omega\tau} d\tau = \pi \delta(\omega) - i\mathcal{P}\frac{1}{\omega}$$
(B.10)

contains the Dirac  $\delta$ -function and the main value generalized function  $\mathcal{P}^{1}_{\omega}$  [58].

In terms of the Fourier transform of the Winger function [56]  $\rho_{pp'} = f_{p-p'} \left(\frac{p'+p}{2}\right)$ or  $f_{k}(p) = \rho_{p+k/2,p-k/2}$  Eq. (B.9) is written as

$$\partial_{\eta} f_{\boldsymbol{k}}(\boldsymbol{p}) + i(E_{\boldsymbol{p}+\boldsymbol{k}/2} - E_{\boldsymbol{p}-\boldsymbol{k}/2})f_{\boldsymbol{k}}(\boldsymbol{p}) = m^{2}\sum_{\boldsymbol{q}}\int \tilde{S}(q,\omega) \Big( (\Delta(\omega - E_{\boldsymbol{p}-\boldsymbol{k}/2} + E_{\boldsymbol{p}-\boldsymbol{k}/2+\boldsymbol{q}}) + \Delta(-\omega + E_{\boldsymbol{p}+\boldsymbol{k}/2} - E_{\boldsymbol{p}+\boldsymbol{k}/2+\boldsymbol{q}}))f_{\boldsymbol{k}}(\boldsymbol{p}+\boldsymbol{q}) - (\Delta(\omega + E_{\boldsymbol{p}+\boldsymbol{k}/2+\boldsymbol{q}} - E_{\boldsymbol{p}+\boldsymbol{k}/2}) + \Delta(-\omega + E_{\boldsymbol{p}-\boldsymbol{k}/2} - E_{\boldsymbol{p}-\boldsymbol{k}/2+\boldsymbol{q}}))f_{\boldsymbol{k}}(\boldsymbol{p}) \Big) d^{3}\boldsymbol{q} \, d\omega.$$
(B.11)

Summation over q could be changed by the integration. Hence, the Dirac delta-functions in (B.10) produce zero contribution to the integral. Actually, since the minimal value of  $\omega$  is restricted by q according to (36), the value  $\omega + \frac{(p+q)^2}{2m} - \frac{p^2}{2m} > q + \frac{pq}{m} + q^2 > 0$ by the virtue of  $p = m\mathcal{V} < m$ . Thus, only the second main value term in (B.10) gives contribution.

In a diffusion approximation, one may expand  $f_{\mathbf{k}}(\mathbf{p}+\mathbf{q}) - f_{\mathbf{k}}(\mathbf{p}) \approx \mathbf{q} \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{p}} + \frac{1}{2}q_i q_j \frac{\partial^2 f_{\mathbf{k}}(p)}{\partial p_i \partial p_j}$ , and the following integrals arise

$$\begin{split} \int_{q < k_{max}} \int_{q}^{2k_{max}} \left( \frac{\mathbf{q}}{\omega + E(\mathbf{p} + \mathbf{q}) - E(\mathbf{p})} - \frac{\mathbf{q}}{\omega + E(\mathbf{p}' + \mathbf{q}) - E(\mathbf{p}')} \right) d\omega \, d^{3}\mathbf{q} \approx \\ -(\mathbf{p} - \mathbf{p}') \tilde{K}_{1}(m, k_{max}) + O(p^{4}), \quad (B.12) \\ \tilde{K}_{1} &= \frac{4}{3}m \left( k_{max}(3k_{max} + 4m) - 8k_{max}^{3/2}\sqrt{m} \arctan\left(\frac{1}{2}\sqrt{\frac{k_{max}}{m}}\right) + \\ & 8m^{2}\ln\left(\frac{2m}{k_{max} + 2m}\right) \right), \\ \int_{q < k_{max}} \int_{q}^{2k_{max}} \left( \frac{\mathbf{q} \otimes \mathbf{q}}{E(\mathbf{p} + \mathbf{q}) - E(\mathbf{p}) + \omega} - \frac{\mathbf{q} \otimes \mathbf{q}}{E(\mathbf{p}' + \mathbf{q}) - E(\mathbf{p}') + \omega} \right) d\omega \, d^{3}\mathbf{q} \approx \\ & (p^{2} - p'^{2}) \tilde{K}_{2}(m, k_{max}) \mathbf{I} + (\mathbf{p} \otimes \mathbf{p} - \mathbf{p}' \otimes \mathbf{p}') \tilde{K}_{3}(m, k_{max}) + O(p^{4}), \quad (B.13) \\ \tilde{K}_{2} &= \frac{8}{15}m \left( k_{max} \left( \frac{2k_{max}m}{k_{max}^{2} + 6k_{max}m + 8m^{2}} + 3 \right) - 10\sqrt{k_{max}m} \arctan\left(\frac{1}{2}\sqrt{\frac{k_{max}}{m}}\right) + 8m \right) \end{split}$$

$$8m^{2}\left(5\ln\left(\frac{2m}{k_{max}+2m}\right)+6\operatorname{arctanh}\left(\frac{k_{max}}{k_{max}+4m}\right)\right)\right),$$
(B.14)

+

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$$\tilde{K}_{3} = \frac{16}{15}m\left(k_{max}\left(k_{max}\left(\frac{2k_{max}m}{k_{max}^{2}+6k_{max}m+8m^{2}}+3\right)-10\sqrt{mk_{max}}\arctan\left(\frac{1}{2}\sqrt{\frac{k_{max}}{m}}\right)+\frac{18}{10}\exp\left(-\frac{k_{max}}{m}\right)\right)\right)$$
(P.15)

$$8m \bigg) - 4m^2 \bigg( 5\ln \bigg( \frac{2m}{k_{max} + 2m} \bigg) + 18 \operatorname{arctanh} \bigg( \frac{k_{max}}{k_{max} + 4m} \bigg) \bigg) \bigg).$$
(B.15)

Finally, we have:

$$\partial_{\eta} f_{\boldsymbol{k}}(\boldsymbol{p}) + i \frac{\boldsymbol{p} \boldsymbol{k}}{m} f_{\boldsymbol{k}}(\boldsymbol{p}) = \frac{i m^2}{32\pi^2 M_p^4} \left( -\tilde{K}_1 \, \boldsymbol{k} \frac{\partial f_{\boldsymbol{k}}(\boldsymbol{p})}{\partial \boldsymbol{p}} + 2\tilde{K}_2 \, \boldsymbol{k} \boldsymbol{p} \, \Delta_{\boldsymbol{p}} f_{\boldsymbol{k}}(\boldsymbol{p}) + 2 \, \tilde{K}_3 \, p_i k_j \frac{\partial^2 f_{\boldsymbol{k}}(\boldsymbol{p})}{\partial p_j \partial p_i} \right).$$
(B.16)

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