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Corrigendum

Corrigendum to "Quantum phases for point-like charged particles and for electrically neutral dipoles in an electromagnetic field" [Ann. Phys. 392 (2018) 49–62]

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Three years ago, our paper [1] has been published, where we advanced the idea of explaining quantum phase effects for electric/magnetic dipoles, expressed as the sum of four terms [1,2]

$$\delta_{\text{dipoles}} \approx \frac{1}{\hbar c}\int (\boldsymbol{m}\times\boldsymbol{E})\cdot d\boldsymbol{s} - \frac{1}{\hbar c}\int ((\boldsymbol{p}\times\boldsymbol{B}))\cdot d\boldsymbol{s} - \frac{1}{\hbar c^2}\int (\boldsymbol{p}\cdot\boldsymbol{E})\,\boldsymbol{v}\cdot d\boldsymbol{s} - \frac{1}{\hbar c^2}\int (\boldsymbol{m}\cdot\boldsymbol{B})\,\boldsymbol{v}\cdot d\boldsymbol{s} \quad (1)$$

(Eq. (11) of [1]) through the superposition of the corresponding quantum phases for point-like charged particles. Hereinafter $\boldsymbol{m}$ ($\boldsymbol{p}$) denotes the magnetic (electric) dipole moment, $\boldsymbol{E}$($\boldsymbol{B}$) is the electric (magnetic) field, $\boldsymbol{v}$ is the velocity, and $d\boldsymbol{s} = \boldsymbol{v}dt$ is the path element.

Developing this idea, we disclosed two new quantum phases for point-like charges – next to the known electric and magnetic Aharonov–Bohm (A–B) phases [3] – which we named as complementary electric and magnetic A–B phases, correspondingly [1].

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Further on, we found that complementary A–B phases for electric charges can be described via fundamental equations of quantum mechanics only in the case [1], where we abandon the customary definition of the momentum operator via the canonical momentum $\hat{\boldsymbol{P}}_c$ of a particle in an EM field, *i.e.*

$$\hat{\boldsymbol{P}}_c = \hat{\boldsymbol{p}} + \frac{e\hat{\boldsymbol{A}}}{c} \rightarrow \hat{\boldsymbol{P}}_c = -i\hbar\nabla \tag{1}$$

(Eq. (30) of [1]), and adopt its new definition via the sum of the mechanical and interactional EM momentum for a particle in an external EM field,

$$\hat{\boldsymbol{p}} + \hat{\boldsymbol{P}}_{EM} \rightarrow \hat{\boldsymbol{P}} = -i\hbar\nabla \tag{2}$$

(Eq. (35) of [1]). The proposed redefinition of the momentum operator (2) has a number of important implications, and their analysis essentially depends on the particular expression for the interactional EM field momentum $\boldsymbol{P}_{EM}$ for various physical problems (see, e.g., Ref. [3]).

Now, we would like to point out an unfortunate error committed in [1] under determination of $\boldsymbol{P}_{EM}$ for the system "point-like charged particle in an external EM field" as a function of the scalar φ and vector $\boldsymbol{A}$ potentials of the external EM field.

Namely, the expression

$$\boldsymbol{P}_{EM} = \frac{e\boldsymbol{A}}{c} + \frac{\boldsymbol{v}e\varphi}{c^2} + \frac{e\boldsymbol{v}\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)}{c^3}, \tag{3}$$

obtained in Ref. [1] see (Eq. (38) of [1]) must be corrected as

$$\boldsymbol{P}_{EM} = \frac{e\boldsymbol{A}}{c} + \frac{\boldsymbol{v}e\varphi}{c^2} - \frac{e\boldsymbol{v}\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)}{c^3}. \tag{3a}$$

Indeed, we start with the known expression for the interactional EM field momentum via the Poynting vector [4]

$$\boldsymbol{P}_{EM} = \frac{1}{4\pi c}\int_V \left(\boldsymbol{E}\times\boldsymbol{B}_e\right)dV + \frac{1}{4\pi c}\int_V \left(\boldsymbol{E}_e\times\boldsymbol{B}\right)dV \tag{4}$$

(Eq. (A.1) of [1]), where $\boldsymbol{E}$ ($\boldsymbol{B}$) denotes the external electric (magnetic) field, while $\boldsymbol{E}_e$ ($\boldsymbol{B}_e$) stands for the electric (magnetic) field of a charged particle. Then we obtain, respectively, the first and second integrals on the rhs of Eq. (4) as follows:

$$\frac{1}{4\pi c}\int_V \left(\boldsymbol{E}\times\boldsymbol{B}_e\right)dV = \frac{\boldsymbol{v}e\varphi}{c^2} \tag{5}$$

(Eq. (A.7) of [1]), and

$$\frac{1}{4\pi c}\int_V \left(\boldsymbol{E}_e\times\boldsymbol{B}\right)dV = -\frac{1}{4\pi c}\int_V \left(\boldsymbol{A}\times\left(\nabla\times\boldsymbol{E}_e\right)\right)dV + \frac{e\boldsymbol{A}}{c} \tag{6}$$

(Eq. (A.11) of [1]). Further manipulations with the remaining integral on the rhs of Eq. (6) yield:

$$-\frac{1}{4\pi c}\int_V \left(\boldsymbol{A}\times\left(\nabla\times\boldsymbol{E}_e\right)\right)dV = -\frac{e\boldsymbol{v}\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)}{c^3} - \frac{\boldsymbol{v}}{4\pi c^2}\int_V \left(\boldsymbol{A}\cdot\left(\left(\boldsymbol{v}\cdot\nabla\right)\boldsymbol{E}_e\right)\right)dV \tag{7}$$

(Eq. (A.20) of Ref. [1]), and we show in Ref. [1] that for the system "point-like charge in an external EM field", the second integral on the rhs of Eq. (7) vanishes. Hence, combining Eqs. (4)–(7), we arrive at Eq. (3a), instead of the sign wise erroneous equation (3) reported in Ref. [1].

Therefore, in the Coulomb gauge, the corresponding Hamiltonian with the momentum operator (2) and the interactional EM field momentum (3a) takes the form

$$\begin{aligned}\hat{H} &= \tfrac{\left(-i\hbar\nabla-\boldsymbol{P}_{EM}\right)^2}{2M} + e\varphi = -\tfrac{\hbar^2}{2M}\Delta + e\varphi - \tfrac{e\boldsymbol{A}\cdot\boldsymbol{v}}{c} - \tfrac{e\varphi v^2}{c^2} + \tfrac{ev^2\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)}{c^3} = \\ &-\tfrac{\hbar^2}{2M}\Delta + \left(1-\tfrac{v^2}{c^2}\right)\left(e\varphi - \tfrac{e\boldsymbol{A}\cdot\boldsymbol{v}}{c}\right)\end{aligned}. \tag{8}$$

The quantum phase of charged particle in an EM field is defined as [5]

$$\delta_{\mathrm{EM}} = -\frac{1}{\hbar}\int\left(\hat{H}-\hat{H}_0\right)dt, \tag{9}$$

where $\hat{H}_0 = -\frac{\hbar^2}{2M}\Delta$ stands for the Hamiltonian of a particle outside an EM field. Substituting Eq. (8) into Eq. (9), we obtain the quantum phase of a charged particle in an EM field as follows:

$$\delta_{EM} = -\frac{1}{\hbar}\int e\varphi dt + \frac{1}{\hbar c}\int e\boldsymbol{A}\cdot d\boldsymbol{s} + \frac{1}{\hbar c^2}\int e\varphi\boldsymbol{v}\cdot d\boldsymbol{s} - \frac{1}{\hbar c^3}\int e\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)\boldsymbol{v}\cdot d\boldsymbol{s}, \tag{10}$$

where we designated $d\boldsymbol{s} = \boldsymbol{v}dt$ the element of the path of particle.

The first and the second terms on the rhs of Eq. (10) correspond to the known electric and magnetic A–B phases [3], while the third and the firth terms stand respectively for the complementary electric and complementary magnetic A–B phases.

The reported corrections do not influence the conclusions of Ref. [1] and open a way to some new insights in the physical meaning of quantum phases.

The obtained Eq. (10) allows presenting the resultant quantum phase of a charged particle in an EM field in general form,

$$\delta_{EM} = -\frac{1}{\hbar}\left(1-\frac{v^2}{c^2}\right)\int e\varphi dt + \left(1-\frac{v^2}{c^2}\right)\frac{1}{\hbar c}\int e\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)dt = -\frac{1}{\hbar\gamma^2}\int L_{\mathrm{int}}dt = \frac{1}{\hbar\gamma^2}S_{\mathrm{int}}, \tag{11}$$

where we have used the equality $d\boldsymbol{s}$=$\boldsymbol{v}dt$, and designated

$$L_{\mathrm{int}} = -e\varphi + \frac{e}{c}\boldsymbol{A}\cdot\boldsymbol{v} \tag{12}$$

the component of the Lagrangian of a charged particle, responsible for its interaction with the EM field. Then, we have used the definition of the interactional component of the action $S_{\mathrm{int}} = \int L_{\mathrm{int}}dt$

Hence, in the non-relativistic limit, where we can put $\gamma\approx 1$, the total phase of a charged particle occurs proportional to the total action S_{total} for a charged particle, which is composed as the sum of the mechanical S_{M} and the interactional field component S_{int}, *i.e.*,

$$\delta_{\mathrm{total}} = \frac{1}{\hbar}\left(S_{\mathrm{M}} + S_{\mathrm{int}}\right) = \frac{1}{\hbar}S_{\mathrm{total}}. \tag{13}$$

Thus, the known semi-classical limit for the wave function of a freely moving particle [5]

$$\psi = \psi_0 e^{iS_M/\hbar}$$

keeps its shape for charged particle in an EM field, too, with the replacement of mechanical component of action S_{M} by the total action S_{total}, *i.e.*, $\psi = \psi_0 e^{iS_{total}/\hbar}$.

Next, we re-address Eq. (10) for the phase of a charged particle in an EM field and consider only the velocity-dependent phase components expressed by the last three terms of this equation. Using Eq. (3a), we can write for these phase components

$$\delta_{EM}(\boldsymbol{v}) = \frac{1}{\hbar c}\int e\boldsymbol{A}\cdot d\boldsymbol{s} + \frac{1}{\hbar c^2}\int e\varphi\boldsymbol{v}\cdot d\boldsymbol{s} - \frac{1}{\hbar c^3}\int e\left(\boldsymbol{A}\cdot\boldsymbol{v}\right)\boldsymbol{v}\cdot d\boldsymbol{s} = \frac{1}{\hbar}\int \boldsymbol{P}_{EM}\cdot d\boldsymbol{s}. \tag{14}$$

Concurrently we remind that the phase for a freely moving particle is given by the equation

$$\delta_{\mathrm{free}}\left(\boldsymbol{v}\right) = \frac{1}{\hbar}\int \boldsymbol{P}_M\cdot d\boldsymbol{s} \tag{15}$$

due to the de Broglie relationship, where $\boldsymbol{P}_M$ denotes the mechanical momentum of a particle.

The total velocity-dependent phase of charged particle is given as the sum of Eqs. (14) and (15), and is equal to

$$\delta_{\mathrm{total}}\left(\boldsymbol{v}\right) = \frac{1}{\hbar}\int\left(\boldsymbol{P}_M + \boldsymbol{P}_{EM}\right)\cdot d\boldsymbol{s}. \tag{16}$$

Taking also into account that the total phase $\delta_{\text{total}}(\boldsymbol{v})$ of a moving charged particle can be presented via the corresponding wave vector $\boldsymbol{k}$ as

$$\delta_{\text{total}}(\boldsymbol{v}) = \frac{1}{\hbar}\int \boldsymbol{k}\cdot d\boldsymbol{s}, \tag{17}$$

we obtain through comparison of Eqs. (16) and (17) $\boldsymbol{k} = (\boldsymbol{P}_M + \boldsymbol{P}_{EM})/\hbar$.

Hence the wavelength of charged particle moving in the EM field is equal to

$$\lambda\!\!\!^{-} = \hbar/|\boldsymbol{P}_M + \boldsymbol{P}_{EM}|\,. \tag{18}$$

Eq. (18) shows that the de Broglie wavelength of a charged particle moving in the presence of an EM field depends not only on its mechanical momentum, but rather on the modulus of the vector sum of mechanical $\boldsymbol{P}_M$ and interactional electromagnetic $\boldsymbol{P}_{EM}$ momenta.

Detailed analysis of the physical implications of Eqs. (13) and (18) will be done elsewhere.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] A.L. Kholmetskii, O.V. Missevitch, T. Yarman, Ann. Phys. 392 (2018) 49.
[2] A.L. Kholmetskii, O.V. Missevitch, T. Yarman, Europhys. Lett. 113 (2016) 14003.
[3] A.L. Kholmetskii, T. Yarman, O.V. Missevitch, M. Arik, Sci. Rep. 8 (2018) 11937.
[4] J.D. Jackson, Classical Electrodynamics, third ed., Wiley, New York, 1998.
[5] L.D. Landau, E.M. Lifshitz, Quantum Mechanics. Non-Relativistic Theory, Elsevier, 1981.