NUMERICAL METHODS FOR NONCONVEX OPTIMAL CONTROL PROBLEMS

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Let us address the following state-linear control system

$$\dot{x}(t) = A(t)x(t) + B(u(t), t) \quad \overset{\circ}{\forall} t \in T :=]t_0, t_1[, \; x(t_0) = x_0; \tag{1}$$

$$u(\cdot) \in \mathcal{U} := \{u(\cdot) \in L^r_\infty(T) \mid u(t) \in U \quad \overset{\circ}{\forall} t \in T\}. \tag{2}$$

under standard assumptions ensuring the existence of the unique absolutely continuous solution $x(\cdot, u) \in AC_n(T), \; x(t) = x(t, u), \; t \in \overline{T}$ of the ODEs system (1) for any feasible control $u(\cdot) \in \mathcal{U}$ [1–5]. In addition, consider the functionals

$$J_i(x, u) = \varphi_{1i}(x(t_1)) + \int\limits_T \varphi_i(x(t), u(t), t)dt, \; i \in \{0\} \cup I, \; I = \{1, \ldots, m\}, \tag{3}$$

where $\varphi_{1i}(x) := g_{1i}(x) - h_{1i}(x) \quad \forall x \in \Omega_1 \subset I\!R^n$, $\varphi_i(x, u, t) := g_i(x, u, t) - h_i(x, t), \; i \in \{0\} \cup I$, with the state-convex functions $g_{1i}(x), \; h_{1i}(x)$, and $x \to g_i(x, u, t)$, $x \to \; h_i(x, t) \quad \forall (u, t) \in U \times T$.

We address now the following optimal control (OC) problem

$$(\mathcal{P}): \qquad \left.\begin{array}{l} J_0(u) := J_0(x(\cdot, u), u(\cdot)) \downarrow \min\limits_u, \; u(\cdot) \in \mathcal{U}, \\ J_i(u) := J_i(x(\cdot, u), u(\cdot)) \leq 0, \; i \in I. \end{array}\right\} \tag{4}$$

It is clear that this OC problem is nonconvex due to nonconvexity of the data, which implies that in $(\mathcal{P})$ there might exist a big number of locally optimal and stationary (say, in the sense of PMP) processes that may be rather far from the set $Sol(\mathcal{P})$ of global solutions of $(\mathcal{P})$.

Further, we employ the penalty function $\pi(x, u) := \pi(u) := \max\{0, J_1(u), \ldots, J_m(u)\}$ and address the auxiliary (penalized) problem

$$(\mathcal{P}_\sigma): \qquad J_\sigma(u) := J_\sigma(x(\cdot, u), u(\cdot)) \downarrow \min_u, \; u(\cdot) \in \mathcal{U}, \tag{5}$$

with the cost (merit) function as follows $J_\sigma(u) := J_0(x(\cdot, u), u(\cdot)) + {}+\sigma\pi(x(\cdot, u), u(\cdot))$, where $\sigma \geq 0$ is a penalty parameter. Recall that the key feature of the Exact Penalization Theory consists in the existence of threshold value $\sigma_* > 0$ of the penalty parameter for which Problems $(\mathcal{P})$ and $(\mathcal{P}_\sigma)$ are equivalent in the sense that $\mathcal{V}(\mathcal{P}) = = \mathcal{V}(\mathcal{P}_\sigma)$ and $Sol(\mathcal{P}) = Sol(\mathcal{P}_\sigma) \quad \forall \sigma > \sigma_*$.

Furthermore, on account of the obvious presentation $J_i(u) = G_i(x, u) - F_i(x),\ i \in \{0\} \cup I$, with the state-convex $G_i(\cdot)$ and $F_i(\cdot)$, one can decompose the merit function $J_\sigma(x, u)$ as follows [6] $J_\sigma(x, u) := G_\sigma(x, u) - F_\sigma(x)$, where $G_\sigma(x, u)$ and $F_\sigma(x)$ are state-convex.

Using this decomposition, we can address the (partially) linearized (at $y(\cdot) \in AC_n(T)$) OC problem [6]

$$(\mathcal{P}_\sigma L(y)): \quad \Phi_{\sigma y}(u) := G_\sigma(x(\cdot), u(\cdot)) - \\ - \langle\langle \nabla F_\sigma(y(\cdot)), x(\cdot)\rangle\rangle \downarrow \min_u,\ u(\cdot) \in \mathcal{U}, \quad (6)$$

with the help of which we can formulate the so-called Global Optimality Conditions for Problem $(\mathcal{P}_\sigma)$.

In addition, we developed a Scheme of Local and Global Searches for the nonconvex OC Problem $(\mathcal{P}_\sigma)$. Combining these procedures with the corresponding updates of the penalty parameter $\sigma_k > 0$, we developed Numerical Methods for the nonconvex Problem $(\mathcal{P})$ that allows us not only to escape local pitfalls of $(\mathcal{P})$, but to reach the globally optimal controls in nonconvex OC problems of the kind.

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