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EXACT PENALTY OF HIGH ORDER FOR EXTREME PROBLEMS OF DIFFERENTIAL INCLUSIONS

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In this paper, using theorems on the continuous dependence of the solution of differential inclusions on the perturbation, we obtain high-order exact penalty theorems for nonconvex extremal problems of differential inclusions in the space of Banach-valued absolutely continuous functions.

Let X be separable Banach space, $a : [0,T] \times X \to comp\, X \bigcup\{\emptyset\}$, $T > 0$, $R_\infty = (-\infty, +\infty]$, $f : [0,T] \times X \times X \to R_\infty$ normal integrant, $\varphi : X \times X \to R_\infty$ function, $M \subset X$ nonempty compact set, $1 \le p < +\infty$.

The symbol $W_p^1([0,T], X)$ denotes Banach space of absolutely continuous functions from $[0,T]$ in X with the first derivative according to Freshet which belongs $L_p([0,T], X)$ with the norm $\|x(\cdot)\|_{W_p^1} = \|x(0)\| + \left(\int_0^T \|\dot{x}(t)\|^p dt\right)^{1/p}$.

A solution $\bar{x}(\cdot) \in W_p^1([0,T], X)$ of the system

$$\dot{x}(t) \in a(t, x(t)), \quad x(0) \in M, \tag{1}$$

minimizing functional

$$J(x) = \varphi(x(0), x(T)) + \int_0^T f(t, x(t), \dot{x}(t))dt \tag{2}$$

among all solutions of (1) in $W_p^1([0,T], X)$ will be called the solution of problems (1), (2) in $W_p^1([0,T], X)$. Let's assume that $|J(\bar{x})| < +\infty$.

Let's set $\psi(s,x,y)=\inf\{\|z-y\|:z\in a(s,x)\}$, $q(x)=\inf\limits_{y\in M}\|y-x\|$, where $\inf\emptyset=+\infty$, $B(\bar{x}(t),\alpha)=\{x\in X:\|x-\bar{x}(t)\|\le\alpha\}$, where $t\in[t_0,T]$, $\alpha>0$ and (see [1])

$$J_r(x)=\varphi(x(0),x(T))+\int_0^T f(t,x(t),\dot{x}(t))dt+$$

$$+r\Bigg(\Big[q(x(0))+(\int_0^T\psi^{\beta}(t,x(t),\,\dot{x}(t))dt)^{\frac{1}{\beta}}\Big]^{\beta}+$$

$$+\|x(\cdot)-\bar{x}(\cdot)\|^{\beta-\nu}_{W^1_\beta}\Big[q(x(0))+(\int_0^T\psi^{\beta}(t,x(t),\,\dot{x}(t))dt)^{\frac{1}{\beta}}\Big]^{\nu}\Bigg),$$

where $x(\cdot)\in W^1_\beta([0,T],X)$, $\beta\ge\nu>0$, $\beta\ge1$.

Let $\gamma>e^{m(T)}(2+m(T))^2$, $m(t)=\int\limits_0^t k(s)ds$, $h=\max\{1,\,T^{\frac{\beta-1}{\beta}}\}$.

Theorem 1. *Let* $a:[0,T]\times X\to comp\,X\bigcup\{\emptyset\}$ *be a multiple-valued map,* $a(t,x)$ *in the domain* $t\in[0,T]$, $x\in B(\bar{x}(t),\alpha)$ *the nonempty compact set and is measurable on* t, $M\subset X$ *nonempty compact set,* $f:[0,T]\times X\times X\to R_\infty$ *normal integrant,* $\varphi:X\times X\to R_\infty$ *function,* $\beta\ge\nu>0$, $\beta\ge1$, *there exist function* $k(\cdot)\in L_\beta[0,T]$ *and numbers* $k_1>0$ *and* $k_2>0$ *such that* $\rho_X(a(t,x),a(t,y))\le k(t)\,\|x-y\|$ *at* $x,y\in B(\bar{x}(t),\alpha)$ *and*

$$|f(t,x_1,z_1)-f(t,x_2,z_2)|\le k_1(\|x_1-x_2\|+\|z_1-z_2\|)^{\nu}((\|x_2-\bar{x}(t)\|+$$
$$+\|z_2-\dot{\bar{x}}(t)\|)^{\beta-\nu}+(\|x_1-x_2\|+\|z_1-z_2\|)^{\beta-\nu})$$

at $x_1,\,x_2\in B(\bar{x}(t),\alpha)$, $z_1,\,z_2\in X$,

$$|\varphi(u_1,\upsilon_1)-\varphi(u_2,\upsilon_2)|\le k_2(\|u_1-u_2\|+\|\upsilon_1-\upsilon_2\|)^{\nu}((\|u_2-\bar{x}(0)\|+$$
$$+\|\upsilon_2-\bar{x}(T)\|)^{\beta-\nu}+(\|u_1-u_2\|+\|\upsilon_1-\upsilon_2\|)^{\beta-\nu})$$

at $u_1,u_2\in B(\bar{x}(0),\alpha)$, $\upsilon_1,\upsilon_2\in B(\bar{x}(T),\alpha)$ *and* $\bar{x}(\cdot)\in W^1_\beta([0,T],X)$ *is the solution of the problems* (1),(2) *in* $W^1_\beta([0,T],X)$. *Then there exists* $r_0>0$ *such that* $\bar{x}(t)$ *are minimized by functional* $J_r(x)$ *at* $r\ge r_0$ *in* $D_\beta=\{x(\cdot)\in W^1_\beta([0,T],X):\|x(\cdot)-\bar{x}(\cdot)\|_{W^1_\beta}\le\frac{\alpha}{\gamma h}\}$.

References

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