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INPUT RECONSTRUCTION PROBLEM UNDER THE LACK OF INFORMATION IN A QUASI-LINEAR STOCHASTIC EQUATION

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1. Problem Statement. The problem of reconstructing unknown inputs in a quasi-linear stochastic differential equation (SDE) is investigated on the basis of the approach of the theory of dynamic inversion suggested in the works by Kryazhimskii, Osipov, and their colleagues, see [1] and its bibliography. We consider the statement when the simultaneous reconstruction of disturbances in the deterministic and stochastic terms of the equation is performed with the use of discrete incomplete information on a number of realizations of the stochastic process. The work actually continues studies [2], where a similar problem was solved for a linear SDE via a partially observed system of linear ordinary differential equations (ODEs) obtained by the method of moments.

A SDE with diffusion depending on the phase state is of the form

$$dx(t,\omega) = (A(t)x(t,\omega) + B(t)u_1(t) + f(t))\,dt + U_2(t)\,x(t,\omega)\,d\xi(t,\omega). \quad (1)$$

Here, $t \in T = [0,\vartheta]$, $x \in \mathbb{R}^n$, $x(0,\omega) = x_0$ is a known deterministic or random (normally distributed) vector; $\omega \in \Omega$, (Ω, F, P) is a probability space, $\xi(t,\omega) \in \mathbb{R}$ is a standard scalar Wiener process; $A(t)$, $B(t)$, and $f(t)$ are continuous matrix functions of dimension $n \times n$, $n \times r$, and $n \times 1$, respectively. Two external disturbances act on the system: vectors $u_1(t) \in \mathbb{R}^r$ and $u_2(t) \in \mathbb{R}^n$ (the main diagonal of a diagonal matrix $U_2(t) \in \mathbb{R}^{n\times n}$) with values from given convex compact sets; both functions are of bounded variation. The input u_1 enters into the deterministic term and influences the mathematical expectation of the desired process, whereas the vector u_2 regulates the amplitude of random noises.

A solution of equation (1) is defined as a stochastic process satisfying corresponding integral identity containing the Ito integral for any t with probability 1. Under the assumptions above, there exists a unique solution, which is a normal Markov process with continuous realizations.

The problem under discussion is as follows. At discrete, frequent enough, times $\tau_i \in T$, $\tau_i = i\delta$, $\delta = \vartheta/l$, $i \in [0 : l]$, the information on some number N of realizations of the stochastic process $x(\tau_i)$ is received, at that only q ($q \leq n$) first coordinates are measurable. It is required to design an algorithm for the dynamical reconstruction of unknown disturbances $u_1(t)$ and $u_2(t)$ generating $x(t)$ from the information above. The probability of an arbitrarily small deviation of approximations from the desired inputs in L_2-metric should be close to 1 for sufficiently large N and the time discretization step $\delta = \delta(N) = \vartheta/l(N)$ concordant with N in an appropriate way.

2. Outline of Solving Algorithm. The specific properties of SDE (1) admit the reduction of the problem to an inverse problem for a nonlinear system of ODEs describing the mathematical expectation and covariance matrix of the desired process. To solve the latter problem, a finite-step ($l(N)$ identical steps) solving algorithm based on the method of auxiliary controlled models [1] is designed. In connection with the lack of incoming information, Block 1 for the dynamical reconstruction of unmeasured coordinates is introduced to get the information on the whole phase state. This information is fed to Block 2 forming (by the feedback law) model controls to approximate real disturbances synchronously with the process. We denote the output of the algorithm by $(u_1^N(\cdot), u_2^N(\cdot))$ emphasizing the dependence of all its parameters on the number N of available trajectories of (1). In the report, we discuss different variants for additional assumptions on the structure of equation (1), on its solutions and measurements, which are sufficient to prove

Theorem 1. *The accuracy of the algorithm is estimated as*

$$P\left(\max_{i=1,2;\, n_1=r,\, n_2=n}\{\, \|u_i^N(\cdot) - u_i(\cdot)\|_{L_2(T;\mathbb{R}^{n_i})}\} \leq h(N)\right) = 1 - g(N), \quad (2)$$

where $h(N)$, $g(N) \to 0$ *as* $N \to \infty$ *and can be written explicitly.*

The algorithm, the choice of its parameters, and the character of convergence in estimate (2) are illustrated by a model example.

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EXACT PENALTY OF HIGH ORDER FOR EXTREME PROBLEMS OF DIFFERENTIAL INCLUSIONS

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In this paper, using theorems on the continuous dependence of the solution of differential inclusions on the perturbation, we obtain high-order exact penalty theorems for nonconvex extremal problems of differential inclusions in the space of Banach-valued absolutely continuous functions.

Let X be separable Banach space, $a : [0,T] \times X \to comp\, X \bigcup \{\emptyset\}$, $T > 0$, $R_\infty = (-\infty, +\infty]$, $f : [0,T] \times X \times X \to R_\infty$ normal integrant, $\varphi : X \times X \to R_\infty$ function, $M \subset X$ nonempty compact set, $1 \le p < +\infty$.

The symbol $W_p^1([0,T], X)$ denotes Banach space of absolutely continuous functions from $[0,T]$ in X with the first derivative according to Freshet which belongs $L_p([0,T], X)$ with the norm $\|x(\cdot)\|_{W_p^1} = \|x(0)\| + \left(\int_0^T \|\dot{x}(t)\|^p dt\right)^{1/p}$.

A solution $\bar{x}(\cdot) \in W_p^1([0,T], X)$ of the system

$$\dot{x}(t) \in a(t, x(t)), \quad x(0) \in M, \tag{1}$$

minimizing functional

$$J(x) = \varphi(x(0), x(T)) + \int_0^T f(t, x(t), \dot{x}(t))dt \tag{2}$$

among all solutions of (1) in $W_p^1([0,T], X)$ will be called the solution of problems (1), (2) in $W_p^1([0,T], X)$. Let's assume that $|J(\bar{x})| < +\infty$.