Theorem 1. *The pair of r-strategies exists $\bar{u}^0(U_0), \bar{v}^0$ which forms the equilibrium situation and gives birth to the equilibrium trajectory $x^0(t)$.*

The number of the moments of the information receipt for the motion $x^0(t)$ is a finite number.

Let $U_{01}(x^0(T)) > U_{02}(x^0(T)) > 0$. The pair $\bar{u}^0(U_{01}), \bar{v}^0$ forms the equilibrium situation and gives birth to the equilibrium trajectory $x^0(t)$ as well as the pair $\bar{u}^0(U_{02}), \bar{v}^0$.

Theorem 2. *Let the pair $\bar{u}^0(U_{01}), \bar{v}^0$ gives birth to the motion $x^0(t)$. The amount of the moments of the information receipt by the player 1 about the motion $x^0(t)$ is not more than the amount of the moments of information receipt for the same motion $x^0(t)$ which is born by the pair $\bar{u}^0(U_{02}), \bar{v}^0$.*

References

1. *Kononenko A.F., Mokhonko E.Z.* Processes of information reception in non-antagonistic differential game. Reports on applied mathematics. M: CC AS USSR, 1982, 20 p.
2. *Kononenko A.F.* Structure of optimum strategies in dynamical control systems // J. Comput. Math. and Math. Phys. 1980. Vol. 20. No 5. P. 1105–1116.
3. *Krasovsky N.N., Subbotin A.N.* Positional differential games. Moscow: Nauka, 1974. 456 p.

VARIATIONAL ANALYSIS IN NONSMOOTH NUMERICAL OPTIMIZATION

B. Mordukhovich

Wayne State University, USA

aa1086@wayne.edu

In this lecture we discuss recent applications of advanced variational analysis and generalized differentiation to the design, justification of numerical algorithms of nonsmooth optimization with applications to practical modeling. Our main attention is paid to developing generalized Newton-type algorithms to solve nonsmooth optimization problems and subgradient systems that are based mainly on constructions and results of second-order variational analysis. Solvability of these algorithms is proved in rather broad settings, and then verifiable conditions for their local and global superlinear convergence are obtained. We consider in more detail problems convex composite optimization for which a generalized damped Newton algorithm exhibiting global superlinear convergence is designed. The efficiency of the designed algorithm is demonstrated by solving a class of Lasso problems that are well-recognized in applications to machine learning and statistics. For this class of nonsmooth optimization problems, we conduct numerical experiments and compare the obtained results with those achieved by using other first-order and second-order methods.

This talk is based on recent joint works with P. D. Khanh (HCMUE, Vietnam), V. T. Phat (WSU), M. E. Sarabi (Miami Univ., USA), and D. B. Tran (WSU).

ON DECOUPLING TRANSFORMATION FOR THREE TIME-SCALE LINEAR TIME-INVARIANT SINGULARLY PERTURBED CONTROL SYSTEMS WITH STATE DELAYS

C.A. Naligama, O.B. Tsekhan

Yanka Kupala State University of Grodno, Grodno, Belarus
naligama_ch_19@student.grsu.by, tsekhan@grsu.by

Consider the three time-scale singularly perturbed linear time-invariant control system with state delays (TSPLTISD):

$$\dot{x}(t) = \sum_{j=0}^{l} A_{11j}x(t-jh) + A_{12}y(t) + A_{13}z(t) + B_1u(t),\ x \in R^{n_1}, u \in R^r,$$
$$\varepsilon_1\dot{y}(t) = \sum_{j=0}^{l} A_{21j}x(t-jh) + A_{22}y(t) + A_{23}z(t) + B_2u(t),\ y \in R^{n_2},$$
$$\varepsilon_2\dot{z}(t) = \sum_{j=0}^{l} A_{31j}x(t-jh) + A_{32}y(t) + A_{33}z(t) + B_3u(t),\ z \in R^{n_3}, t \geq 0,$$

where A_{i1j}, A_{i2}, A_{i3}, B_i, $i = \overline{1,3}$, $j = \overline{0,l}$ are constant matrices with appropriate dimensions, $h = const > 0$ is a delay, $0 < \varepsilon_2 \ll \varepsilon_1 \ll \frac{\varepsilon_2}{\varepsilon_1} \ll 1$ are the small parameters, that describe the time-scale separation, $u(t)$ ia a piecewise continuous on T r-vector control function. Let $p \triangleq \frac{d}{dt}$ be the differentiation operator, e^{-ph} the delay operator: $e^{-ph}v(t) = v(t-h)$. Similar to [1],[2] a generalization of Chang's non-degenerate decoupling transformation [3], under change of variables $T(\varepsilon_1, \mu, e^{-ph})$,