**Theorem 2.** *Let the functions* $L(\cdot)$ *and* $L_x(\cdot)$ *be continuously differentiable in totality of variables and the admissible function* $\bar{x}(\cdot)$ *be an extremal of problem (1), (2), and along it for the vectors* $\eta \neq 0$ *and* $\left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta$, *where* $\bar{\lambda} \in (0,1)$ *the Weierstrass condition degenerate at any point* $t$ *of the interval* $(\bar{t}_0, \bar{t}_1) \subset [t_0, t_1]$ , *i.e. we have the equalities*

$$\mathscr{E}\left(\bar{L}\right)(t,\eta) = \mathscr{E}\left(\bar{L}\right)\left(t, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta\right) = 0. \tag{5}$$

*Furthermore, let the extremal* $\bar{x}(\cdot)$ *be twice continuously differentiable on the interval* $(\bar{t}_0, \bar{t}_1)$. *Then: (i) if the extremal* $\bar{x}(\cdot)$ *is a strong local minimum in problem (1),(2), then the following inequality is fulfilled*

$$\eta^T\left[\bar{\lambda}\bar{L}_{xx}(t,\eta) + \left(1-\bar{\lambda}\right)\bar{L}_{xx}\left(t, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta\right)\right]\eta - \frac{d}{dt}\Delta\bar{L}_x^T(t,\eta)\,\eta \geq 0,$$
$$\forall t \in (\bar{t}_0, \bar{t}_1)\,, \tag{6}$$

*where* $\bar{L}_{xx}(t,\xi) := L_{xx}(t, \bar{x}(t), \dot{\bar{x}}(t) + \xi)$, $\xi \in \left\{\eta, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta\right\}$,
$\Delta\bar{L}_x(t,\eta) := L_x(t, \bar{x}(t), \dot{\bar{x}}(t) + \eta) - \bar{L}_x(t)$;

*(ii) if the extremal* $\bar{x}(\cdot)$ *is a weak local minimum in problem (1),(2), then there exists such a number* $\delta > 0$, *at which for each point* $\left(\eta, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta, \bar{\lambda}\right) \in \mathrm{B}_\delta(0) \times \mathrm{B}_\delta(0) \times (0{,}1)$ *satisfying condition (5), the inequality (6) is fulfilled.*

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# ADDITIONAL PAYMENT IN NON-ANTAGONISTIC DIFFERENTIAL GAME


**E.Z. Mokhonko**

Dorodnicyn Computing Center FRC CSC RAS, Moscow, Russia
ezmokhon@mail.ru


**Introduction.** Chernousko F.L., Melikjan A.A., Kononenko A.F., Mokhonko E.Z. [1] investigated how to receive the same result using the sample data information instead of the continuous reception of information.

In this paper some differential game is considered. The first player is able to pay additional payment to the second player, if the second player does not deviate from the agreed trajectory. The equilibrium situation is constructed in $r$-strategies with a possibility to pay an additional payment. $r$-strategies permit to receive information about position as sample data or in continuous way. The payment changes the character of information receipt about the trajectory. For example, it is getting possible to receive information about the equilibrium trajectory not countable times but the finite number times only.

The aim of the article is to clarify the character of changes of the information receipt about equilibrium trajectory under changes of additional payment.

**1. Description of the differential game.** Let us consider some differential non-antagonistic game of two players

$$\dot{x} = f(x,t,u,v), \quad t_0 \le t \le T, \quad x(t_0) = x^0, \quad u \in P, \quad v \in Q,$$

$$I_1(u,v) = g_1(x(T)), \qquad I_2(u,v) = g_2(x(T)) + U(x(T)).$$

Here $x$ is $n$-dimensional vector of state, $u$ and $v$ are $p$- and $q$- dimensional vector-functions of control. Players 1 and 2 choose the meaning of the functions in order to maximize the appropriate cost functions $I_1(u,v) = g_1(x(T))$, $I_2(u,v) = g_2(x(T)) + U(x(T))$, $g_1$ and $g_2$ are continuous functions. The sets $P$ and $Q$ are compacts in the appropriate vector spaces. The vector-function $f(x,t,u,v)$ is a continuous function of all its arguments and satisfies restrictions which are imposed on it in [2]. We use the concepts of the Eiler's broken line and the motion [3].

$$U(x(T)) = \begin{cases} U_0 \ge 0, & x(T) = x^0, \\ 0, & x(T) \ne x^0. \end{cases}$$

If $U_0 > 0$, then $U(x(T))$ is the additional payment. The player 1 pays it to the second player if the trajectory of game is $x^0$ at the end of the game. If $U_0 = 0$, then the game without the additional payment is considered.

The set of permissible strategies of every player $\overline{U}, \overline{V}$ is the set of measurable for every argument positional $u(x,t), v(x,t)$ and program $u(t), v(t)$ controls. In addition to this some special strategies $\bar{u}, \bar{v}$ are permissible. They are called $r$- strategies [1]. $r$-strategies permit to receive information about position as sample data or in continuous way.

In this paper we consider the case $U_0(x^0(T)) > 0$, $x^0 = x^0(T)$, that is the game with the additional payment. The next theorem is proved.

**Theorem 1.** *The pair of $r$-strategies exists $\bar{u}^0(U_0), \bar{v}^0$ which forms the equilibrium situation and gives birth to the equilibrium trajectory $x^0(t)$.*

*The number of the moments of the information receipt for the motion $x^0(t)$ is a finite number.*

Let $U_{01}(x^0(T)) > U_{02}(x^0(T)) > 0$. The pair $\bar{u}^0(U_{01}), \bar{v}^0$ forms the equilibrium situation and gives birth to the equilibrium trajectory $x^0(t)$ as well as the pair $\bar{u}^0(U_{02}), \bar{v}^0$.

**Theorem 2.** *Let the pair $\bar{u}^0(U_{01}), \bar{v}^0$ gives birth to the motion $x^0(t)$. The amount of the moments of the information receipt by the player 1 about the motion $x^0(t)$ is not more than the amount of the moments of information receipt for the same motion $x^0(t)$ which is born by the pair $\bar{u}^0(U_{02}), \bar{v}^0$.*

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## VARIATIONAL ANALYSIS IN NONSMOOTH NUMERICAL OPTIMIZATION


**B. Mordukhovich**

Wayne State University, USA
aa1086@wayne.edu


In this lecture we discuss recent applications of advanced variational analysis and generalized differentiation to the design, justification of numerical algorithms of nonsmooth optimization with applications to practical modeling. Our main attention is paid to developing generalized Newton-type algorithms to solve nonsmooth optimization problems and subgradient systems that are based mainly on constructions and results of second-order variational analysis. Solvability of these algorithms is proved in rather broad settings, and then verifiable conditions