$$= -I'_{ix}\left(x\left(t_i\right), v_i\right)\left(I'_{ix}\left(x\left(t_i\right), v_i\right) + E\right)^{-1}\psi\left(t_i\right), \quad i = 1, 2, ..., p, \tag{7}$$

with the boundary conditions

$$\psi(0) = A'\lambda + \frac{\partial\Phi}{\partial x(0)}, \ \psi(T) = -\frac{\partial\Phi}{\partial x(T)}.$$

Theorem 2. *Le the conditions of theorem 1 be fulfilled. Then for the optimality of the control $u_* \in U$ in problem (1)-(5) it is necessary that the inequality*

$$\int_0^T \langle H_u(t, x_*(t), u_*(t), \psi_*(t)), u(t) - u_*(t)\rangle \, dt \geq 0$$

to be fulfilled for any $u_* \in U$.

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NECESSARY MINIMUM CONDITIONS IN CALCULUS OF VARIATIONS PROBLEMS IN THE PRESENCE OF VARIOUS DEGENERATIONS

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We consider the following problem:

$$J\left(x\left(\cdot\right)\right) = \int_{t_0}^{t_1} L\left(t, x\left(t\right), \dot{x}\left(t\right)\right)dt \to \min_{x(\cdot)}, \tag{1}$$

$$x\left(t_0\right) = x_0, \ x\left(t_1\right) = x_1, \ x\left(\cdot\right) \in PC^1\left(I, R^n\right), \tag{2}$$

where R^n is n-dimensional Euclidean space, t_0, t_1, x_0, x_1 are the given points, $I := [t_0, t_1]$, while $L(t, x, \dot{x}) : I \times R^n \times R^n \to R$ is a given function, $PC^1(I, R^n)$ is the set of piecewise-smooth functions $x(t) : I \to R^n$.

We call the functions $x(\cdot) \in PC^1(I, R^n)$ satisfying the boundary condition (2), admissible functions. An admissible function that satisfies the Euler equation is called extremal. We study the problem (1),(2) in the case when along the extremal the Weierstrass condition and the Legendre condition degenerate at separate points or on some intervals. The method of the study is based on the introduction of Weierstrass type special variations [1, p.125], characterized by a number parameter. Two types of new necessary conditions are obtained: of equality type and of inequality type for strong and weak local minimum. It is shown on specific examples that these minimum conditions are not corollaries of necessary optimality conditions obtained in [2, p. 111, 179, 146] and have their own application area.

We give some theorems proved in this paper.

Theorem 1. *Let the functions $L(\cdot)$ and $L_{\dot{x}}(\cdot)$ be twice continuously differentiable in totality of variables and the admissible function $\bar{x}(\cdot)$ be an extremal of the problem (1), (2). Furthermore, let at the point $\theta \in (t_0, t_1)$ the function $\bar{x}(\cdot)$ be twice differentiable and along it for the vector $\eta \neq 0$ the Weierstrass and Legendre conditons degenerate at the poit θ, i.e. we have the equalities*

$$\mathscr{E}\left(\bar{L}\right)(\theta, \eta) = \eta^T \bar{L}_{\dot{x}\dot{x}}(\theta)\, \eta = 0. \tag{3}$$

Then: (i) if the extremal $\bar{x}(\cdot)$ is a strong local minimum in problem (1), (2), then the following equalities are fulfilled

$$\eta^T \left[L_x(\theta, \bar{x}(\theta), \dot{\bar{x}}(\theta) + \eta) - \bar{L}_x(\theta) - \bar{L}_{x\dot{x}}(\theta)\, \eta\right] = 0, \tag{4}$$

$$\bar{L}_{\dot{x}\dot{x}\dot{x}}(\theta)\,[\eta, \eta, \eta] = 0,$$

where $\mathscr{E}\left(\bar{L}\right)(t, \eta) = L(t, \bar{x}(t), \dot{\bar{x}}(t) + \eta) - \bar{L}(t) - \bar{L}^T_{\dot{x}}(t)\,\eta$ is a Weierstrass function [1, p.124], calculated along the extremal $\bar{x}(\cdot)$, where $\bar{L}(t) := L(t, \bar{x}(t), \dot{\bar{x}})$, then the symbols $\bar{L}_x(\cdot), \bar{L}_{\dot{x}}(\cdot), \bar{L}_{x\dot{x}}(\cdot)$ and so on, are determined in a similar way;

(ii) if the extremal $\bar{x}(\cdot)$ is a weak local minimum in problem (1), (2), then there exists such a number $\delta > 0$, at which for each point $\eta \in B_\delta(0)$ satisfying condition (3), equalities (4) are valid, where the symbol $B_\delta(0)$ is a closed ball of radius δ centered at the point $0 \in R^n$.

Theorem 2. *Let the functions* $L(\cdot)$ *and* $L_x(\cdot)$ *be continuously differentiable in totality of variables and the admissible function* $\bar{x}(\cdot)$ *be an extremal of problem (1), (2), and along it for the vectors* $\eta \neq 0$ *and* $\left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta$, *where* $\bar{\lambda} \in (0,1)$ *the Weierstrass condition degenerate at any point* t *of the interval* $(\bar{t}_0, \bar{t}_1) \subset [t_0, t_1]$ *, i.e. we have the equalities*

$$\mathscr{E}\left(\bar{L}\right)(t,\eta) = \mathscr{E}\left(\bar{L}\right)\left(t, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta\right) = 0. \tag{5}$$

Furthermore, let the extremal $\bar{x}(\cdot)$ *be twice continuously differentiable on the interval* $(\bar{t}_0, \bar{t}_1)$. *Then: (i) if the extremal* $\bar{x}(\cdot)$ *is a strong local minimum in problem (1),(2), then the following inequality is fulfilled*

$$\eta^T\left[\bar{\lambda}\bar{L}_{xx}(t,\eta) + \left(1-\bar{\lambda}\right)\bar{L}_{xx}\left(t, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta\right)\right]\eta - \frac{d}{dt}\Delta\bar{L}_x^T(t,\eta)\,\eta \geq 0,$$
$$\forall t \in (\bar{t}_0, \bar{t}_1)\,, \tag{6}$$

where $\bar{L}_{xx}(t,\xi) := L_{xx}(t, \bar{x}(t), \dot{\bar{x}}(t) + \xi)$, $\xi \in \left\{\eta, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta\right\}$,
$\Delta\bar{L}_x(t,\eta) := L_x(t, \bar{x}(t), \dot{\bar{x}}(t) + \eta) - \bar{L}_x(t)$;

(ii) if the extremal $\bar{x}(\cdot)$ *is a weak local minimum in problem (1),(2), then there exists such a number* $\delta > 0$, *at which for each point* $\left(\eta, \left(\bar{\lambda}-1\right)^{-1}\bar{\lambda}\eta, \bar{\lambda}\right) \in \mathrm{B}_\delta(0) \times \mathrm{B}_\delta(0) \times (0{,}1)$ *satisfying condition (5), the inequality (6) is fulfilled.*

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ADDITIONAL PAYMENT IN NON-ANTAGONISTIC DIFFERENTIAL GAME

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Introduction. Chernousko F.L., Melikjan A.A., Kononenko A.F., Mokhonko E.Z. [1] investigated how to receive the same result using the sample data information instead of the continuous reception of information.