analytical solution of the inverse kinematics problem. The presence of a rigorous mathematical model makes it possible to use all the results of mathematical control theory in delta robot control problems. In particular, it is possible to determine the stabilizing control in the problem of stabilizing a given position of the gripper of a delta robot [8] by the method of N.N. Krasovskii [9].

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FIRST ORDER OPTIMALITY CONDITIONS FOR AN OPTIMAL CONTROL PROBLEM WITH NONLOCAL CONDITIONS UNDER IMPULSE ACTIONS

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We consider the following nonlocal boundary value problem under impulse actions:

$$\frac{dx}{dt} = f(t, x, u(t)),\ 0 \le t \le T,\ t \ne t_i,\ i = 1, 2, ..., p, \tag{1}$$

$$Ax(0) + \int_0^T m\,(t)x\,(t)\,dt = C, \tag{2}$$

$$x(t_i^+) = x(t_i) + I_i(x(t_i)),\ i = 1, 2, ..., p,\ 0 < t_1 < t_2 < ... < t_p < T, \tag{3}$$

$$u\,(\cdot) \in U \subset L_2^r\,([0, T])\,, \tag{4}$$

where $x(t) \in R^n$; $f(t, x, u)$ is n-dimensional continuous and differentiable function with respect to (x, u); A, $m(t) \in R^{n \times n}$, $C \in R^{n \times 1}$ are the given matrices, moreover $\det N \ne 0$, $N = A + \int_0^T m(t)dt$; $I_i : R^n \to R^n$, $i = 1, 2, \ldots, p$, are some continuously differentiable given functions; u are control parameters; $x(t_i^+)$ denotes the right limit of $x(t)$ at $t = t_i$.

On the solutions of boundary value problem (1)-(4) it is required to minimize the functional

$$J(u) = \Phi(x(0), x(T)), \tag{5}$$

where $\Phi(x, y)$ is a given scalar function with continuous first derivatives with respect to (x, y) .

Note that an optimal control problem with two-point boundary conditions under impulse actions was studied in [1].

At first, under some additional conditions on the impact data of the problem we prove that boundary value problem (1)-(3) has a unique solution for each fixed admissible control (4).

Theorem 1. *Let the above conditions be fulfilled and furthermore,* $\det\left(E + I_{ix}\left(x\left(t_i\right)\right)\right) \ne 0,\ i = 1, 2, ..., p.$

Then the functional (5) is differentiable under the constraints (1)-(4) and its gradient is of the form

$$J'(u) = f_u'(t, x, u)\psi(t) \in L_2^r[0, T]$$

where $\psi(t)$ is the solution of the difference-differential system

$$\frac{d\psi(t)}{dt} = -f_x'(t, x, u)\psi(t) - m'(t)\lambda,\ t \ne t_i, \tag{6}$$

$$\psi(t_i^+) - \psi(t_i) =$$

$$= -I'_{ix}\left(x\left(t_i\right), v_i\right)\left(I'_{ix}\left(x\left(t_i\right), v_i\right) + E\right)^{-1} \psi\left(t_i\right), \quad i = 1, 2, ..., p, \tag{7}$$

with the boundary conditions

$$\psi(0) = A'\lambda + \frac{\partial \Phi}{\partial x(0)}, \;\; \psi(T) = -\frac{\partial \Phi}{\partial x(T)}.$$

Theorem 2. *Le the conditions of theorem 1 be fulfilled. Then for the optimality of the control $u_* \in U$ in problem (1)-(5) it is necessary that the inequality*

$$\int_0^T \left\langle H_u(t, x_*(t), u_*(t), \psi_*(t)), u(t) - u_*(t) \right\rangle dt \geq 0$$

to be fulfilled for any $u_* \in U$.

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NECESSARY MINIMUM CONDITIONS IN CALCULUS OF VARIATIONS PROBLEMS IN THE PRESENCE OF VARIOUS DEGENERATIONS

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We consider the following problem:

$$J\left(x\left(\cdot\right)\right) = \int_{t_0}^{t_1} L\left(t, x\left(t\right), \dot{x}\left(t\right)\right) dt \to \min_{x(\cdot)}, \tag{1}$$

$$x\left(t_0\right) = x_0, \;\; x\left(t_1\right) = x_1, \;\; x\left(\cdot\right) \in PC^1\left(I, R^n\right), \tag{2}$$