DISCRETE VOLTERRA OPERATOR AND ITS APPLICATIONS

M.P. Dymkov

Belarus State Economic University, Minsk, Belarus
dymkov_m@bseu.by

In this paper in framework of the uniformed view based on operator approach the main control problems such as stability, stabilizability, controllability, linear-quadratic optimization and feedback control problems are considered for linear discrete Volterra equations. It is shown that Volterra operator properties play a key role to study the main structural characteristics of the considered control system. Using the presentation of this operator in the ring of power series allows us to apply some algebraic methods for research.

Let E be a finite dimensional normed space over the complex field C with norm $|\cdot|_E$, and Z_+ be the set of nonnegative integers. Denote by $s(Z_+, E)$ the linear space of all sequences on E, i.e. the functions $f : Z_+ \to E$. Let $b(Z_+, E)$ be the subspace from $s(Z_+, E)$ of all bounded functions, i.e. the functions $f : Z_+ \to E$ such that $\sup\limits_{k \in Z_+} |f(k)|_E < +\infty$. Now let V be another finite dimensional normed spaces over complex field C, $A_t \in \mathcal{L}(E, E), t \in Z_+, B \in \mathcal{L}(V, E)$, where $\mathcal{L}(E, V)$ denotes the Banach space of all linear operators from E to V.

Define the Volterra operator $\mathcal{V} : s(Z_+, E) \to s(Z_+, E)$ as follows

$$(\mathcal{V}\varphi)(t) = \sum_{i=0}^{t} A_i \varphi(t-i), \ t \in Z_+. \tag{1}$$

Associate with each Volterra operator $\mathcal{V}$ its representation $\mathcal{V}(z)$ in the ring of power series defined by

$$\mathcal{V}(z) = \sum_{i=0}^{\infty} A_i z^i, \quad z \in C. \tag{2}$$

It is obvious that the matrix function $\mathcal{V}(z)$ is a linear map $E \to V$ for each z from the unit disk $\mathcal{D} = \{z \in C : |z| \le 1\}$. Now, suppose that the operators A_i are such that the power series (2) converge in some domain $\mathcal{G}$ containing the unit disk $\mathcal{D}$. Under given assumptions, $\mathcal{V}$ is a linear bounded operator. It can be proved the following prepositions.

Lemma 1. *The Volterra operator $\mathcal{V} : b(Z_+, E) \to b(Z_+, E)$ is i)surjective if, and only if, $rank\mathcal{V}(z) = n$ $(n = dim\ E)$ for all $z \in C, |z| \leq 1$; ii) injective if, and only if, $rank\mathcal{V}(z) = n$ $(n = dim\ E)$ for some $z \in C, |z| \leq 1$.*

The spectrum of the Volterra operator can be given as follows

Lemma 2. *The spectrum $\Sigma(\mathcal{V})$ of the operator V can be evaluated by the formulae*

$$\Sigma(\mathcal{V}) = \bigcup_{|z|\leq 1} \sigma(\mathcal{V}(z)), \tag{3}$$

where $\sigma(\mathcal{V}(z))$ denotes eigenvalues of the matrix $\mathcal{V}(z)$.

Consider the following system of equations

$$\sum_{i=0}^{s} A_i x(s-i) + B_s y = \beta_s \quad \text{for all} \quad s \in Z_+ \tag{4}$$

with respect to unknown $x \in b(Z_+, E)$, $y \in W$ under the given $\beta \in b(Z_+, E)$. This system is equivalent to the following equation in the ring of power series

$$\mathcal{V}(z)x(z) + B(z)y = \beta(z), \quad z \in \mathcal{D}. \tag{5}$$

Using this representation the following result can be proved

Theorem 1. *Equation* (4) *solvable in class $b(Z_+, E)$ for any $\beta \in b(Z_+, V)$ if and only if*

1) $\operatorname{rank}\mathcal{V}(z) = n_2 \quad (n_2 = \dim V) \quad$ for all $\quad |z| = 1$;

2) $\dim\{\mathcal{L}_{z_1}[\mathcal{V}, B] + ... + \mathcal{L}_{z_r}[\mathcal{V}, B]\} = \rho(\mathcal{V})$,

$z_i \in R_V,\ i = 1, 2, ..., r$, where the linear spaces $\mathcal{L}_{z_i}[\mathcal{V}, B]$ are constructed by special procedure with help of matrices $\mathcal{V}(z)$, $B(z)$ and the number $\rho(\mathcal{V})$ is the singularity power of the matrix $\mathcal{V}(z)$ in the unit disk $\mathcal{D}$.

In the paper on the base of the obtained results the main control problems such as stability, stabilizability, controllability, linear-quadratic optimization and feedback control problems for some classes of linear discrete Volterra equations are investigated.

This work was supported in part by the State Programm of Scientific Research (grant 1-30/2021 B).

References

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