Theorem 2. *Under the conditions* ***(A1)*** *–* ***(A4)*** *and* ***(A6)*** *the following statements hold:*

1. *for any* $u \in I_f$ *the solution* $\varphi(t,u,f)$ *of equation (1) belonging to compact global attractor* $\{I_g|\ g \in H(f)\}$ *and it is Bohr/Levitan almost periodic;*
2. *for any* $u \in \mathbb{R}^n \setminus I_f$ *there exists a point* $u_f \in I_f$ *such that*
   (a) *the solution* $\varphi(t,u_f,f)$ *of equation (1) belonging to compact global attractor* $\{I_g|\ g \in H(f)\}$ *and it is Bohr/Levitan almost periodic;*
   (b) *the solution* $\varphi(t,u,f)$ *of equation (1) is asymptotic to solution* $\varphi(t,u_f,f)$*, i.e.,*

$$\lim_{t\to+\infty} \rho(\varphi(t,u,f),\varphi(t,u_f,f)) = 0. \qquad (4)$$

Denote by $\mathfrak{A}$ (respectively, $\mathfrak{B}$) the family of all equations (1) satisfying conditions **(A1)** – **(A5)** (respectively, **(A1)** – **(A4)** and **(A6)**).

Remark. Note that

1. $\mathfrak{B} \subseteq \mathfrak{A}$;
2. the inclusion $\mathfrak{A} \subseteq \mathfrak{B}$ is not true even for scalar ($n = 1$) differential equations (see [1, Ch.XII]).

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TIME STRETCHING IN THE GAME METHOD OF RESOLVING FUNCTIONS

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In the development of the first direct method of L.S. Pontryagin [1], the method of resolving functions was developed [2]. These methods provide a guaranteed result without worrying about optimality that is quite justified from a practical point of view.

The deciding factor in study of dynamic games is availability of information on current state of the process, its prehistory or various-kind discrimination of the players. It was shown that the general quasi-linear game of pursuit with variable delay of information is equivalent to the pursuit game with complete information and somewhat changed dynamics and terminal set [3]. On the one hand, this made it possible to apply classic methods to analyze a wide class of pursuit games with delayed information. On the other hand, this turns out to be useful to tackle game problems of pursuit, for which Pontryagin's condition [1], lying at the heart of the first direct method and reflecting an advantage of the pursuer over the evader in control resources, does not hold. These are the problems of soft meeting (simultaneous coincidence of states and velocities of the players) and problems of pursuit for oscillatory processes [2].

M.S. Nikolskij in [4] provided a deep insight into the condition, resulted in its modification by D. Zonnevend [5], which incorporates certain function later on called the function of time stretching. It was shown that the modified condition is closely related with the passage from original game to the game with special kind information delay [3]. This gave impetus to the development of efficient approach — the principle of time stretching to analyze the games that do not meet Pontryagin's condition. The concept of time stretching function was extended to the function being a sum of piecewise continuous and absolutely continuous functions [6]. This made it feasible to expand the range of problems susceptible to analytical solution, in particular, at the account of oscillatory systems. In the frames of Pontryagin's first direct method, the time stretching principle was applied for solving the problems of soft meeting in various cases of second-order dynamics. Formulas of the function of time stretching in explicit form were obtained for oscillatory processes and in the case of different-kind dynamics of the players [6]. This approach was implemented for analysis of the dynamic games, described by a system of general form, which encompasses a wide range of the functional-differential systems [7]. Also, it was applied to the method of resolving functions and its modifications related with the shift function and the upper and the lower resolving functions. This made it feasible to terminate the game using quasi- and stroboscopic strategies. As an illustration, one problem of approaching two conflict-controlled oscillatory systems was analyzed in detail.

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ERROR BOUNDS REVISITED

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We propose a unifying general framework of quantitative primal and dual sufficient error bound conditions covering linear and nonlinear, local and global settings. We expose the roles of the assumptions involved in the error bound assertions, in particular, on the underlying space: general metric, Banach or Asplund. Employing special collections of slope operators, we introduce a succinct form of sufficient error bound conditions, which allows one to combine in a single statement several different assertions: nonlocal and local primal space conditions in complete metric spaces, and subdifferential conditions in Banach and Asplund spaces. In the nonlinear setting, we cover both the conventional and the 'alternative' error bound conditions.

Dedicated to the memory of Rafail Fedorovich Gabasov.

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