ON THE STRUCTURE OF THE LEVINSON CENTER FOR MONOTONE DISSIPATIVE NON-AUTONOMOUS DYNAMICAL SYSTEMS

D. Cheban

Moldova State University, Chişinău, Republic of Moldova
cheban@usm.md, davidcheban@yahoo.com

This talk is dedicated to the study the structure compact global attractor (Levinson center) of monotone dynamical systems.

Let $W \subseteq \mathbb{R}^n$. Consider the differential equation

$$u' = f(t,u) \quad (f \in C(\mathbb{R} \times W, \mathbb{R}^d)). \tag{1}$$

Along with the equation (1) we consider its H-class, i.e., the family of the equations

$$v' = g(t,v) \quad (g \in H(f) = \overline{\{f_\tau : \tau \in \mathbb{R}\}}), \tag{2}$$

where $f_\tau(t,u) = f(t+\tau,u)$ and by bar is indicated the closure in the compact-open topology.

Below we will use the following conditions.

Condition (A1). The function $f \in C(\mathbb{R} \times W, \mathbb{R}^d)$ is said to be regular if for every equation (2) the conditions of existence, uniqueness and extendability on $\mathbb{R}_+$ are fulfilled.

We will suppose that the function f is regular. Denote by $\varphi(\cdot, v, g)$ the solution of (2) passing through the point $v \in W$ for $t = 0$. Then the mapping $\varphi : \mathbb{R}_+ \times W \times H(f) \to W$ satisfies the following conditions:

1) $\varphi(0, v, g) = v$ for all $v \in W$ and $g \in H(f)$;
2) $\varphi(t, \varphi(\tau, v, g), g_\tau) = \varphi(t+\tau, v, g)$ for each $v \in W$, $g \in H(f)$ and $t, \tau \in \mathbb{R}_+$;
3) $\varphi : \mathbb{R}_+ \times W \times H(f) \to W$ is continuous.

Denote by $Y := H(f)$ and $(Y, \mathbb{R}, \sigma)$ a dynamical system of translations on Y, induced by the dynamical system of translations $(C(\mathbb{R} \times W, \mathbb{R}^d), \mathbb{R}, \sigma)$. The triple $\langle W\varphi, (Y, \mathbb{R}, \sigma)\rangle$ is a cocycle over $(Y, \mathbb{R}_+, \sigma)$ with the fiber $\mathbb{R}^d$. Hence, the equation (1) generates a cocycle $\langle W, \varphi, (Y, \mathbb{R}, \sigma)\rangle$ and the non-autonomous dynamical system $\langle (X, \mathbb{R}_+, \pi), (Y, \mathbb{R}, \sigma), h\rangle$, where $X := W \times Y$, $\pi := (\varphi, \sigma)$ and $h := pr_2 : X \to Y$.

Condition (A2). The function $f \in C(\mathbb{R} \times W, \mathbb{R}^d)$ is Bohr/Levitan almost periodic in $t \in \mathbb{R}$ uniformly in u on every compact subset $K \subset W$.

Condition (A3). Equation (1) is monotone, i.e., the cocycle $\langle W, \varphi, (H(f), \mathbb{R}, \sigma)\rangle$ (or shortly φ) generated by (1) is monotone (this means that, if $u, v \in W$ and $u \leq v$ then $\varphi(t, u, g) \leq \varphi(t, v, g)$ for all $t \geq 0$ and $g \in H(f)$) and $\mathbb{R}^n_+$ is positively invariant with respect to cocycle φ ($\varphi(t, u, g) \in \mathbb{R}^n_+$ for any $(t, u, g) \in \mathbb{R}_+ \times \mathbb{R}^n_+ \times H(f)$).

Condition (A4). Equation (1) with regular right hand side f admits a compact global attractor (Levinson center), i.e., for any $g \in H(f)$ there exists a nonempty compact subset I_g of $\mathbb{R}^n$ such that the family $\{I_g|\ g \in H(f)\}$ possesses the following properties:

1. there exists a nonempty compact subset K of $\mathbb{R}^n$ such that $I_g \subseteq K$ for any $g \in H(f)$ and, consequently, $\mathbf{I} = \bigcup\{I_g|\ g \in H(f)\}$ is pre-compact;
2. $\{I_g|\ g \in H(f)\}$ is invariant, i.e., $\varphi(t, I_g, g) = I_{\sigma(t,g)}$ for any $g \in H(f)$ and $t \geq 0$;
3. $\{I_g|\ g \in H(f)\}$ uniformly attracts every bounded subset of $\mathbb{R}^n$, i.e., for any bounded subset M of $\mathbb{R}^n$ we have

$$\lim_{t \to +\infty} \sup_{g \in H(f)} \beta(\varphi(t, M, g), \mathbf{I}) = 0, \tag{3}$$

   where $\beta(A, B) := \sup_{a \in A} \rho(a, B)$, $\rho(a, B) := \inf_{b \in B} \rho(a, b)$ and ρ is the distance defined by norm $|\cdot|$ on the space $\mathbb{R}^n$.

Condition (A5). For every $g \in H(f)$ and $v_0 \in \mathbb{R}^n_+$ the solution $\varphi(t, v_0, g)$ of equation (2) is positively uniformly stable, i.e., for any positive number ε there exists a positive number $\delta = \delta(\varepsilon, v_0, g)$ ($v \in \mathbb{R}^n_+$) such that $\rho(\varphi(t_0, v, g), \varphi(t_0, v_0, g)) < \delta$ ($t_0 \geq 0$) implies $\rho(\varphi(t, v, g), \varphi(t, v_0, g)) < \varepsilon$ for any $t \geq t_0$.

The main result of this talk we formulate in the following Theorems.

Theorem 1. *Under the conditions **(A1)-(A5)** there exists at least two points $u \in I_f$ such that the solution $\varphi(t, u, f)$ of equation (1) belonging to compact global attractor $\{I_g|\ g \in H(f)\}$ and it is Bohr/Levitan almost periodic.*

Condition (A6). Equation (1) has a strongly monotone first integral, i.e., there exists a function $V \in C^1(\mathbb{R}^n, \mathbb{R})$ such that $\nabla V(x) \gg 0$ (or equivalently, $V'_{x_i}(x) > 0$ for any $i = 1, \dots, n$ and $x \in \mathbb{R}^n$).

Theorem 2. *Under the conditions **(A1)** – **(A4)** and **(A6)** the following statements hold:*

1. *for any $u \in I_f$ the solution $\varphi(t,u,f)$ of equation (1) belonging to compact global attractor $\{I_g|\ g \in H(f)\}$ and it is Bohr/Levitan almost periodic;*
2. *for any $u \in \mathbb{R}^n \setminus I_f$ there exists a point $u_f \in I_f$ such that*
   (a) *the solution $\varphi(t,u_f,f)$ of equation (1) belonging to compact global attractor $\{I_g|\ g \in H(f)\}$ and it is Bohr/Levitan almost periodic;*
   (b) *the solution $\varphi(t,u,f)$ of equation (1) is asymptotic to solution $\varphi(t,u_f,f)$, i.e.,*

$$\lim_{t\to+\infty} \rho(\varphi(t,u,f), \varphi(t,u_f,f)) = 0. \quad (4)$$

Denote by $\mathfrak{A}$ (respectively, $\mathfrak{B}$) the family of all equations (1) satisfying conditions **(A1)** – **(A5)** (respectively, **(A1)** – **(A4)** and **(A6)**).

Remark. Note that

1. $\mathfrak{B} \subseteq \mathfrak{A}$;
2. the inclusion $\mathfrak{A} \subseteq \mathfrak{B}$ is not true even for scalar ($n = 1$) differential equations (see [1, Ch.XII]).

References

1. *Cheban D.N.* Global Attractors of Nonautonomous Dynamical and Control Systems. 2nd Edition. Interdisciplinary Mathematical Sciences. Vol.18. River Edge, NJ: World Scientific, 2015, xxv+589 pp.

TIME STRETCHING IN THE GAME METHOD OF RESOLVING FUNCTIONS

G.TS. Chikrii, A.A. Chikrii

V.M. Glushkov Institute of Cybernetics, National Academy of Sciences of Ukraine, Kyiv, Ukraine

{g.chikrii, ark.chikrii}@gmail.com

In the development of the first direct method of L.S. Pontryagin [1], the method of resolving functions was developed [2]. These methods provide a guaranteed result without worrying about optimality that is quite justified from a practical point of view.