ON DISCRETE APPROXIMATION OF SET-MEMBERSHIP ESTIMATION FOR CONTINUOUS DYNAMICAL SYSTEMS

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Introduction. Set-membership approaches to estimation problems have been studied since long time [1, 2]. More widely applicable and universal guaranteed approach was elaborated in monographs [3, 4]. In this talk, we continue the investigation of discrete approximations of state estimation problems for continuous dynamical systems [5].

1. Problem Formulation. Consider non-observable dynamical system with noisy measurement

$$\dot{x} = f(t,x),\ t \in [0,T],\ y(t) = g(t,x(t)) + w(t),\ x \in \mathbb{R}^n,\ y \in \mathbb{R}^m, \quad (1)$$

where the unknown function $w \in L_2^m[0,T]$ is bounded by the norm: $\|w\|_{L_2^m} \leq 1$. Continuous functions f, g are supposed to have continuous gradients f_x, g_x. The initial state x_0 is also unknown and $x_0 \in X_0$, where X_0 is a bounding set. According to [4] let us introduce the *information set* (IS) $\mathbf{X}(t, X_0, y)$ containing all the states $x(t)$ for which there exists an admissible pair x_0, w that generates this state by virtue of system (1). If $X_0 = \mathbb{R}^n$ the IS is denoted by $\mathbf{X}(t, y)$.

Given $y(\cdot)$, our problem is *to find IS* $\mathbf{X}(t, X_0, y)$ *and elaborate numerical schemes for approximations of IS.*

2. Discrete Approximation Scheme. First of all we give the theorem that helps in finding of IS especially for linear case. Let $\mathcal{X}(t, X_0)$ be the image of X_0 at the time t according to system (1).

Theorem 1. *The IS* $\mathbf{X}(t, X_0, y)$ *is the intersection* $\mathcal{X}(t, X_0) \bigcap \mathbf{X}(t, y)$. *The set* $\mathbf{X}(t, y) = \{x \in \mathbb{R}^n \mid W(t, x, y(\cdot)) \leq 1\}$, *where function* W *is defined by Bellman's equation* $W_t + W_x f(t,x) = \|y(t) - g(t,x)\|^2$ *with initial condition* $W(0, x, y(\cdot)) = 0$.

The proof of the theorem is based on the equality

$$\|w\|^2_{L_2^m[0.t]} = \int_0^t \|y(s) - g(s, x(s))\|^2 ds.$$

Let N be a natural number and $\Delta = T/N$ be a step on the time axis. Consider the Euler approximation of system (1):

$$x_{k+1} = x_k + \Delta f(k\Delta, x_k), \quad k \in 0 : N-1. \tag{2}$$

For system (2), we form recurrently the following sets ($\mathbf{X}_0 = X_0,\ J_0 = 0$):

$$\begin{gathered} \hat{X}_k = \{x \in \mathbf{X}_k \mid J_{k+1}(x) \le 1\}, \quad \mathbf{X}_{k+1} = \hat{X}_k + \Delta f(k\Delta, \hat{X}_k), \\ J_{k+1}(x) = J_k(x) + \int\limits_{k\Delta}^{(k+1)\Delta} \|y(t) - g(t,x)\|^2 dt, \quad k \in 0 : N-1. \end{gathered} \tag{3}$$

Theorem 2. *Let X_0 be a compact set and IS $\mathbf{X}(T, y)$ be bounded. Then the sets $\mathbf{X}_N$ from (3) converge to IS $\mathbf{X}(T, X_0, y)$ in Hausdorff metric as $N \to \infty$.*

Various cases. Examples. If the system (1) is linear, i.e. $f(t,x) = A(t)x$, $g(t,x) = G(t)x$, then IS $\mathbf{X}(T, y)$ is an ellipsoid provided that the system is observable. In this case, it is better to approximate the initial set with an ellipsoid $\{x \mid (x-\bar{x})'P_0(x-\bar{x}) \le 1\} \supset X_0$ and to use equations from [5]. The system of the more complicated form $\dot{x} = f(t,x) + bv(t)$, $y(t) = g(t,x) + cv(t)$, where $\|v\|_{L_2^q} \le 1$, is also taken into consideration. Examples are considered. Among them: an oscillator, double integrator and others.

The work was performed as part of research conducted in the Ural Mathematical Center with the financial support of the Ministry of Education and Science of the Russian Federation.

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