$$+ x_1'(t) + \alpha_1 x_1(t) + \alpha_2 x_2(t) + \alpha_3 x_1^2(t) + \alpha_4 x_1(t)x_2(t) + \alpha_5 x_2^2(t) = f_1(t),$$

$$\int_0^t \int_0^t K_{22}(t-s_1, t-s_2) x_2(s_1) x_2(s_2)\, ds_1\, ds_2 + \int_0^t K_2(t-s_1) x_2(s_1)\, ds_1 +$$

$$+ x_2'(t) + \beta_1 x_1(t) + \beta_2 x_2(t) + \beta_3 x_1^2(t) + \beta_4 x_1(t)x_2(t) + \beta_5 x_2^2(t) = f_2(t), \qquad (1)$$

где f_1, f_2 – обобщенные функции с носителем на замкнутой положительной полуоси, $\alpha_k, \beta_k \in \mathbb{R}$,

$$\int_0^t K_1(t-s) x_1(s)\, ds + x_1'(t) + \alpha_1 x_1(t) + \alpha_2 x_2(t) + \alpha_3 x_1^2(t) + \alpha_4 x_1(t)x_2(t) + \alpha_5 x_2^2(t) = f_1(t),$$

$$\int_0^t K_2(t-s) x_2(s)\, ds + x_2'(t) + \beta_1 x_1(t) + \beta_2 x_2(t) + \beta_3 x_1^2(t) + \beta_4 x_1(t)x_2(t) + \beta_5 x_2^2(t) = f_2(t) \qquad (2)$$

Нахождение компонент асимптотически обратного оператора второй кратности выполняется рекуррентно, и построение компонент сводится к решению систем определенного количества соответствующих линейных уравнений. Такой подход позволяет описать динамическую систему на разных этапах ее состояния, а достоинство заключается в том, что можно применять достаточно простой алгоритм для прямого построения асимптотически обратного оператора.

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ON APPROXIMATE CALCULATION OF MOMENTS OF SDE

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In this report the new method of approximate calculation of moments of the equation (1) is proposed:

$$X_t = X_0 + \int_0^t \alpha(X_{s-})\, ds + \int_0^t \beta(X_{s-})\, dW_s. \qquad (1)$$

Here we suppose, that $X_0 \in \mathbb{R}$, W_t is the Wiener process, $t \in [0,T]$ and the integral on W_s in the right side of the equation (1) is the integral in Ito sense. Also the functions α and β are satisfies to conditions of the existence of a strong solution of the equation (see, e.g. [1, 2]).

The Monte Carlo method is now often used for the calculation of moments of SDE. Usually, this method requires significant computing power every time, then we need to calculate some parameters, e.g. moments, of the equation.

The proposed method is more economical, since the main power are needed only at the preliminary stage of calculating functionals, the values of which are stored and subsequently used for the moments calculation.

The method is based on the use of weak approximations of the SDE's solution (see e.g. [3]). The approximate formula for functionals on a solution of SDE (see [4]) is used for calculation of moments of the solution.

$$\mathbb{E}_{\hat{X}_{(\cdot)}}[f(\hat{X}_0, \hat{X}_{(\cdot)})] \approx J(f,Y) = \frac{1}{2}\sum_{j=1}^{2} A_j \int_0^1 \int_0^1 \int_{-1}^1 f(\hat{X}_0, Y_j(\,\cdot\,, u_1, u_2, v))\, du_1\, du_2\, dv, \qquad (2)$$

where

$$Y_t = X_0 + \alpha\Bigg(X_0 + \alpha\left(X_0 + \beta(X_0,|v|)\mathrm{sign(v)}1_{(|v|,1]}(u_2), u_2\right) a_{j2}1_{(u_2,1]}(u_1) +$$

$$+ \beta\left(X_0 + \alpha(X_0,u_2)a_{j2}1_{(u_2,1]}(|v|), |v|\right)\mathrm{sign(v)}1_{(|v|,1]}(u_1), u_1\Bigg)\rho_{j,1}(t,u_1) +$$

$$+ \alpha\Bigg(X_0 + \alpha\left(X_0 + \beta(X_0,|v|)\mathrm{sign(v)}1_{(|v|,1]}(u_1), u_1\right) a_{j1}1_{(u_1,1]}(u_2) +$$

$$+ \beta\left(X_0 + \alpha(X_0,u_1)a_{j1}1_{(u_1,1]}(|v|), |v|\right)\mathrm{sign(v)}1_{(|v|,1]}(u_2), u_2\Bigg)\rho_{j,2}(t,u_2) +$$

$$+ \beta\Bigg(X_0 + \alpha\left(X_0 + \alpha(X_0,u_2)a_{j2}1_{(u_2,1]}(u_1), u_1\right) a_{j1}1_{(u_1,1]}(|v|) +$$

$$+ \alpha\left(X_0 + \alpha(X_0,u_1)a_{j1}1_{(u_1,1]}(u_2), u_2\right) a_{j2}1_{(u_2,1]}(|v|), |v|\Bigg)\rho(t,v),$$

$$\rho_{jk}(s,u_k) = a_{jk}1_{[u_k,1]}(s), \quad k=1,2, \quad \rho(s,v) = \mathrm{sign(v)}1_{[|v|,1]}(s), \quad A_1 + A_2 = 1,$$

$$a_{11} = \frac{1}{2}\left(1 - \sqrt{-\frac{A_2}{A_1}}\right), \quad a_{12} = \frac{1}{2}\left(1 + \sqrt{-\frac{A_2}{A_1}}\right),$$

$$a_{21} = \frac{1}{2}\left(1 - \sqrt{-\frac{A_1}{A_2}}\right), \quad a_{22} = \frac{1}{2}\left(1 + \sqrt{-\frac{A_1}{A_2}}\right),$$

At the next step estimated values of moments are used for approximating of probability density function of the solution at some fixed moment, which in turn makes it possible to calculate the values used as initial values for the equation on the next time interval.

The report presents the results of a numerical experiment for SDE with linear and nonlinear drift and dispersion parts for the case of value of $T = 1$.

Examples of calculation in the case of

$$\alpha(x) = \cos x, \quad \beta(x) = \cos x, \quad X_0 = 1, \quad \Delta t = 0.1$$

are shown at pictures («blue» line-proposed method, «green» – Monte-Carlo).

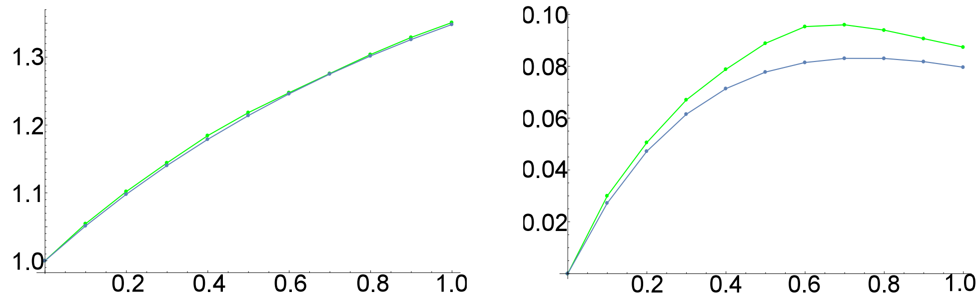

Figure. Mean (at the left) and variance (at the right) of (1).

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