ON EQUATIONS FOR THIN ELASTIC NANOSIZE STRIP TAKING INTO ACCOUNT NONLOCAL EFFECTS IN THICKNESS DIRECTION

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A plane isotropic elastic strip of thickness h and length l with a traction-free faces is considered. It is assumed that $\varepsilon = h/l$ and $\varepsilon_1 = a/h$ are small parameters, where a is the internal characteristic dimension of the nanosize strip in the thickness direction. Vibrations of the plane strip are governed by equations

$$\begin{aligned} &\frac{\partial \sigma_{13}}{\partial z} + \varepsilon \frac{\partial \sigma_{11}}{\partial x} - \rho \frac{\partial^2 u}{\partial t^2} + f_1(x,t) = 0, \\ &\frac{\partial \sigma_{33}}{\partial z} + \varepsilon \frac{\partial \sigma_{13}}{\partial x} - \rho \frac{\partial^2 w}{\partial t^2} + f_3(x,t) = 0, \end{aligned} \tag{1}$$

written in the dimensionless form. Here u, w are the longitudinal and transverse displacements in the x- and z-directions $(0 \le x \le l, 0 \le z \le 1)$ respectively, ρ is the density, f_k are the volume forces, and σ_{ij} are stresses coupled to the macroscopic counterparts $\sigma_{ij}^{(m)}$ by means of relations [1]:

$$\sigma_{ij} - \varepsilon_1^2 \frac{\partial^2 \sigma_{ij}}{\partial z^2} = \sigma_{ij}^{(m)} - \varepsilon_1^2 \xi \frac{\partial^2 \sigma_{ij}^{(m)}}{\partial z^2}, \tag{2}$$

where ξ is the local volume fraction in the Eringen's two-phase nonlocal theory of elasticity.

Assuming that $w \sim \varepsilon^{-4}$, $u \sim \varepsilon^{-3}$, $\sigma_{13} \sim \varepsilon^{-1}$, $\sigma_{33} \sim 1$ [2], and $\varepsilon_1^2 = \varepsilon\kappa$, where $\kappa \sim 1$, all required functions in (1) are expended into series by a small parameter ε. In particular, $w = \varepsilon^{-4} \sum\limits_{k=0}^{\infty} \varepsilon^k W_k$. Considering step-by-step the sequence of arising boundary-value problems, we derive the sequence of partial differential equations for the functions $W_k(x,z,t)$. We detected that the partial differential equation for $W_0(x,t)$ is the classical beam equation, the differential equation for $W_0 + \varepsilon^2 W_2(x,z,t)$, taking into account shears, looks like the Timoshenko–Reissner one [2], while a differential equation for $W_0 + \varepsilon^2 W_2 + \varepsilon^3 W_3(x,z,t)$ is a new equation accounting additionally for the nonlocality effect in the thickness direction.

References

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ДОПУСТИМЫЕ ВОЗМУЩЕНИЯ ОБОБЩЁННОЙ СИСТЕМЫ ЛЭНГФОРДА В ОДНОМ ЧАСТНОМ СЛУЧАЕ

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В [1] изучена обобщённая система Лэнгфорда:

$$\begin{aligned} \dot{x} &= ax + by + xz, \\ \dot{y} &= cx + dy + yz, \\ \dot{z} &= ez - \left(x^2 + y^2 + z^2\right); \quad x, y, z, a, b, c, d, e \in \mathbb{R}, \end{aligned} \tag{1}$$

где a, b, c, d, e — параметры модели.

Если к правой части системы (1) добавить допустимые возмущения, т.е. такие допущения, которые не изменяют отражающей функции системы (см. [2, 3]), то многие качественные свойства решений допустимо возмущённой системы сохранятся. Таким образом, результаты исследований системы (1) можно использовать для изучения допустимо возмущённых систем (см. [4, 5]).