FIRST MIXED PROBLEM FOR GENERAL TELEGRAPH EQUATION WITH VARIABLE COEFFICIENTS ON A SEGMENT

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We derive the global correctness theorem on $\dot{Q}_n =]0, d[\times]0, d_{n+1}[$ to the problem:

$$\mathcal{L}u \equiv u_{tt}(x,t) - a^2(x,t)u_{xx}(x,t) + b(x,t)u_t(x,t) +$$

$$+c(x,t)u_x(x,t) + q(x,t)u(x,t) = f(x,t), (x,t) \in \dot{Q}_n, \quad (1)$$

$$u|_{t=0} = \varphi(x),\ u_t|_{t=0} = \psi(x),\ 0 < x < d, \quad (2)$$

$$u|_{x=0} = \mu_1(t),\ u|_{x=d} = \mu_2(t),\ 0 < t < d_{n+1},\ d_n = (n-1)h^{(2)}[d/2, g_2(0.0)], \quad (3)$$

The characteristic equations $dx - (-1)^i a(x,t)dt = 0$ give the implicit characteristics $g_i(x,t) = C_i$, $i = 1, 2$. If $a(x,t) \geq a_0 > 0$, then they decrease strictly in t at $i = 1$ and increase at $i = 2$ with increasing x. Therefore, the implicit functions $y_i = g_i(x,t)$ have the inverse functions $x = h_i\{y_i, t\}$, $t = h^{(i)}[x, y_i]$. If coefficient $a \in C^2(G_\infty)$, then functions g_i, h_i, $h^{(i)} \in C^2$ with respect to the variables x, t, y_i, $i = 1, 2$ [1].

It was only first proved the existence of a unique and stable classical solution to corresponding auxiliary mixed problem for general telegraph equation on half-line by the Schauder's method of continuation with respect to a parameter and the author's theorems on increasing the smoothness of strong solutions in article [1].

Then without explicitly continuing the problem data f, φ, ψ, μ_1, μ_2 outside a set Q_n we have proved the global correctness theorem to this mixed problem by author novel "method of auxiliary mixed problems for a semi-bounded string (wave equation on a half-line)" [2] and to the global correctness theorem for auxiliary mixed problem on half-line in the work [3]. The statement of mixed problem (1)–(3) and definition of its classical solutions imply the necessity for some smoothness requirements and all matching conditions. To find matching conditions, we differentiate equalities (3) twice in t, and calculate the values of the derivatives of solutions u for $x = 0, t = 0$ and $x = d, t = 0$ using the initial conditions (2) and the equation (1). The additional corresponding integral requirements is deduced by author correction method.

References

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ПОЛНОЕ ПРЕОБРАЗОВАНИЕ РАДОНА–КИПРИЯНОВА. СВЯЗЬ С ПРЕОБРАЗОВАНИЕМ БЕССЕЛЯ–КИПРИЯНОВА–КАТРАХОВА

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В [1] введен следующий интегральный оператор, называемый преобразованием Радона–Киприянова:

$$K_\gamma[f](\xi;p) = \int\limits_{\mathbb{R}_1} \mathcal{P}^\gamma_{ev,x}\,\delta(p - x\xi)f(x)x^\gamma dx, \quad \mathbb{R}_1 = \{-\infty < x < +\infty\},$$