$a(0) = 0$; $\mathbf{K}^\infty \subset \mathbf{K}$ — подмножество функций $a \in \mathbf{K}$ таких, что если $a(r) \to +\infty$, то $r \to +\infty$; $\dot{V}(x) = \liminf\limits_{t\to 0} \dfrac{V(xt) - V(x)}{t}$, если $V(xt) - V(x)$ определено для всех достаточно малых $t > 0$ и предел существует, а также определение асимптотической компактности [2, 3] полудинамической системы в замыкании $\overline{B(M, \Delta)}$.

Теорема. Пусть M – замкнутое инвариантное подмножество X. Предположим, что существуют число $\Delta > 0$, функция $\dot{V} \in \mathbf{C}(B(M,\Delta),\mathbb{R})$, функции $a \in \mathbf{K}^\infty$ и $b \in \mathbf{K}$ такие, что выполняются условия:

1) $V(x) \le a(d(M,x))\ \forall x \in B(M,\Delta)$ и $V(x) = 0\ \forall x \in M$;

2) множество $G = \{x \in B(M,\Delta)\backslash M : V(x) \ge 0\}$ содержит последовательность $(q_n) \subset B(M,\Delta)\backslash M$ такую, что $V(q_n) = 0$ и $d(M,q_n) \to 0$ при $n \to \infty$;

3) $\dot{V}(x) > b(d(M,x))\ \forall x \in B(M,\Delta)\backslash M$.

Тогда множество M неустойчиво.

Если, кроме того, полудинамическая система $(X, \mathbf{R}^+, \pi)$ асимптотически компактна в $\overline{B(M, \Delta)}$, то в области G существует отрицательная полутраектория $\gamma^-(y)$ такая, что:

a) $d(M, \gamma^-(x)) = 0$;

b) $V(yt) \to 0$ при $t \to -\infty$.

При доказательстве используются методы качественной теории устойчивости движения [1] и монографии [4].

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DIFFERENTIAL EQUATIONS IN THE VALUATION OF DERIVATIVES

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The theory of derivatives valuation is based on a continuous stochastic model of changes in asset prices. The basic formula for the dynamics of the share price, on which all further constructions and estimates are based, has the form:

$$dS(t) = S(t)(\mu dt + \sigma dW(t)),$$

$S(t)$ is the stock price at time t, μ is its expected return, σ is the volatility of the stock price, and the element $dW(t)$ containing the randomness factor that affects the asset price is known as the Wiener process or Brownian motion [2].

The value of the option at time t is given by a continuously differentiable function $f(t, S^1, \dots, S^n)$ and is replicated by the value of the portfolio containing the bond and n underlying shares. Replicating strategy $\phi = \left(\phi^1(t), \dots, \phi^n(t)\right)'$ has the form

$$\phi^i(t) = f'_{S^i}, \quad \phi^0(t) = \frac{f - \sum_{i=1}^n f'_{S^i} S^i}{S^0},$$

$\phi^i(t)$ is the number of items of the corresponding underlying asset, and the function f is the function of the option value [1]. Let's denote the «delta matrix» of the portfolio $\Delta(t)$ (delta is an indicator of the sensitivity of the calculated option value to insignificant fluctuations in the price of the underlying asset), where $\Delta_{ij}(t) = f^{(i)}_{S^j}$, consequently, $f^{(i)} = \sum_{j=0}^{n} \Delta_{ij}(t) S^j$. Then when

$$\Delta_{i0} = \phi^0(t)$$

the value of the portfolio has the form

$$X(t) = \left(\varphi^0(t) + \sum_{j=0}^{n} \varphi^i(t)\Delta_{i0}(t)\right) S^0 + \sum_{j=1}^{n} \left(\sum_{i=1}^{n} \varphi^i(t)\Delta_{ij}(t)\right) S^j. \quad (1)$$

$\Delta(t)$ should be regular, since this condition ensures the completeness of the market formed by the bond and n options. Let π_{stock} be the optimal trading strategy of the corresponding problem for a stock portfolio, $X^*(T)$ is an optimal portfolio cash flow at the end of the period. Consequently $\varphi^i_{stock}(t) = \pi^i_{stock}(t) X^*(t)/S^i$, $\varphi^0_{stock}(t) = \left(1 - \sum_{i=1}^{n} \pi^i_{stock}(t)\right) X^*(t)/S^0$. Then the corresponding portfolio value process must satisfy the stochastic differential equation $dX(t) = \left(r\varphi^0_{stock} S^0 + \sum\limits_{i=1}^{n} \mu_i \varphi^i_{stock} S^i\right) dt + \sum\limits_{i,j=1}^{n} \varphi^i_{stock} S^i \sigma_{ij} dW^j$. On the other hand, if there is a trading strategy φ with value $X(t)$ in the option market, then based on (1)

$$dX(t) = \left(r\varphi^0 S^0 + \sum_{i=0}^{n} \varphi^i \left(r\Delta_{i0} S^0 + \sum_{j=1}^{n} \mu_j \Delta_{ij} S^j\right)\right) dt + \sum_{i,j,k=1}^{n} \varphi^i \Delta_{ik} S^k \sigma_{kj} dW^j.$$

By equating the right-hand sides of the resulting equations and comparing the coefficients at W, we obtain the optimal strategy for the option portfolio

$$\varphi^i(t) = (\Delta')^{-1} \varphi^i_{stock}(t), \quad \varphi^0(t) = \frac{X(t) - \sum_{i=1}^{n} \varphi^i_{stock}(t)\, f^{(i)}}{S^0(t)}.$$

As shown by numerous studies, the authors of which tried to find a pattern in the change of financial asset prices, prices change in an unpredictable way. Random price movements indicate that the market works well as an information processing system, it is efficient. The forecast of favorable future behavior of the exchange rate leads to favorable current behavior. Prices rise and fall only in response to new, unpredictable information, because the information that could have been predicted has already been reflected in prices. Therefore, at present, the sections of portfolio theory related to the use of stochastic models undergoing intensive development. These are the methods of the general theory of random processes that are best suited for an adequate description of the evolution of the main (stocks and bonds) and derivatives (forwards, futures, options, etc.) of securities in conditions of uncertainty, and also allow us to introduce dynamics into consideration.

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SOME CASES OF MIXED JOINT DISCRETE UNIVERSALITY FOR CLASS OF ZETA–FUNCTIONS

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The universality property says roughly that any analytic non-vanishing function can be approximated uniformly on any compact subsets of certain vertical strip by shifts of the zeta– or L–functions.