Unlike in [1], we place in Eqs.(1) not only the potential V under the gradient sign, but the quotient V/T, because T is now not constant. We verify that the system (1) obeys the first and the second laws of thermodynamics (the requirement of thermodynamic consistency).

The mean velocity of the Brownian particle is

$$v = \frac{\partial}{\partial t}\int xf(x,t)\mathrm{d}x = \int x\frac{\partial f}{\partial t}\mathrm{d}x = -\int x\frac{\partial j}{\partial x}\mathrm{d}x = -\Big[xj(x,t)\Big]_{-\infty}^{\infty} + \int j(x,t)\mathrm{d}x.$$

We impose the conditions $j(\pm\infty) = 0$ and $f(\pm\infty) = 0$. Thus we have a mixed Dirichlet-Neumann condition. So

$$v = \int j\mathrm{d}x = -\int D\left[\frac{\partial f}{\partial x} + f\frac{\partial}{\partial x}\left(\frac{V}{T}\right)\right].$$

If $D, \partial V/\partial x$ and T are constant, we get Einstein's formula

$$v = -\frac{D}{T}\frac{\partial V}{\partial x} = \frac{D}{T}\mathbf{F}.$$

Then we consider the state of stationary diffusion in one and three dimensions. We show that even in the stationary case it is not Gibbsian.

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MULTI-TERM FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS IN POWER GROWTH FUNCTION SPACES

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Definition 1. *By $BSA_p(\mathbb{R}_+)$, $p \geq 0$, we denote the set of locally integrable functions f on $\mathbb{R}_+$ such that*

$$\sup_{T>0} \frac{1}{(T+1)^p}\int_0^T |f(t)|^2\,dt < \infty. \tag{1}$$

Theorem 1. *A function $F(s)$ is the Laplace transform of $f \in BSA_p(\mathbb{R}_+)$ if and only if $F(s)$ is holomorphic in the right-half plane* $\operatorname{Re} s > 0$, *and*

$$\sup_{x>0}\left(\frac{x}{x+1}\right)^p \int_{-\infty}^{\infty} |F(x+iy)|^2\,dy < \infty. \tag{2}$$

Theorem 2. *Let $k > 0$, $f_0 \in \mathbb{R}$, $g, h \in L^1(\mathbb{R}_+)$, be given, and $\|g\|_1 < k$. Then the multi-term Riemann–Liouville fractional integro-differential equation*

$$D_{0+}^{\alpha_0} f(t) + \sum_{j=1}^{n} a_j D_{0+}^{\alpha_j} f(t) + kf(t) + \int_0^t g(t-\tau)f(\tau)d\tau = h(t), \quad I_{0+}^{1-\alpha_0} f(0+) = f_0, \tag{3}$$

$$\frac{1}{2} < \alpha_0 \leq 1,\ 0 < \alpha_n < \ldots < \alpha_1 < \alpha_0,$$

where $k, a_1, a_2, ..., a_n \in \mathbb{R}_+$, $g, h \in L^1(\mathbb{R}_+)$ are given, and f is the unknown, and D_{0+}^{α} is the Riemann-Liouville fractional derivative

$$D_{0+}^{\alpha} f(t) = \frac{d^n}{dt^n} I_{0+}^{n-\alpha} f(t), \quad I_{0+}^{n-\alpha} f(t) = \int_0^t \frac{(t-\tau)^{n-\alpha-1}}{\Gamma(n-\alpha)} f(\tau)\,d\tau, \quad \alpha < n, \tag{4}$$

has a unique solution f from $BSA_{2\alpha_0-1}(\mathbb{R}_+)$.

Theorem 3. *Let* $g \in L^1(\mathbb{R}_+) \cup L^\infty(\mathbb{R}_+), f \in H_0^m(\Omega)$, $\partial\Omega \in C^m$ *with* $m > \dfrac{3d+3}{2}$, *and* $\|g\|_1 < k\lambda_1$, *where* λ_1 *is the first eigenvalue of the Dirichlet Laplacian on* Ω. *Then the boundary value problem*

$$\begin{cases} D_{0+}^{\alpha} u(x,t) = k\Delta u(x,t) - \int_0^t g(t-\tau)u(x,\tau)d\tau, & (x,t) \in Q = \Omega \times \mathbb{R}^+, \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times \mathbb{R}^+, \\ I_{0+}^{1-\alpha} u(x,0) = f(x), & x \in \Omega, \quad \frac{1}{2} < \alpha \le 1, \end{cases} \tag{5}$$

where $\Omega \subset \mathbb{R}^d$ $(d \ge 1)$ *is a bounded domain with smooth boundary* $\partial\Omega \in C^{[\frac{d}{2}]+1}$, *has a unique classical solution in* $C^2(\overline{\Omega}) \times BSA_1^{\alpha}(\mathbb{R}_+)$.

By $f(t) \in BSA_1^{\alpha}(\mathbb{R}_+)$ we mean both $f(t), D_{0+}^{\alpha} f(t) \in BSA_1(\mathbb{R}_+)$. Similar results are also obtained for the Caputo fractional derivative. This is a joint work with Dinh Thanh Duc and Tran Dinh Phung.

BLOW-UP PROBLEM FOR NONLOCAL NONLINEAR PARABOLIC EQUATION WITH NONLOCAL NONLINEAR BOUNDARY CONDITION

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We consider nonlinear nonlocal parabolic equation

$$u_t = \Delta u + a(x,t)u^r \int_\Omega u^p(y,t)\,dy - b(x,t)u^q,\ x \in \Omega,\ t > 0, \tag{1}$$

with nonlinear nonlocal boundary condition

$$u(x,t) = \int_\Omega k(x,y,t)u^l(y,t)\,dy,\ x \in \partial\Omega,\ t > 0, \tag{2}$$

and initial data

$$u(x,0) = u_0(x),\ x \in \Omega, \tag{3}$$

where r, p, q, l are positive constants, Ω is a bounded domain in $\mathbb{R}^n$ for $n \ge 1$ with smooth boundary $\partial\Omega$. We suppose that $a(x,t),\ b(x,t),\ k(x,y,t)$ and $u_0(x)$ satisfy the following conditions:

$$a(x,t),\ b(x,t) \in C_{loc}^{\alpha}(\overline{\Omega} \times [0,\infty)),\ 0 < \alpha < 1,\ a(x,t) \ge 0,\ b(x,t) \ge 0;$$

$$k(x,y,t) \in C(\partial\Omega \times \overline{\Omega} \times [0,\infty)),\ k(x,y,t) \ge 0;$$

$$u_0(x) \in C(\overline{\Omega}),\ u_0(x) \ge 0,\ x \in \overline{\Omega},\ u_0(x) = \int_\Omega k(x,y,0)u_0^l(y)\,dy,\ x \in \partial\Omega.$$

We prove some global existence and blow-up results for (1) – (3). Criteria on this problem which determine whether the solutions blow up in finite time for large or for all nontrivial initial data are also given. Our global existence and blow-up results depend on the behavior of $a(x,t),\ b(x,t)$ and $k(x,y,t)$ as $t \to \infty$. The initial boundary value problem (1) – (3) with $a(x,t) \equiv 0$ has been considered in [1,2].

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О МНОЖЕСТВЕ УПРАВЛЯЕМОСТИ В ОДНОЙ ЗАДАЧЕ С ФАЗОВЫМ ОГРАНИЧЕНИЕМ

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Рассмотрим управляемый объект, поведение которого описывается дифференциальным уравнением второго порядка

$$\ddot{x} + \omega^2 x = u, \tag{1}$$