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АНАЛИТИЧЕСКАЯ МОДЕЛЬ ДВИЖЕНИЯ СКИПА С УЧЕТОМ НАЛИЧИЯ ГОЛОВНОГО И УРАВНОВЕШИВАЮЩЕГО КАНАТОВ

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Приведена процедура построения аналитической модели, описывающей динамику движения шахтного скипа с учетом наличия головного и уравновешивающего канатов и криволинейности проводников. Выведены формулы для сил, действующих на скип со стороны проводников. Показано, что частоты собственных колебаний скипа зависят от вертикального ускорения и пройденного пути при подъеме сосуда. Построен график (диаграмма) вертикальной скорости скипа, соблюдение которого не вызывает появления значимых вертикальных колебаний скипа из-за упругости канатов. Разработан алгоритм нахождения главного вектора и главного момента системы сил, действующих на скип, по показаниям трех акселерометров, регистрирующих горизонтальные ускорения сосуда при его движении.

Ключевые слова: скип; проводники; динамика шахтного скипа; вертикальные колебания; главный вектор системы сил; главный момент системы сил; горизонтальные ускорения.

ANALYTICAL MODEL OF SKIP MOTION TAKING INTO ACCOUNT INFLUENCE OF HEAD AND BALANCING ROPES

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The article describes an analytical model of mine skip dynamics taking into account the presence of the head and balancing ropes and the existing curvilinearity of the guides. Expressions for the forces acting on the skip from the side of the guides have been constructed. It is shown, that the frequencies of natural vibrations of skip depend on the vertical acceleration and the distance traveled during its lifting. A graph (diagram) of skips vertical speed which observance does not lead to the appearance of skips vertical vibrations due to elasticity of the ropes is developed. An algorithm for finding the forces principal vector and the forces principal moment acting on the skip based on the reading of three accelerometers recording horizontal accelerations of skip during its movement is presented.

Keywords: skip; guides; mine skip dynamics; vertical vibrations; forces principal vector; forces principal moment; horizontal accelerations.

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Introduction

A simplified scheme of shaft lifting rig adopted in the article consists of a friction pulley driven by the lifting machine, two skips moving along the guides and the main and the counterweight rope. Four spring-loaded rollers are installed at the end of the skip frame and they copy profiles of the guides during its movement. Guides are welded box-section beams with local deviations from the vertical in two horizontal directions. Ascent of the loaded skip and the descent of the empty one is carried out by the head rope slung over the friction pulley. Unloading the skip in its upper position and loading the empty skip in its lower position are performed simultaneously after which the pulley changes direction of rotation and the working cycle repeats.

System lifting vessel – reinforcement dynamics investigation during the lifting vessel movement and this system dynamic behaviour diagnostics is an urgent applied problem [1–3]. At the same time this problem is very complex in terms of correct mechanics and mathematical models construction and an adequate model analysis performing.

The research objectives presented in this article were:

- construction of the skip motion analytical model with an acceptable degree of accuracy. That model must allow to determine the force effect on the skip from the side of rigid curved guides or according to their known profile, or according to the readings of accelerometers installed on the skip;
- determination of the skip natural horizontal vibrations frequencies;
- the hoisting machine torque change graph obtaining, which excludes skip vertical vibrations occurrence in an elastic rope.

Skip motion vector equations

The skip motion as a mechanical system can be represented as a rigid body complex motion (*cxyz* coordinate system), consisting of translational motion of $O_1X_1Y_1Z_1$ coordinate system with a given speed $v(t)$ along OZ axis of the stationary system $OXYZ$ ($v(t)$ is the vertical speed of the skip mass center) and of five independent movements of system *cxyz*: two translational movements along O_1X_1 and O_1Y_1 axes and three rotations around axes CX', CY', CZ' of Koenig coordinate system (directions of the corresponding axes $OXYZ$, $O_1X_1Y_1Z_1$ and $CX'Y'Z'$ coincide). The resulting complex movement which occurs relative to the given bulk motion of the system $O_1X_1Y_1Z_1$ is obtained as a result of the superposition of these five motions. Each of these motions and consequently the resulting one arises due to external horizontal influences on skip from the side of the guides and vertical influences at points M_0 and M_5 of the main and balancing ropes suspension (fig. 1).

The skip – guide contact node at contact points M_i, $i = \overline{1, 4}$, is schematically represented as consisting of three independent springs in contact with three guide surfaces through rollers that roll along the conductor during the skip motions (fig. 2).

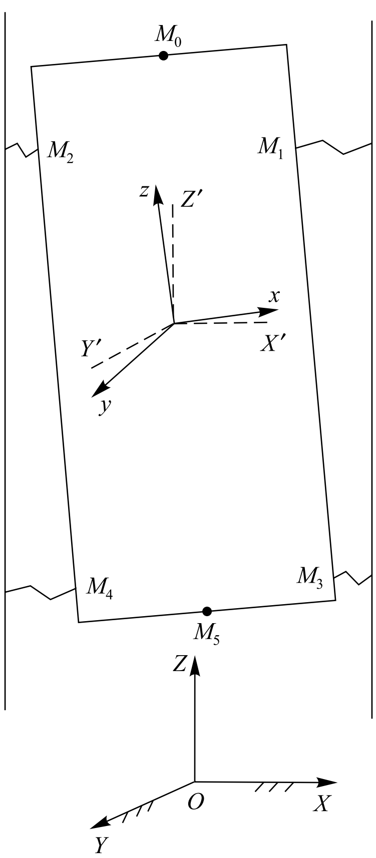

Fig. 1. Coordinate systems and skip with guides and ropes contact points

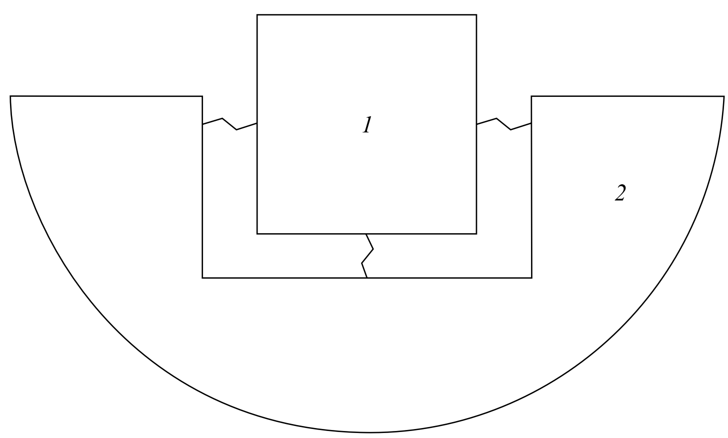

Fig. 2. Skip with guides contact node scheme: *1* – guide; *2* – skip

One can use the following system of equations to describe the skip with mass M relative motion dynamic under the action of the forces $\bar{F}_i$ applied at points M_i, $i = \overline{0, 5}$:

$$M\bar{W}_c = \sum_{i=1}^{4} \bar{F}_i, \tag{1}$$

$$\frac{d\bar{K}'_c}{dt} = \sum_{i=0}^{5} \overline{CM'_i} \times \bar{F}_i. \tag{2}$$

Equations (1), (2) are vector notation of the skip mass center horizontal motion relative to $O_1X_1Y_1Z_1$ coordinate system and the change in angular momentum $\bar{K}'$ relative to the Koenig coordinate system [4].

In equations (1), (2)

$$\bar{F}_i = \left(F_{ix}, F_{iy}, 0\right),\ i = \overline{1, 4},\ \bar{F}_0 = \left(0, 0, F_{0Z}\right),\ \bar{F}_5 = \left(0, 0, F_{5Z}\right),$$

$$\overline{CM'_i} = \overline{CM_i} + \left(\varphi_x, \varphi_y, \varphi_z\right) \times \overline{CM_i},\ i = \overline{0, 5},$$

$$\overline{CM_0} = \left(0, 0, z_0\right),\ \overline{CM_5} = \left(0, 0, z_5\right),\ \overline{CM_1} = \left(x_1, 0, z_1\right),$$

$$\overline{CM_2} = \left(-x_1, 0, z_1\right),\ \overline{CM_3} = \left(x_1, 0, z_3\right),\ \overline{CM_4} = \left(-x_1, 0, z_3\right),$$

where φ_x, φ_y, φ_z are the angles between the corresponding axes of coordinate systems $cxyz$ and $CX'Y'Z'$. Dashes indicate that the vectors belong to the coordinate system $CX'Y'Z'$.

Considering skip as an absolutely rigid body symmetric with respect to planes cxz, cyz, and angles φ_x, φ_y, φ_z small one can replace the moments of inertia of skip I'_x, I'_y, I'_z relative to the axes of system $CX'Y'Z'$ in expression for $\bar{K}'_c$ with the moments of inertia I_x, I_y, I_z relative to the axes of the coordinate system $cxyz$: $\bar{K}'_c = \left(I_x\dot{\varphi}_x, I_y\dot{\varphi}_y, I_z\dot{\varphi}_z\right)$.

Forces acting on a skip during its motion

The spring-loaded rollers of the contact node copy the guide cylindrical surfaces from its three sides during skip motion. The equations of these surfaces in the $OXYZ$ fixed coordinate system can be written for the first guide as

$$X = h_x + f_{1x}(z),\ Y = h_y + f_{1y}(z),\ Y = -h_y + f_{1y}(z)$$

and for the second guide as

$$X = -h_x + f_{2x}(z),\ Y = h_y + f_{2y}(z),\ Y = -h_y + f_{2y}(z).$$

Functions $f_{1x}(z)$, $f_{1y}(z)$, $f_{2x}(z)$, $f_{2y}(z)$ give the deviations algebraic value of the guides points from the corresponding vertical planes $X = \pm h_x$, $Y = \pm h_y$.

The force values F_{ix}, F_{iy}, $i = \overline{1, 4}$, are equal to the corresponding springs compression (tension) to the multiplied by the stiffness coefficient c which is considered the same for all springs since the guides act on the skip through the springs. Small displacements $\Delta\rho_i$, $i = \overline{1, 4}$, of the skip points M_i relative to the coordinate system $O_1X_1Y_1Z_1$ occur to the mass center displacement by a vector $\left(X_c, Y_c, 0\right)$ and three rotations around the mass center by angles φ_x, φ_y, φ_z. Thus

$$\Delta\bar{\rho}_1 = \left(X_c, Y_c, 0\right) + \left(\varphi_x, \varphi_y, \varphi_z\right) \times \left(x_1, 0, z_1\right) = \left(X_c + z_1\varphi_y, Y_c + x_1\varphi_z - z_1\varphi_x, -x_1\varphi_y\right).$$

Similarly, we have

$$\Delta\bar{\rho}_2 = \left(X_c + z_1\varphi_y, Y_c - x_1\varphi_z - z_1\varphi_x, x_1\varphi_y\right),$$

$$\Delta\bar{\rho}_3 = \left(X_c + z_3\varphi_y, Y_c + x_1\varphi_z - z_3\varphi_x, -x_1\varphi_y\right),$$

$$\Delta\bar{\rho}_4 = \left(X_c + z_3\varphi_y, Y_c - x_1\varphi_z - z_3\varphi_x, x_1\varphi_y\right).$$

Taking into account the conductors deviations, we obtain

$$F_{1x} = c\left(f_{1x}(s + h) - X_c - z_1\varphi_y\right),\ F_{2x} = c\left(f_{2x}(s + h) - X_c - z_1\varphi_y\right),$$

$$F_{3x} = c\left(f_{1x}(s) - X_c - z_3\varphi_y\right),\ F_{4x} = c\left(f_{2x}(s) - X_c - z_3\varphi_y\right),$$
$$F_{1y} = 2c\left(f_{1y}(s+h) - Y_c - x_1\varphi_z + z_1\varphi_x\right),\ F_{2y} = 2c\left(f_{2y}(s+h) - Y_c + x_1\varphi_z + z_1\varphi_x\right),$$
$$F_{3y} = 2c\left(f_{1y}(s) - Y_c - x_1\varphi_z + z_3\varphi_x\right),\ F_{4y} = 2c\left(f_{2y}(s) - Y_c + x_1\varphi_z + z_3\varphi_x\right).$$

Here $s(t) = \int_0^t v(\tau)d\tau$ is the distance passed by the skip during by the time t after the its movement start, $h = z_1 - z_3$ is the distance between points M_1 and M_3.

A part of the balancing rope from the suspension point to the loop in the sump, which length is approximately $s(t)$, moves translationally with a speed $v(t)$ together with the skip (fig. 3). Thus

$$F_{0Z} = \left(M + \rho_b s(t)\right)\left(\dot{v}(t) + g\right),\ F_{5Z} = -\rho_b s(t)\left(\dot{v}(t) + g\right),$$

where ρ_b is the rope density and g is the gravity acceleration.

Skip motion scalar equations and their solution

We project the vector equation (1) on the horizontal axes of $OXYZ$ coordinate system and equation (2) on the Koenig coordinate system axis to obtain a system of skip motion scalar equations

$$M\ddot{X}_c = c\left(F_x - 4X_c - 2z_{13}\varphi_y\right), \tag{3}$$

$$M\ddot{Y}_c = 2c\left(F_y - 4Y_c + 2z_{13}\varphi_x\right), \tag{4}$$

$$I_x\ddot{\varphi}_x = 2c\left(\Phi_x + 2z_{13}Y_c - 2\left(z_1^2 + z_3^2\right)\varphi_x\right) - F_{05}\varphi_x, \tag{5}$$

$$I_y\ddot{\varphi}_y = c\left(\Phi_y - 2z_{13}X_c - 2\left(z_1^2 + z_3^2\right)\varphi_y\right) - F_{05}\varphi_y, \tag{6}$$

$$I_z\ddot{\varphi}_z = 2cx_1\left(\Phi_z - 4x_1\varphi_z\right). \tag{7}$$

$$F_x = f_{1x}(s+h) + f_{2x}(s+h) + f_{1x}(s) + f_{2x}(s),$$

$$F_y = f_{1y}(s+h) + f_{2y}(s+h) + f_{1y}(s) + f_{2y}(s),$$

$$\Phi_x = -z_1\left(f_{1y}(s+h) + f_{2y}(s+h)\right) - z_3\left(f_{1y}(s) + f_{2y}(s)\right),$$

$$\Phi_y = z_1\left(f_{1x}(s+h) + f_{2x}(s+h)\right) + z_3\left(f_{1x}(s) + f_{2x}(s)\right),$$

$$\Phi_z = f_{1y}(s+h) + f_{1y}(s) - f_{2y}(s+h) - f_{2y}(s),$$

$$F_{05} = z_0F_{0Z} + z_5F_{5Z},\ z_{13} = z_1 + z_3.$$

Note that equations (3)–(7) approximate such as deriving them, discarded terms of higher order terms to the values X_c, Y_c, φ_x, φ_y, φ_z. In addition note that equations (5), (6) have variable coefficients due to the function $F_{05}(t)$ presence. Analysis of this function form shows that its change during the skip movement is much slower than changing the desired system functions (3)–(7).

An approximate solution of equations with slowly varying coefficients is usually found by the asymptotic averaging method [5]. Following this method in our case an approximate solution can be obtained by considering the variable coefficients «frozen», i. e. constant when performing the solution. The «frozen» coefficients «unfreeze» and their dependence on time is restored after the system analytical solution obtaining.

The system (3)–(7) solution with «frozen» coefficients is easily found by using the integral Laplace transform [6] and has the form

$$\varphi_z(t) = \sqrt{\frac{c}{2I_z}} \cdot \int_0^t \Phi_z(\tau)\sin\omega_1(t-\tau)d\tau,\ \omega_1 = 2x_1\sqrt{\frac{2c}{I_z}},$$

$$
\begin{aligned}
X_c(t) &= \int_0^t A_1(t-\tau)F_x(\tau)\,d\tau - \int_0^t A_2(t-\tau)\Phi_y(\tau)\,d\tau,\\
\varphi_y(t) &= \int_0^t A_3(t-\tau)\Phi_y(\tau)\,d\tau - \int_0^t A_2(t-\tau)F_x(\tau)\,d\tau,\\
Y_c(t) &= \int_0^t B_1(t-\tau)F_y(\tau)\,d\tau + \int_0^t B_2(t-\tau)\Phi_x(\tau)\,d\tau,\\
\varphi_x(t) &= \int_0^t B_3(t-\tau)\Phi_x(\tau)\,d\tau + \int_0^t B_2(t-\tau)F_y(\tau)\,d\tau;
\end{aligned}
\tag{8}
$$

$$A_i(t) = a_{i1}\sin\omega_2 t + a_{i2}\sin\omega_3 t,\ i = \overline{1,\,3},$$

$$a_{11} = \frac{c\left(2c\left(z_1^2 + z_3^2\right) - I_y\omega_2^2 + F_{05}\right)}{a_5},$$

$$a_{12} = \frac{c\left(2c\left(z_1^2 + z_3^2\right) - I_y\omega_3^2 + F_{05}\right)}{a_6},$$

$$a_5 = \omega_2\left(a_2 - 2a_1\omega_2^2\right),\ a_6 = \omega_3\left(a_2 - 2a_1\omega_3^2\right),$$

$$a_1 = MI_y,\ a_2 = 2Mc\left(z_1^2 + z_3^2\right) + 4cI_y + MF_{05},$$

$$a_3 = 4c^2\left(z_1 - z_3\right)^2 + 4cF_{05},\ \omega_2 = \sqrt{\frac{a_2 - a_4}{2a_1}},\ \omega_3 = \sqrt{\frac{a_2 + a_4}{2a_1}},$$

$$a_4 = \sqrt{a_2^2 - 4a_1a_3},\ a_{21} = \frac{2c^2z_{13}}{a_5},\ a_{22} = \frac{2c^2z_{13}}{a_6},$$

$$a_{31} = c\frac{4c - M\omega_2^2}{a_5},\ a_{32} = c\frac{4c - M\omega_3^2}{a_6};$$

$$B_i(t) = b_{i1}\sin\omega_4 t + b_{i2}\sin\omega_5 t,\ i = \overline{1,\,3},$$

$$b_{11} = 2c\frac{4c\left(z_1^2 + z_3^2\right) - I_x\omega_4^2 + F_{05}}{b_5},\ b_{12} = 2c\frac{4c\left(z_1^2 + z_3^2\right) - I_x\omega_5^2 + F_{05}}{b_6},$$

$$b_1 = MI_x,\ b_2 = 4Mc\left(z_1^2 + z_3^2\right) + 8cI_x + MF_{05},\ b_3 = 16c^2\left(z_1 - z_3\right)^2 + 8cF_{05},$$

$$b_4 = \sqrt{b_2^2 - 4b_1b_3},\ \omega_4 = \sqrt{\frac{b_2 - b_4}{2b_1}},\ \omega_5 = \sqrt{\frac{b_2 + b_4}{2b_1}},$$

$$b_5 = \omega_4\left(b_2 - 2b_1\omega_4^2\right),\ b_6 = \omega_5\left(b_2 - 2b_1\omega_5^2\right),$$

$$b_{21} = \frac{8c^2z_{13}}{b_5},\ b_{22} = \frac{8c^2z_{13}}{b_6},\ b_{31} = 2c\frac{8c - M\omega_4^2}{b_5},\ b_{32} = 2c\frac{8c - M\omega_5^2}{b_6}.$$

The formulas obtained allow us to find the force effect to the skip at points M_i, $i = \overline{0, 5}$, according to the guides known profiles, the acceleration $\dot{v}(t)$ and the path $s(t)$. Note that only ω_1 of skip natural vibrations five frequencies ω_i, $i = \overline{1, 5}$, has a constant value. The remaining frequencies depend on the functions $\dot{v}(t)$, $s(t)$, which in turn are determined by the moment $M_{\text{cr}}(t)$ applied on the mine hoisting machine drum.

The relationship between speed v and torque M_{cr}

An approximate relationship between the quantities $v(s)$ and $M_{\text{cr}}(s)$ considering the head and the balancing ropes inextensible can be obtained from the theorem on the change in kinetic energy [4] according to which the kinetic energy differential of the mine hoisting mechanism is equal to the elementary work sum of the gravity forces and the torque $M_{\text{cr}}(s)$ (see fig. 3):

$$\frac{1}{2} M_{\text{eff}}\, d\left(v^2(s)\right) = \left(\frac{M_{\text{cr}}(s)}{r} + \left(m - M + l\rho_{\text{b}} - l\rho_{\text{h}}\right) g - 2g\left(\rho_{\text{b}} - \rho_{\text{h}}\right) s \right) ds. \quad (9)$$

Here $M_{\text{eff}} = m + M + \left(\rho_{\text{b}} + \rho_{\text{h}}\right) l + \frac{I}{r^2}$, m is the empty skip mass; ρ_{b}, ρ_{h} are the balancing and the head ropes linear densities, respectively; I, r are the moment of inertia and drum radius; l is the the ropes length; $0 \le s(t) \le l$.

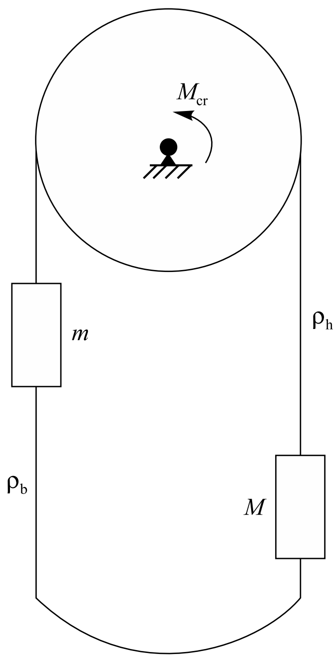

Fig. 3. Mine hoist scheme

From equality (9) it follows that the loaded skip will begin to rise under the condition

$$M_{\text{cr}}(0) > rg\left(M - m + l\rho_{\text{h}} - l\rho_{\text{b}}\right).$$

After dividing equality (9) by dt we obtain an equation

$$M_{\text{eff}}\ddot{s}(t) = \frac{M_{\text{cr}}(t)}{r} + \left(m - M + l\rho_{\text{b}} - l\rho_{\text{h}}\right) g - 2g\left(\rho_{\text{b}} - \rho_{\text{h}}\right) s(t). \quad (10)$$

Assuming that the loaded skip lifting takes place under the constant torque M_{cr} action from a rest state at $s = 0$ and with a stop at the upper point $s = l$ then from equation (9) we obtain that this is possible only at

$$M_{\text{cr}} = (M - m) gr$$

and

$$v^2(s) = \frac{2gs\left(\rho_{\text{b}} - \rho_{\text{h}}\right)(l - s)}{M_{\text{eff}}}. \quad (11)$$

From (11) it follows that condition $\rho_{\text{b}} > \rho_{\text{h}}$ must be satisfied and the maximum speed value is reached when $s = \frac{l}{2}$:

$$v_{\max} = l\sqrt{\frac{g\left(\rho_{\text{b}} - \rho_{\text{h}}\right)}{2M_{\text{eff}}}}.$$

Solving equation (10) at $M_{\text{cr}} = (M - m) gr$ taking into account $s(0) = 0$, $\dot{s}(0) = 0$ we obtain

$$s(t) = \frac{l}{2}\left(1 - \cos\left(\sqrt{\frac{2g\left(\rho_{\text{b}} - \rho_{\text{h}}\right)}{M_{\text{eff}}}} \cdot t\right)\right). \quad (12)$$

From (12) we receive the skip motion time t_0 to a complete stop:

$$t_0 = \pi\sqrt{\frac{M_{\text{eff}}}{2g\left(\rho_{\text{b}} - \rho_{\text{h}}\right)}}.$$

In general case $M_{\text{cr}}(t)$ is obtained from equation (10) according to the given motion law $s(t)$ and is processed by the mine hoisting machine digital control system. Therefore, it is necessary to ensure the second derivative $\ddot{s}(t)$ continuity on the interval $[0, t_0]$ and its equality to zero at the segment extreme points to obtain a continuous change in $M_{\text{cr}}(t)$.

A smooth function $s(t)$ constructing example

Let us construct function $s(t)$ with five sections of its second derivative linear variation:

$$\ddot{s}(t) = 4v_{\max}\begin{cases} \dfrac{t}{t_1^2},\ 0 \le t \le \dfrac{t_1}{2}, \\ \dfrac{t_1 - t}{t_1^2},\ \dfrac{t_1}{2} \le t \le t_1, \\ 0,\ t_1 \le t \le t_0 - t_2, \\ \dfrac{t_0 - t_2 - t}{t_2^2},\ t_0 - t_2 \le t \le t_0 - \dfrac{t_2}{2}, \\ \dfrac{t - t_0}{t_2^2},\ t_0 - \dfrac{t_2}{2} \le t \le t_0. \end{cases}$$

Here $v_{\max}$ is the maximum motion speed; $t_1 = \dfrac{2v_{\max}}{w_1}$, $t_2 = \dfrac{2v_{\max}}{w_2}$ are the time of acceleration and deceleration, t_0 is the motion time; w_1, w_2 are the largest acceleration modules during acceleration and deceleration.

Integration gives the speed $v(t)$ and path $s(t)$:

$$v(t) = v_{\max}\begin{cases} \dfrac{2t^2}{t_1^2},\ 0 \le t \le \dfrac{t_1}{2}, \\ 1 - \dfrac{2(t_1 - t)^2}{t_1^2},\ \dfrac{t_1}{2} \le t \le t_1, \\ 1,\ t_1 \le t \le t_0 - t_2, \\ 1 - \dfrac{2(t_0 - t_2 - t)^2}{t_2^2},\ t_0 - t_2 \le t \le t_0 - \dfrac{t_2}{2}, \\ \dfrac{2(t - t_0)^2}{t_2^2},\ t_0 - \dfrac{t_2}{2} \le t \le t_0. \end{cases}$$

$$s(t) = v_{\max}\begin{cases} \dfrac{2t^3}{3t_1^2},\ 0 \le t \le \dfrac{t_1}{2}, \\ t + \dfrac{2(t_1 - t)^3}{3t_1^2} - \dfrac{t_1}{2},\ \dfrac{t_1}{2} \le t \le t_1, \\ t - \dfrac{t_1}{2},\ t_1 \le t \le t_0 - t_2, \\ t + \dfrac{2(t_0 - t_2 - t)^3}{3t_2^2} - \dfrac{t_1}{2},\ t_0 - t_2 \le t \le t_0 - \dfrac{t_2}{2}, \\ \dfrac{2(t - t_0)^3}{3t_2^2} - \dfrac{t_2}{2} + t_0 - \dfrac{t_1}{2},\ t_0 - \dfrac{t_2}{2} \le t \le t_0. \end{cases}$$

If l is the distance of the skip lift then $s(t_0) = l$ and the motion time is $t_0 = \dfrac{l}{v_{\max}} + \dfrac{t_1 + t_2}{2}$.

The principal vector and the principal moment of the forces acting on the skip

If twice differentiate function (8) we obtain the principal vector $\bar{F}_c$ and the principal moment $\bar{M}_c$ of the forces acting on a skip in its relative motion:

$$\bar{F}_c = \left(M\ddot{X}_c,\ M\ddot{Y}_c,\ 0\right),\ \ \bar{M}_c = \left(I_x\ddot{\varphi}_x,\ I_y\ddot{\varphi}_y,\ I_z\ddot{\varphi}_z\right). \tag{13}$$

In this case the guides profiles and all skip mechanical characteristics must be known. Also the components of the mass center acceleration $\bar{W}_c = \left(\ddot{X}_c,\ \ddot{Y}_c,\ 0\right)$ and angular acceleration $\bar{\varepsilon} = \left(\ddot{\varphi}_x,\ \ddot{\varphi}_y,\ \ddot{\varphi}_z\right)$ in equalities (13) can be expressed in terms of the accelerations horizontal components $\bar{W}_i$, $i = \overline{6,\ 8}$, of any three skip points M_i, $i = \overline{6,\ 8}$, that do not lie on one straight line and accelerations $\bar{W}_i$ can be obtained from the readings of the accelerometers in points M_i. Indeed if we take as a pole for example a point M_6 then taking into account the first quantities order we obtain [4]

$$\bar{W}_c = \bar{W}_6 + \bar{\varepsilon} \times \overline{M_6C},$$

$$\bar{\varepsilon} \times \overline{M_6M_7} = \bar{W}_7 - \bar{W}_6, \tag{14}$$

$$\bar{\varepsilon} \times \overline{M_6M_8} = \bar{W}_8 - \bar{W}_6. \tag{15}$$

Vector equalities (14), (15) give the following four scalar equalities when projected onto horizontal axes:

$$\begin{aligned}
&\ddot{\varphi}_y\left(z_7 - z_6\right) - \ddot{\varphi}_z\left(y_7 - y_6\right) = W_{7x} - W_{6x},\\
&\ddot{\varphi}_z\left(x_7 - x_6\right) - \ddot{\varphi}_x\left(z_7 - z_6\right) = W_{7y} - W_{6y},\\
&\ddot{\varphi}_y\left(z_8 - z_6\right) - \ddot{\varphi}_z\left(y_8 - y_6\right) = W_{8x} - W_{6x},\\
&\ddot{\varphi}_z\left(x_8 - x_6\right) - \ddot{\varphi}_x\left(z_8 - z_6\right) = W_{8y} - W_{6y}.
\end{aligned} \tag{16}$$

One can compose a uniquely solvable system for finding $\ddot{\varphi}_x$, $\ddot{\varphi}_y$, $\ddot{\varphi}_z$ from these equalities. In particular, from the first three equations (16) we obtain

$$\ddot{\varphi}_x = \frac{\Delta_x}{\Delta},\ \ddot{\varphi}_y = \frac{\Delta_y}{\Delta},\ \ddot{\varphi}_z = \frac{\Delta_z}{\Delta},$$

$$\Delta = \left(z_7 - z_6\right)\left(\left(y_7 - y_6\right)\left(z_8 - z_6\right) - \left(y_8 - y_6\right)\left(z_7 - z_6\right)\right),$$

$$\Delta_x = -\left(W_{7x} - W_{6x}\right)\left(z_8 - z_6\right)\left(x_7 - x_6\right) + \left(W_{8x} - W_{6x}\right)\left(z_7 - z_6\right) \times$$

$$\times \left(x_7 - x_6\right) + \left(W_{7y} - W_{6y}\right)\left(\left(z_7 - z_6\right)\left(y_8 - y_6\right) - \left(z_8 - z_6\right)\left(y_7 - y_6\right)\right),$$

$$\Delta_y = \left(z_7 - z_6\right)\left(\left(W_{8x} - W_{6x}\right)\left(y_7 - y_6\right) - \left(W_{7x} - W_{6x}\right)\left(y_8 - y_6\right)\right),$$

$$\Delta_z = \left(z_7 - z_6\right)\left(\left(W_{8x} - W_{6x}\right)\left(z_7 - z_6\right) - \left(W_{7x} - W_{6x}\right)\left(z_8 - z_6\right)\right).$$

Coordinates of points M_i, $i = \overline{6,\ 8}$, must not vanish the determinant Δ.

Conclusion

An approximate analytical model for the mine skip motion has been developed. It is important that this model takes into account the head and balance ropes influence as well as the curvature of the guides.

Expressions that determine the force interaction of skip with guides during the lifting vessel motion are obtained. The expressions obtained make it possible to determine the principal vector and the principal moment of the forces acting on the skip using data coming from three accelerometers installed on the skip fixing the horizontal accelerations values.

An analysis of the skip natural vibrations frequencies has been carried out. Expressions that determine the dependence of the natural vibrations frequencies on the vertical acceleration and the path covered by the skip are given. A diagram of the speed, that does not provoke the vertical vibrations occurrence of skip on the elastic rope is proposed.

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