Секция 2. Геометрия и топология

Homogeneous spaces with symmetries[1]

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Homogeneous spaces are an important object of the modern differential geometry. In particular, the investigation of the homogeneous (pseudo)riemannian manifolds with some restrictions on isometry group of these manifolds or homogeneous spaces with symmetries is actual. We have obtained the following results in this direction.

1. Conformally flat polyhedral metrics of bounded curvature were researched. The theorems about monotonicity and extrapolation for such classes metrics were proved.

2. Conformally semi-flat homogeneous Riemannian spaces were investigated. A classification of the conformally semi-flat 4-dimensional Lie algebras was given. Some new examples of the conformally semi-flat but non conformally flat Riemannian manifolds were constructed.

3. It is proved that the Riemannian manifold with the trivial integer part into Coulkarni-Nomidzu decomposition of the curvature tensor is Einstein manifold. As a result we proved that homogeneous Riemannian manifold of such type is an, isometric to the direct Riemannian product of the Euclidean space and flat torus.

4. In addition to the well known results on triple Lie-Einstein algebras the structure of the triple Lie-Einstein algebras of F-homogeneous spaces with Einstein-Killing metric was researched, new examples of Einstein manifolds were constructed.

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5. A classification of the 3-dimensional and 4-dimensional Lie groups with left invariant (pseudo)Riemannian metric and almost harmonic Schouten-Weyl and Weyl tensors was given. The structure of the homogeneous F-spaces with harmonic Weyl (Schouten-Weyl) tensor was investigated.

6. In addition to the classical results of J.Milnor full classification of possible signature of the one-dimensional curvature operator on 3-dimensional Lie groups with left invariant Riemannian metric was obtained. Series theorems, about 3-dimensional Lie groups with left invariant Riemannian metric of sign-defined Ricci and one dimensional curvature were proved.

7. It was proved that all of the base canonical f–structures on naturally reductive k–symmetric spaces ($k \geq 3$) are nearly Kähler f–structures [1]. Besides, a criterion for a canonical f–structure being the sum or the difference of the two base f–structures to be nearly Kähler was also given. We describe those of base f–structures, which are also Hermitian f–structures, and give a criterion for the remaining base structures to be Hermitian [4]. These are the results of the general character and imply some of the previous ones related to homogeneous k–symmetric spaces of orders $k = 4$ and $k = 5$. Specifically, the particular case $k = 3$ leads to some classical results by N.A. Stepanov, J.A. Wolf, A. Gray, V.F. Kirichenko and others, obtained in the 1960-1980s. As an example, we apply these general results for the canonical f–structures on homogeneous 6–symmetric spaces.

8. Invariant distributions on naturally reductive k–symmetric spaces ($k\geq3$) were considered. We indicate those of the base invariant distributions, which are determined by the canonical almost product structures on homogeneous k-symmetric spaces and generate totally geodesic foliations on these spaces. For particular cases $k = 5$ and $k = 7$ all the canonical distributions were investigated in detail (the cases $k = 4$ and $k = 6$ were studied before by direct methods). It provides new invariant examples of Riemannian almost product structures in the sense of A.M. Naveira's classification.

We have used essentially ideas and methods of papers [1]—[6] under the proofs of general theorems.

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Canonical structures and distributions on spaces with symmetries of order k[1]

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Introduction. Idea of symmetry is very important and fruitful in natural sciences, specifically, in mathematics. In this respect, theory of symmetric spaces plays a remarkable role in many branches of mathematics. More general, among homogeneous manifolds of Lie groups there exists a wide and

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