



# The possibility to measure the magnetic moments of short-lived particles (charm and beauty baryons) at LHC and FCC energies using the phenomenon of spin rotation in crystals



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## ABSTRACT

The use of spin rotation effect in bent crystals for measuring the magnetic moment of short-lived particles in the range of LHC and FCC energies is considered. It is shown that the estimated number of produced baryons that are captured into a bent crystal grows as  $\sim \gamma^{3/2}$  with increasing particle energy. Hence it may be concluded that the experimental measurement of magnetic moments of short-lived particles using the spin rotation effect is feasible at LHC and higher energies (for LHC energies, e.g., the running time required for measuring the magnetic moment of  $\Lambda_c^+$  is  $2 \div 16$  hours).

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## 1. Introduction

The magnetic moment, an important characteristic of elementary particles, still remains unmeasured for many of them (e.g., charm and beauty baryons, and  $\tau$ -leptons). This is because for particles with a short lifetime  $\tau$  ( $\tau = 2 \cdot 10^{-13}$  s for  $\Lambda_c^+$ ,  $\tau = 3.5 \cdot 10^{-13}$  s for  $\Xi_c^+$ , or  $\tau = 2.9 \cdot 10^{-13}$  s for  $\tau$ -leptons), the decay length  $l \sim 3 \div 4$  cm if the energy acquired through the production reaction equals 1 TeV. For this reason, the anomalous magnetic moments of short-lived particles cannot be measured with conventional methods.

The existence of the spin rotation phenomenon for high-energy particles moving in bent crystals in the channeling regime was first established in [1]. The spin rotation angle is determined by the anomalous magnetic moment  $\mu'$ . According to [1], the spin rotation angle can be, e.g., for baryons, as high as several radians, allowing us to hope that the spin rotation effect can be used for measuring  $\mu'$ .

The idea advanced in [1] was experimentally verified and confirmed in Fermilab for  $\Sigma^+$  hyperons with a momentum of 375 GeV/c [2–4]. A detailed analysis of the experiment [2–4], given in [5–7], proves the feasibility of using the spin rotation effect in bent crystals to measure the anomalous magnetic moment of

short-lived charm baryons  $\Lambda_c^+$  and  $\Xi_c^+$  in the momentum range of hundreds of GeV/c. A thorough consideration was given in [5] to charm baryons  $\Lambda_c^+$  produced by a beam of protons whose momentum in a tungsten target was 800 GeV/c. The characteristic momentum of the produced  $\Lambda_c^+$  was  $200 \div 300$  GeV/c (characteristic Lorentz factor  $\gamma \simeq 153$ ). For this reason, it seems pertinent to return to the question of using the spin rotation effect in bent crystals for measuring the magnetic moment of short-lived particles in the light of the increase of proton energy at LHC up to 7 TeV (and more at future FCC; see also the discussion of opportunities of spin physics studies using LHC multi-TeV beams [8]).

At first glance, it may seem that the number of particles captured into channeling regime in a bent crystal reduces as the particle energy is increased, because the Lindhard angle  $\vartheta_L$  decreases ( $\vartheta_L \sim \frac{1}{\sqrt{\gamma}}$ , with  $\gamma$  being the particle Lorentz factor). As a consequence, the usable signal count rate in the detector is reduced, worsening the conditions for using the spin rotation effect in measuring the anomalous magnetic moment of short-lived particles.

Nevertheless, this paper shows that the increase in proton energies at LHC and future FCC facilities provides the conditions under which the number of hyperons captured into channeling regime in a bent crystal will increase in proportion to  $\gamma^{3/2}$ . Hence, the conditions under which the spin rotation effect can be used for measuring the anomalous magnetic moments of short-lived particles will improve appreciably.

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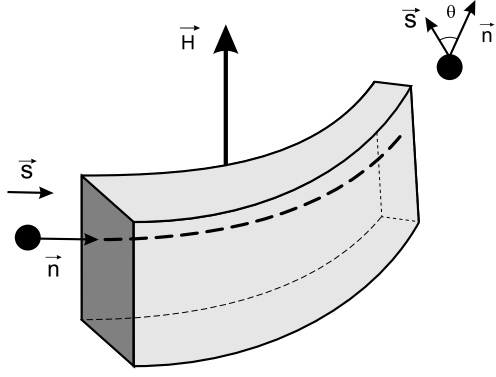


Fig. 1. Spin rotation in a bent crystal.

## 2. Spin rotation of relativistic particles passing through a bent crystal and measurements of the magnetic moment of short-lived particles

Let a high-energy particle be incident onto a bent crystal. As early as in 1976, E. Tsyganov [9] demonstrated that high-energy particles in a bent crystal may move in the channeling regime, thus moving along a curved path and deviating from the initial direction. At present, the experiments on channeling in bent crystals are performed at the world's largest accelerators and a great variety of crystal optical elements are used to manipulate the beams of high-energy particles [10–15].

In a bent crystal, a particle moves along a curved path under the action of the electric field  $\vec{E}$  induced by the crystallographic plane. In the instantaneous rest frame of the particle, due to relativistic effects, the electric field  $\vec{E}$  produces a magnetic field  $\vec{H}$  that acts on the particle magnetic moment, causing spin rotation in this field (Fig. 1) [1].

In the high energy range of interest, the particle wavelength is much less than the distance between the crystallographic planes. Consequently, particle motion in bent crystals is quasi-classical. The quasi-classical character of particle motion allows using the Bargmann–Michel–Telegdi (BMT) equations [16] to describe the evolution of particle spin in crystals [1].

If a crystal is nonmagnetic, the BMT equation for the spin polarization vector  $\vec{\zeta} = \vec{s}/s$  can be written in the form [1,17,18]

$$\frac{d\vec{\zeta}}{dt} = \frac{2\mu'}{\hbar} [\vec{E}(\vec{\zeta}\vec{n}) - \vec{n}(\vec{\zeta}\vec{E})], \quad (1)$$

where  $\vec{E}$  is the electric field at the particle location and  $\vec{n} = \vec{v}/c$  (where  $\vec{v}$  is the particle velocity). Let us recall here that vector  $\vec{\zeta}$  characterizes the polarization of the particle in its “instantaneous” rest frame. Strictly speaking, however, the equations of spin motion in crystals, according to [19], should include the additional terms that take into account the spatial inhomogeneity inherent in the electric fields in crystals. These terms were first obtained by R.H. Good [20]. According to [19], the corrections related to the field inhomogeneity can be rather large for electrons and muons, but insignificant for baryons, so we shall not analyze them here.

Thus, according to Eq. (1), the spin precession frequency  $\omega$  for a particle moving in a bent crystal is

$$\omega = \frac{2\mu'E}{\hbar}. \quad (2)$$

Let us note here that this expression for the frequency is immediately derivable if we consider what field acts on the spin in the particle's instantaneous rest frame. Really, as a result of relativistic transformations, the transverse to particle velocity electric field

$E$  generates in the particle rest frame a magnetic field  $H = \gamma E$  orthogonal to both  $E$  and particle velocity. The spin precession frequency associated with the anomalous magnetic moment in the particle rest frame is (see also [21,22])

$$\omega' = \frac{2\mu'H}{\hbar} = \frac{2\mu'\gamma E}{\hbar}. \quad (3)$$

The spin precession frequency in the laboratory frame is

$$\omega = \frac{\omega'}{\gamma} = \frac{2\mu'E}{\hbar}, \quad (4)$$

which coincides with Eq. (2).

It is noteworthy that with particle's anomalous magnetic moment  $\mu'$  equal to zero the particle spin precession frequency would equal the orbital rotation frequency of the momentum, and the particle spin direction would follow that of the momentum. If  $\mu' \neq 0$ , the angle between the polarization and momentum direction vectors changes. The angle  $\vartheta_s$  of particle spin rotation relative to the particle momentum direction is  $\vartheta_s = \omega T = \omega \frac{L}{c}$ , where the time  $T = \frac{L}{c}$ , with  $L$  being the path length traveled by the channeled particle in the bent crystal. From this follows that the rotation angle per unit path length

$$\vartheta_{s1} = \frac{\omega}{c} = \frac{2\mu'E}{\hbar c}. \quad (5)$$

Let us also remind that the magnetic moment of the particle with spin  $S$  is related to the gyromagnetic (Landé) factor  $g$  (see, e.g., [16]) as

$$\mu = \frac{e\hbar}{2mc} gS, \quad (6)$$

where  $m$  is the particle mass. For  $S = \frac{1}{2}$ , we can write

$$\mu = \frac{e\hbar}{2mc} + \frac{e\hbar}{2mc} \frac{g-2}{2} = \mu_B + \mu', \quad \mu' = \frac{g-2}{2} \mu_B, \quad (7)$$

where  $\mu_B$  is the Bohr magneton. Consequently, we have

$$\vartheta_s = \frac{2\mu'E}{\hbar c} L = \frac{g-2}{2} \frac{eEL}{mc^2}. \quad (8)$$

Here  $eE$  is the force responsible for rotating the particle momentum that should be equal to the centrifugal force  $f_c = \frac{m\gamma c^2}{R}$ , i.e.,  $eE = \frac{m\gamma c^2}{R}$ , where  $R$  is the curvature radius of the channel. From this we have

$$\vartheta_s = \frac{g-2}{2} \gamma \frac{L}{R}. \quad (9)$$

We shall take into account that  $\frac{L}{R}$  is equivalent to the momentum's rotation angle  $\vartheta_p$ . Hence,

$$\vartheta_s = \frac{g-2}{2} \gamma \vartheta_p. \quad (10)$$

Equation (10) was derived by V. Lyuboshits using the BMT equation [23].

It follows from Eq. (9) that the spin rotation angle per unit path length is

$$\vartheta_{s1} = \frac{g-2}{2} \frac{\gamma}{R}. \quad (11)$$

On the other hand, the equality  $eE = \frac{mc^2\gamma}{R}$  yields  $R = \frac{mc^2\gamma}{eE}$ .

The quantity  $|eE| = U'$ . Here  $U' = \frac{dU}{d\rho}$ ,  $U$  is the particle's potential energy in the channel, and  $\rho$  is the transverse distance to the crystallographic plane [10]. As a consequence, we have

$$R = \frac{mc^2}{U'} \gamma. \quad (12)$$

The minimum value of the curvature radius (the critical radius)  $R_{cr}$  at which particle motion in a bent channel is still possible for certain  $\gamma$  is achieved at maximum  $U'_{max}$ :

$$R_{cr} = \frac{mc^2}{U'_{max}} \gamma.$$

Consequently, the maximum rotation angle per unit length is

$$\vartheta_{s1}^{max} = \frac{g-2}{2} \frac{\gamma}{R_{cr}} = \frac{g-2}{2} \frac{U'_{max}}{mc^2}. \quad (13)$$

As is seen, the maximum spin rotation angle  $\vartheta_{s1}^{max}$  is energy-independent. However, at  $R = R_{cr}$ , the capturing acceptance (the fraction of particles captured into channeling regime) vanishes [10]. For certain  $R > R_{cr}$ , the spin rotation angle can be written in the form

$$\vartheta_{s1} = \vartheta_{s1}^{max} \frac{R_{cr}}{R} = \frac{g-2}{2} \frac{U'_{max}}{mc^2} \frac{R_{cr}}{R}. \quad (14)$$

As a result, we can see that for the given ratio  $\frac{R_{cr}}{R}$ , the angle  $\vartheta_{s1}$  is energy-independent, too.

The dechanneling length increases as the energy is increased [10]. This enables us to increase the rotation angle by increasing the length of the bent crystal channel

$$\vartheta_s = \vartheta_{s1} L, \quad L \leq L_D,$$

where  $L_D$  is the dechanneling length. For protons channeled in Si and having energies greater than  $150 \div 200$  GeV, the length  $L_D > 10$  cm.

According to [10], channeling of protons in 15-cm-long Si crystals is currently available.

As reported in [10], for Si,  $U' = 5$  GeV cm<sup>-1</sup>, and hence the rotation angle per 1 cm path length in a Si crystal is

$$\vartheta_{s1} = \frac{g-2}{2} \frac{5 \text{ GeV cm}^{-1}}{mc^2} \frac{R_{cr}}{R}. \quad (15)$$

According to the estimates [5], for a particle with a mass  $\Lambda_c^+ \simeq 2.29$  GeV, the  $g$ -factor ranges within  $1.36 \div 2.45$ . From this we get the minimum value of the ratio  $\frac{g-2}{2} \simeq 0.32$ .

Taking  $\frac{R_{cr}}{R} = \frac{1}{3}$  (in which case the dechanneling length is maximum [10]), we obtain that for  $\Lambda_c^+$ , the rotation angle per unit length in a bent silicon crystal equals

$$\vartheta_{s1} \simeq 0.46 \text{ rad.}$$

For the relation  $\frac{R_{cr}}{R} = \frac{1}{10}$ , we have  $\vartheta_{s1} \simeq 0.1$  rad. As a result, for  $L = 10$  cm, we have  $\vartheta_{s10} \simeq 1$  rad, for  $L = 15$  cm, the spin rotation angle  $\vartheta_{s15} \simeq 1.5$  rad.

Thus, if the ratio  $\frac{R_{cr}}{R}$  remains constant, the value of the spin rotation angle appears to be constant and rather large in a wide range of energies. The decrease in the Lindhard angle  $\vartheta_L \sim \frac{1}{\sqrt{\gamma}}$  with increasing energy seemingly worsens the observation conditions. However, as the energy is increased, the angular width  $\delta\vartheta$  in which the produced particles move is  $\delta\vartheta \sim \frac{1}{\gamma}$ , and hence the fraction of particles captured into the channel is increased with energy as  $\frac{\vartheta_L}{\delta\vartheta} \sim \sqrt{\gamma}$ . According to [24,25], as the energy is increased, the probability of  $\Lambda_c^+$  production also increases in proportion to  $\gamma$ . For this reason, the increase in energy leads to an increasing number of usable events  $N \sim \gamma^{3/2}$ , i.e., to an increasing number of deflected  $\Lambda_c^+$  entering the detector. For example, if the energy of  $\Lambda_c^+$  is increased from 300 GeV to 3 TeV, i.e., tenfold, then the number of usable events  $N$ , which is increased as  $\gamma^{3/2}$ , increases by

more than a factor of 30. As a result, the time required to observe the effect is reduced. According to [5], for a proton beam with the intensity  $10^9 \text{ s}^{-1}$  and momentum 800 GeV/c, producing  $\Lambda_c^+ \sim 300$  GeV/c, the running time required for measuring the magnetic moment of  $\Lambda_c^+$  is  $10 \div 80$  hours. In the case considered here, this time is reduced in proportion to  $\sqrt{N}$ , i.e., by more than a factor of 5.

The conclusion drawn here about the possibility to reduce the running time  $T_{run}$  is based on the extrapolation of the estimate given in [5] to the 7 TeV region. The experimental data on the production cross section of  $\Lambda_c^+$  hyperons in  $pp$  collisions at 7 TeV, recently reported by LHCb collaboration [26–28], appeared to be less than the predicted value by a factor of two (about 230  $\mu\text{b}$  instead of the predicted 540  $\mu\text{b}$ ). At the same values of other parameters that we used earlier, this leads to the increase of  $T_{run}$  by a factor of  $\sqrt{2}$ . But we shall bear in mind that there is another influencing parameter—the thickness of the hyperon-producing target. The hyperon yield is increased with the target thickness. In the presented estimate a 5 mm thick tungsten target was used, as in [5]. The increase of the target thickness, e.g., up to 10 mm, reduces the running time by factor of  $\sqrt{2}$  that eliminates  $T_{run}$  increase due to cross section reduction. Therefore we get the same estimate of  $T_{run}$ .

As a result, such experiments should be quite acceptable at LHC and FCC facilities.

### 3. Conclusion

In this paper, we have demonstrated the potential of spin rotation effect in bent crystals for measuring the magnetic moment of positively-charged short-lived particles in the range of LHC and FCC energies. The estimates show that despite the decrease in the Lindhard angle with the increase in the particle energy, the number of produced baryons that are captured into a bent crystal grows as  $\sim \gamma^{3/2}$ . Hence, the experimental measurement of magnetic moments of short-lived particles using the spin rotation effect seems quite feasible at LHC and higher energies.

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