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Yahor Dydyshka, Vitaly Yermolchyk

National Scientific and Educational Center of Particles and High Energy Physics BSU, Pervomayskaya str. 18, 220088 Minsk, Belarus

E-mail: `Yahor.Dydyshka@cern.ch,vitaly.yermolchyk@cern.ch`

Abstract. We present the Monte Carlo event generator `LePaProGen` for lepton pair production at hadron colliders. `LePaProGen` focuses on the description of higher-order electroweak radiative corrections. The generator is implementing a new algorithm for the selection of the optimal variables for phase space parametrization.

1. Introduction

The processes of single electroweak boson production are very significant for physical studies at LHC. There are many groups working on them to provide adequate theoretical predictions. `FEWZ` [1], `POWHEG` [2–4], `SANC` [5], `HORACE` [6] et al. programs has been developed for this purpose. At energies 1-5 TeV charged and neutral current Drell-Yan processes are background to the processes of new physics. To make distinguish between different models it is important to have both fast and accurate tool for Monte-Carlo simulation. Designed for this purpose code `READY` [7] based on Sudakov and collinear logarithms approximation working only at extra large dilepton invariant mass. It is convenient to have a tool which works equally good at all kinematic regions and nevertheless is very fast. For this reason we introduce our event generator `LePaProGen`.

2. LePaProGen

LePaProGen (Lepton Pair Production Generator) is a Monte-Carlo generator for the neutral and charged current Drell-Yan process. It is intended for fast and accurate generation of events and uses independent generation algorithm. Now it implements one loop radiative corrections with the exact hard QED Bremsstrahlung contribution. But the higher order QCD or electroweak corrections could be easily added by design. LePaProGen is written in C++ using modular architecture. To overcome the C++ templates limitations we use a Python module which calculates all the constants and preprocesses the input settings. A glue layer between `Python` and C++ parts of the program is `Mako` template library which performs optimized code generation for the process chosen by user. LePaProGen can be used as a `Pythia8` plug-in. It has all needed interfaces for convenient work: it generates events in the Les Houches Accord (LHA) format, the `HepMC` output format; it uses the `LHAPDF` interface for parton density functions. It supports a variety of renormalization schemes and performs the `POWHEG`-like matching.

In the following sections we will describe the main algorithms implemented in LePaProGen in detail.

3. Generation of phase space points

First of all we start with phase space points generation. All the variables can be divided into two groups: the "slow" and the "fast". The "slow" variables can be generated uniformly but the "fast" ones require importance sampling. In tree level amplitude all "fast" variables are due to propagators. In order to generate a point in phase space we need to parametrize it by the invariant variables, which appear in the propagators. It can be done by effectively replacing all the propagators by δ-functions

$$\int dR_n \left(\frac{1}{p^2 - m^2}... \right) = \int ds' \frac{1}{s' - m^2} \left[\int dR_n \delta(p^2 - s')... \right]. \tag{1}$$

Such integrals appear in generalized unitarity methods and can be expressed by Feynman graphs (Fig. 1) with the help of the optical theorem. In fact all intermediate and final particles in such
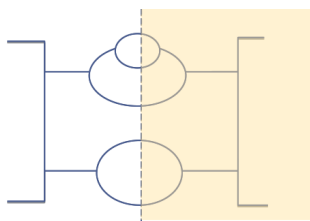

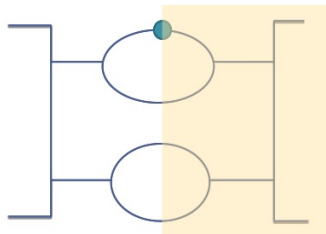**Figure 1.** The Feynman graph for integral.

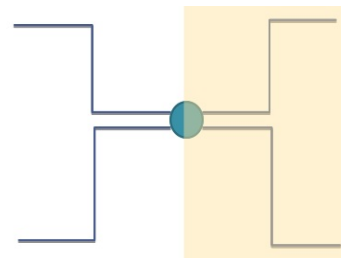**Figure 2.** The contraction of the first loop.

Figure 3. The result of the final contraction.

graph are on-shell. To calculate the integral we subsequently contract one loop subdiagrams starting with ones containing lesser number of propagators (Fig. 2-3). As a result we obtain a Jacobian determinant of transformation. This algorithm is a generalization of the recurrence relation for phase space volume from Byckling and Kajantie [8].

4. Particles Momenta Reconstruction

The cross-section for the process $a + b \to 1 + \ldots + n$ is the following phase space integral:

$$\sigma_n = \frac{(2\pi)^4}{2\lambda^{1/2}(s, m_a^2, m_b^2)} \frac{1}{(2\pi)^{3n}} \int \prod_{i=1}^n \frac{d^3 p_i}{2E_i} |M|^2 \delta(p_a + p_b - \sum_i p_i). \tag{2}$$

For now lets formulate the problem of phase space reconstruction.

- We select some set of invariant valiables sutable for generation.
- Then we sample these variables according to some importance density.
- We must evaluate the matrix element at the selected point of phase space. If the matrix element is in the invariant form then we need only the full set of scalar products $p_i \cdot p_j$.
- If the matrix element is expressed in terms of noninvariant components of vectors in some reference frame, then we must express them in our invariant set of variables. This task we call "reconstruction".

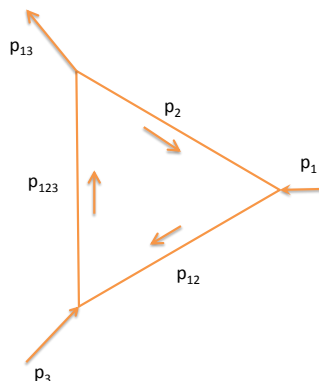Consider reconstruction in the case of 3-propagators phase space integral.

Figure 4. Momentum flow diagram.

4.1. Three-Point Reconstruction

This case corresponds to the interference between the decay and the t-channel scattering, or to the subdiagram of interference between two 3-particle decay channels. Our reconstruction algorithm is universal and works for both interpretations. Differences appear only in the integration limits.

As input parameters we have two 4-momenta. These are the momenta of the particles scattered in t-channel p_1 and p_{13} (Fig. 4). Let the momenta after scattering be p_2 and p_{12}. The masses of the decay products and the intermediate t-channel propagators are known (m_1, m_{12} and m_{123}). We use only the invariant squared masses $s_1 = p_1^2$, $s_{12} = p_{12}^2$ and $s_{123} = p_{123}^2$ because a contracted one-loop subdiagram can appear at the place of a particle. It is evident, that due to the 4-momentum conservation it is enough to reconstruct only the 4-momentum of one of the particles.

The first necessary condition for the decay is time-likeness of the 2-plane spanned by external momenta:

$$\pi_{13} = p_1 \wedge p_3 = p_1 \wedge p_{13}, \qquad\qquad \|\pi_{13}\|^2 > 0. \qquad (3)$$

This condition guarantees finiteness of the phase space volume and is weaker then the corresponding one from [8]. Consequently our approach is more general and can be applicable in cases when relations from [8] does not work.

The scalar products are fixed by the masses of decay products:

$$p_1 \cdot p_2 = \frac{s_{12} - s_1 - s_2}{2}, \qquad\qquad p_2 \cdot p_{13} = \frac{s_{123} - s_2 - s_{13}}{2}. \qquad (4)$$

The next step is to construct reciprocal basis:

$$\mathfrak{p}_1 = p_{13}\pi_{13}^{-1}, \qquad\qquad \mathfrak{p}_{13} = -p_1\pi_{13}^{-1}. \qquad (5)$$

Using this basis we can reconstruct the longitudinal component of p_2 and norm of the transverse component:

$$p_{2L} = (p_1 \cdot p_2)\mathfrak{p}_1 + (p_2 \cdot p_{13})\mathfrak{p}_{13}, \qquad\qquad p_{2T}^2 = s_2 - p_{2L}^2. \qquad (6)$$

Here is the place to apply the second necessary condition on the input parameters: it is evident that the longitudinal part is time-like ($p_{2L}^2 > 0$) so the transverse part should be space-like:

$$p_{2T}^2 < 0. \qquad (7)$$

The direction of the transverse part of p_2 cannot be determined from provided data and should be generated randomly and uniformly. Lets generate a unit 2-vector n uniformly on circle.

In order to use this as the direction of p_{2T} we should Lorentz-transform it to the plane orthogonal to the all external momenta. We are going to construct a Lorentz transformation which maps $\pi_{13}/\|\pi_{13}\|$ to the bivector $\gamma_0 \wedge \gamma_3$. In Clifford algebra this transformation is determined by the rotor R:

$$R = \sqrt{S}, \qquad S = \mathrm{sgn}\big(\langle \pi_{13}\gamma_{03}\rangle\big)\pi_{13}\gamma_{03}, \qquad \gamma_{03} = \gamma_0\gamma_3 = \gamma_0 \wedge \gamma_3. \qquad (8)$$

$$\sqrt{S} \propto \big(\|S\| + S\big)\Big(\langle S\rangle_+ + \sqrt{\langle S\rangle_+\langle S\rangle_-}\Big)\langle S\rangle_-,$$
$$\langle S\rangle_\pm = \|S\| + \langle S\rangle \pm \langle S\rangle_4. \qquad (9)$$

$$p_{2T} = \sqrt{-p_{2T}^2}\,RnR^{-1}, \qquad n = n_x\gamma_1 + n_y\gamma_2. \qquad (10)$$

The above mentioned procedure is correct because Lorentz-transformations preserve orthogonality. Moreover, this 4-rotation has minimum norm in parametric space.

The Jacobian determinant we need is

$$J_3 = \frac{\pi}{2(2\pi)^6\sqrt{\lambda(s_1,s_3,s_{13})}}. \qquad (11)$$

5. The Amplitude

In this section we turn our focus on the hard Bremsstrahlung amplitudes. They are generated using helicity amplitude technique by our MetaAmp package. We use the Kleiss-Stirling convention [9]. Lets choose two basis vectors k_0 and k_1, which satisfy relations:

$$k_0^2 = 0, \qquad k_0 \cdot k_1 = 0, \qquad k_1^2 = -1. \qquad (12)$$

So then we construct basis spinors $u_+(k_0)$ and $u_-(k_0)$, such that

$$u_\lambda(k_0)\bar{u}_\lambda(k_0) = \omega_\lambda \hat{k}_0, \qquad u_\lambda(k_0) = \lambda\hat{k}_1 u_{-\lambda}(k_0), \qquad \omega_\lambda = \frac{1 + \lambda\gamma_5}{2}. \qquad (13)$$

If we take $k_0 = \{1/2,0,0,1/2\}$ and $k_1 = \{0,1,0,0\}$ than in the Weyl representation (up to an arbitrary phase) the basis spinors look like $u_+(k_0) = \{1,0,0,0\}^T$ and $u_-(k_0) = \{0,0,0,-1\}^T$.

Now we can express a spinor for arbitrary external particle with momentum p:

$$u_\lambda(p) = \frac{\hat{p} \pm m}{\sqrt{2p \cdot k_0}}u_{-\lambda}(k_0). \qquad (14)$$

To generate amplitudes MetaAmp uses Fierz identities and Dirac matrix algebra. In that way we reduce the task to calculation of the following expressions:

$$\langle pq\ldots k\rangle = \bar{u}_+(k_0)\hat{p}\hat{q}\ldots\hat{k}\omega_+ u_+(k_0) = \mathrm{Tr}[\hat{p}\hat{q}\ldots\hat{k}\omega_+\hat{k}_0],$$
$$\langle pq\ldots k] = \bar{u}_+(k_0)\hat{p}\hat{q}\ldots\hat{k}\omega_- u_-(k_0) = \mathrm{Tr}[\hat{p}\hat{q}\ldots\hat{k}\omega_-\hat{k}_0\hat{k}_1],$$
$$[pq\ldots k\rangle = \bar{u}_-(k_0)\hat{p}\hat{q}\ldots\hat{k}\omega_+ u_+(k_0) = \mathrm{Tr}[\hat{p}\hat{q}\ldots\hat{k}\omega_+\hat{k}_1\hat{k}_0],$$
$$[pq\ldots k] = \bar{u}_-(k_0)\hat{p}\hat{q}\ldots\hat{k}\omega_- u_-(k_0) = \mathrm{Tr}[\hat{p}\hat{q}\ldots\hat{k}\omega_-\hat{k}_0]. \qquad (15)$$

Normalization factors from external wave-functions are factorized out from the whole amplitude so square roots are not presented. We have chosen $k_0 = \{1/2, 0, 0, 1/2\}$ and $k_1 = \{0, 1, 0, 0\}$, so operations (15) are reduced to taking the component of the resulting matrix making numerical calculation of generated amplitudes really fast.

This is an example of the single bremsstrahlung amplitudes generated by MetaAmp:

$$A^{\gamma}_{+--+-} = 4e^3 \delta^{i_1}_{i_2} \left[Q(l)D_{RR}(p^2_{12}) \left(\frac{1}{p^2_{45}} [p_2 p_3\rangle \langle p_4 p_5\rangle \langle p_1 p_{45}] - \frac{1}{p^2_{35}} [p_3] \langle p_1 p_4] [p_2 p_3 p_5] \right) \right.$$

$$\left. + Q(q)D_{RR}(p^2_{34}) \left(\frac{1}{p^2_{25}} [p_2] \langle p_1 p_4] [p_3 p_2 p_5] - \frac{1}{p^2_{15}} [p_2 p_3\rangle \langle p_1 p_5\rangle \langle p_4 p_{15}] \right) \right], \qquad (16)$$

where $D_{LR}(s) = Q(q)Q(\mu) + g_L(q)g_R(l)\frac{s}{s - M_Z^2 + i\Gamma_Z M_Z}$; $Q(q)(Q(\mu))$ – charge of quark (muon); $g_L(q)$, $g_R(l)$ – weak current coupling constants; i_1, i_2 – color indices.

6. Plots
In order to demonstrate work of our generator we present some plots Fig. 5-7 for the neutral-current Drell-Yan process generated using LePaProGen.
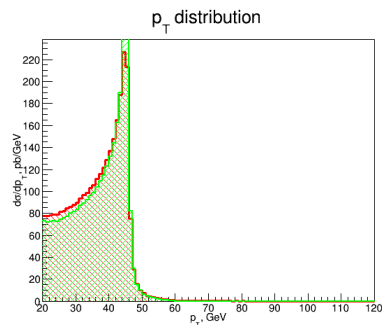

Figure 5. The transverse momentum distribution of the muon. Born – green, Born + EW –guaranty finteness red.
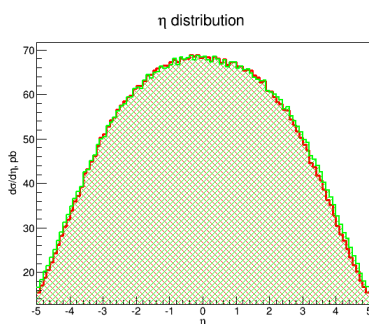
Figure 6. The pseudorapidity distribution of the muon. Born – green, Born + EW – red.
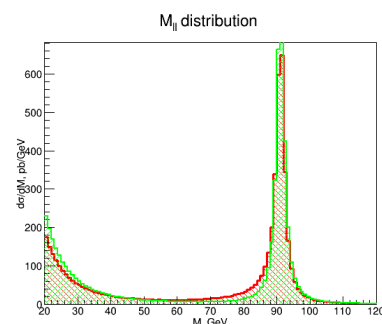
Figure 7. The muon pair invariant mass distribution. Born – green, Born + EW – red.

7. Conclusions
In this paper a new generator `LePaProGen` for Drell-Yan processes is presented. A new algorithm for the selection of the optimal variables for phase space parametrization is implemented. The resulting efficiency is under investigation. A comparison with existing Monte-Carlo algorithms, like `Vegas` and `FOAM`[10], will be performed. A fast modification of helicity amplitude method is used. The detailed numerical comparison between the predictions of `LePaProGen` and other MC event generators for charged and neutral current Drell-Yan processes will be fulfilled in the near future.

Acknowledgments

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