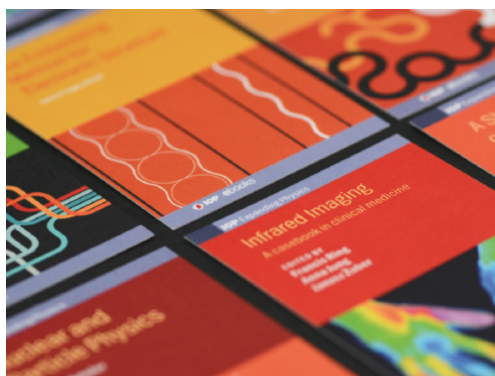


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Non-isothermal motion of viscous fluid layer on outer surface of horizontal rotating cylinder

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Abstract. We study the non-isothermal plane motion of viscous fluid on the outer surface of rotating with constant angular velocity horizontally placed cylinder in the fields of surface tension, gravity and inertia. The problem was solved by direct method and evolution of the free surface was investigated taking into account varying temperature of fluid.

1. Introduction

Today we observe the development of new promising technologies, which are based on the flow of layer of melts on rotating surfaces. Thus it's important to study the motion, instability and decomposition of layers of fluid with variable viscosity, depending on temperature. The non-isothermal plane flow on the outer surface of rotating with constant angular velocity cylinder is investigated. The impact of inertia, surface tension and gravity is taken into account. Such flows occur in formation of mineral and metal fibers from the melts by centrifugal casting and coating application on rotating surfaces.

The solution of the isothermal problem on the outer cylindrical surface in the approximation of a thin film was studied in [1-3]. The motion of a not necessary thin layer of a viscous fluid on the outer surface of a rotating cylinder taking into account inertial forces was explored in [4, 5]. The results of experimental studies of the flow were described in [1, 6]. In [7], the three-dimensional flow of a thin layer was investigated for a slow rotation of the cylinder without taking into account the inertial forces.

2. Model

We consider non-isothermal plane motion of not necessary thin viscous layer on outer surface of cylinder that rotates with constant angular velocity ω_0 . Such flows are used in coating and film applications on rotating surfaces. We use moving coordinate system O, η, φ, τ , that rigidly connected with surface of rotating cylinder, and instead of transversal velocity component w we use relative angular velocity $\omega(\eta, \varphi, \tau) = w\eta^{-1} - 1$, that describes deviation of angular velocity from the velocity when the cylinder and viscous layer move as a whole. The motion of fluid is described by Navier-Stokes equations with variable viscosity, energy equation, continuity equation and free surface equation. These equations are nondimensionalized by cylinder radius R_0 and velocity $\omega_0 R_0$. The dimensionless temperature is determined by $T = (T_f - T_e)T_e^{-1}$, where T_f is dimensional temperature of the fluid and T_e is dimensional temperature of the environment and cylinder. We have:

$$v_\tau + vv_\eta + \omega v_\varphi - (\omega + 1)^2 \eta = -p_\eta + k(T)Re_0^{-1}(\Delta v - 2\eta^{-1}\omega_\varphi - \eta^{-2}v)$$



$$+(dk/dT) Re_0^{-1} (2T_\eta v_\eta + T_\phi (\omega_\eta + \eta^{-2} v_\phi)) - Fr^{-1} \sin(\phi + \tau); \quad (1)$$

$$\omega_\tau + v\omega_\eta + \omega\omega_\phi + 2\eta^{-1}v(\omega + 1) = -\eta^{-2}p_\phi + k(T)Re_0^{-1}(\Delta\omega + 2\eta^{-1}\omega_\eta + 2\eta^{-3}v_\phi) \\ + (dk/dT) Re_0^{-1} (T_\eta \eta^{-2}(v_\phi + \eta^2\omega_\eta) + 2T_\phi \eta^{-3}(v + \eta\omega_\phi)) - Fr^{-1}\eta^{-1} \cos(\phi + \tau); \quad (2)$$

$$T_\tau + vT_\eta + \omega T_\phi = Pe^{-1}(T_{\eta\eta} + \eta^{-1}T_\eta + \eta^{-2}T_{\phi\phi}) + \Phi; \quad (3)$$

$$(\eta v)_\eta + (\eta\omega)_\phi = 0; \quad (4)$$

$$\eta = h(\phi, \tau), \quad h_\tau + \omega h_\phi = v. \quad (5)$$

We require boundary condition of adhesion, the absence of viscous interaction with environment, jump of normal stresses on the free surface caused by the surface tension, third kind boundary conditions for the temperature on the cylinder surface and free surface, and the initial conditions:

$$\eta = 1, \quad v = 0, \quad \omega = 0; \quad (6)$$

$$\eta = h(\phi, \tau), \quad (1 - \eta^{-2}h_\phi^2)(\eta\omega_\eta + \eta^{-1}v_\phi) + 2\eta^{-1}h_\phi(v_\eta - \eta^{-1}v - \omega_\phi) = 0; \quad (7)$$

$$\eta = h(\phi, \tau), \quad (2/R_s) We^{-1} = p - p_a - 2k(T)Re_0^{-1}(1 + \eta^{-2}h_\phi^2)^{-1}(v_\eta - \eta^{-1}h_\theta(\omega_\eta\eta + \eta^{-1}v_\phi) \\ + \eta^{-2}h_\phi^2(\omega_\phi + \eta^{-1}v)); \quad (8)$$

$$\eta = 1, \quad T_\eta = Nu_1 T; \quad (9)$$

$$\eta = h(\phi, \tau), \quad T_\eta = -Nu_2 T; \quad (10)$$

$$\tau = 0, \quad h = h_0(\phi), \quad v = v_0(\eta, \phi), \quad w = w_0(\eta, \phi), \quad T = T_0(\eta, \phi). \quad (11)$$

In equations (1) - (12) subscript denotes the partial derivative in the specified direction, Φ – dissipative function, Δ – Laplace operator, $2/R_s$ – mean curvature of the layer surface:

$$\Phi = 2k(T)Re_0^{-1}Eck \left(v_\eta^2 + (\omega_\phi + \eta^{-1}v)^2 + (\eta^{-1}v_\phi + \eta\omega_\eta)^2 \right), \\ \Delta = \partial^2/\partial\eta^2 + \eta^{-1}\partial/\partial\eta + \eta^{-2}\partial^2/\partial\phi^2, \quad (12) \\ 2/R_s = (h^2 + 2h_\phi^2 - hh_{\phi\phi}) (h^2 + h_\phi^2)^{-1.5}.$$

Relations (1) - (12) represent a nonlinear initial-boundary value problem with respect to functions $v(\eta, \phi, \tau)$, $\omega(\eta, \phi, \tau)$, $p(\eta, \phi, \tau)$, $T(\eta, \phi, \tau)$ and free boundary $\eta = h(\phi, \tau)$. Equations (1) - (12) have following dimensionless parameters: initial Reynolds number, Froude number, Weber number, ration of viscosities, Peclet number, Eckert number and two Nusselt numbers

$$Re_0 = R_0^2\omega_0 v_0^{-1}, \quad Fr = R_0\omega_0^2 g^{-1}, \quad We = \rho R_0^3\omega_0^2\sigma^{-1}, \\ k(T) = \mu(T)\mu_0^{-1}, \quad Pe = \rho c R_0^2\omega_0\lambda^{-1}, \quad Eck = R_0^2\omega_0^2(cT_c)^{-1}, \quad (13) \\ Nu_1 = \alpha_1 R_0\lambda^{-1}, \quad Nu_2 = \alpha_2 R_0\lambda^{-1}.$$

Heat transfer conditions on the cylinder surface and fluid, and also for fluid and environment are determined by empirical formulas [8]:

$$Nu_1 = 0.49Re_1^{0.5}, \quad Re_1 = 2R_0^2\omega_0 v_0^{-1}, \quad Re_1 < 10^3; \\ Nu_1 = 0.245 Re_1^{0.6}, \quad Re_1 \geq 10^3. \quad (14)$$

$$Nu_2 = 0.49Re_2^{0.5}, \quad Re_2 = 2R_0^2\omega_0 v_2^{-1}, \quad Re_2 < 10^3; \\ Nu_2 = 0.245 Re_2^{0.6}, \quad Re_2 \geq 10^3. \quad (15)$$

In equations (1) - (15) we use the next designations: σ – the surface tension of the fluid, g – gravity acceleration, c – specific heat capacity, λ – thermal conductivity of the fluid, α_1 and α_2 – thermal diffusivity on $\eta = 1$ and $\eta = h(\varphi, \tau)$ respectively, v_0 – initial kinematic viscosity of the fluid when $T = T_e$, v_2 – kinematic viscosity of the environment, p_a – ambient pressure, $\mu(T)$ – dynamic viscosity of the fluid, that depends on temperature.

In case of sufficiently fast rotation of the cylinder, the value $Re \sim \varepsilon^{-1} \gg 1$, $Fr \sim \varepsilon^{-2} \gg 1$, $We \sim \varepsilon^{-2} \gg 1$ (by “ \sim ” we denote the order of magnitude). At the same time the relative change in fluid flow in the transversal direction is substantially less than in the radial direction, and the radial velocity component is much less than the transversal component, i.e.

$$Re^{-1} \sim \varepsilon, Fr^{-1} \sim \varepsilon^2, We^{-1} \sim \varepsilon^2,$$

$$\partial/\partial\tau, \partial/\partial\varphi \sim \varepsilon \ll 1, \partial/\partial\eta \sim 1, v \sim \varepsilon, \omega \sim 1, h \sim 1.$$

We exclude the infinitesimals from the system (1) - (12). In equation (1) and boundary conditions (8), (10), (11) we take into account terms of the first order. In the remaining equations we take into account terms with order ε , leaving also in (2) and (9) the effect of gravity and surface tension. We obtain the following system with boundary and initial conditions:

$$p_\eta = (\omega + 1)^2 \eta; \quad (16)$$

$$\begin{aligned} \omega_\tau + v\omega_\eta + \omega\omega_\varphi + 2\eta^{-1}v(\omega + 1) = -\eta^{-2}p_\varphi + k(T)Re_0^{-1}(\omega_{\eta\eta} + 3\eta^{-1}\omega_\eta) \\ + (dk/dT) Re_0^{-1}T_\eta\omega_\eta - \eta^{-1}Fr^{-1}\cos(\varphi + \tau); \end{aligned} \quad (17)$$

$$T_\tau + vT_\eta + \omega T_\varphi = Pe^{-1}(T_{\eta\eta} + \eta^{-1}T_\eta) + 2k(T)Re_0^{-1}Eck\eta^2\omega_\eta^2; \quad (18)$$

$$(\eta v)_\eta + (\eta\omega)_\varphi = 0; \quad (19)$$

$$\eta = h(\varphi, \tau), \quad h_\tau + \omega h_\varphi = v; \quad (20)$$

$$\eta = 1, \quad v = 0, \quad \omega = 0; \quad \eta = h(\varphi, \tau), \quad \omega_\eta = 0; \quad (21)$$

$$\eta = h(\varphi, \tau), \quad We^{-1}(h^{-1} - h^{-2}h_{\varphi\varphi}) = p - p_a; \quad (22)$$

$$\eta = 1, \quad -T_\eta = Nu_1T; \quad \eta = h(\varphi, \tau), \quad T_\eta = Nu_2T; \quad (23)$$

$$\tau = 0, \quad h = h_0(\varphi), v = v_0(\eta, \varphi), w = w_0(\eta, \varphi), T = T_0(\eta, \varphi). \quad (24)$$

Note the initial-boundary value problem (16) - (24) takes into account the influence of physical factors such as gravity forces, surface tension, inertia, viscosity, external pressure on the liquid flow. And the condition of thinness of the layer is not required.

To solve the problem (16) – (24) we use direct Kapitza-Shkadov method [9]. We introduce new variable ζ :

$$\zeta = (\eta - 1)/\delta(\varphi, \tau), \quad 0 \leq \zeta \leq 1, \quad \delta(\varphi, \tau) = h(\varphi, \tau) - 1.$$

Dependency of relative angular velocity from variable ζ has quadratic form, that satisfies boundary conditions (21):

$$\omega(\zeta, \varphi, \tau) = -N(\varphi, \tau)\zeta(1 - 0.5\zeta), \quad (25)$$

where $N(\varphi, \tau)$ is 2π periodic function on φ to be determined. $N(\varphi, \tau) = 0$ corresponds to the rotation of the cylinder and the fluid layer as a whole.

Like the Navier-Stokes equations, the heat equation has a parabolic form. Suppose that the solution of the heat equation can also be sought in the form of a quadratic dependence on the radial coordinate:

$$T(\zeta, \varphi, \tau) = B(\varphi, \tau)(\zeta^2 + b\zeta + c).$$

The constants b and c are determined from the boundary conditions (23):

$$b = -(2Nu_1 + 2\delta Nu_1 + 2Nu_2 + 2\delta^2 Nu_1 Nu_2 + \delta^3 Nu_1 Nu_2 Nu_1) \\ \times (Nu_1 + Nu_2 + \delta^2 Nu_1 Nu_2)^{-1},$$

$$c = (-2 + Nu_1 + 2\delta Nu_1 + Nu_2 - \delta^2 Nu_2 + \delta^2 Nu_1 Nu_2 + \delta^3 Nu_1 Nu_2) \\ \times (Nu_1 + Nu_2 + \delta^2 Nu_1 Nu_2)^{-1}.$$

Next, after switching to a variable ζ we integrate equations (17), (18) over the layer thickness from 0 to 1. The pressure in the layer $p(\zeta, \varphi, \tau)$ is determined from equation (16). Substituting (25) into continuity equation (19) allows us to obtain a formula for the radial velocity component v .

Substituting $v(\delta, \varphi, \tau)$, $w(\delta, \varphi, \tau)$, $p(\delta, \varphi, \tau)$, $T(\delta, \varphi, \tau)$ values into integrated over the layer thickness equations (17), (18), we obtain two equations of evolution. The third equation of evolution is derived from the free surface equation (20). To solve the problem by the direct method, it is necessary to define the dependence of viscosity on temperature. To define $\mu(T)$ we use the second order polynomial function built using empirical tabular data.

In general, the evolution equations have the form

$$N_\tau = f_1(\delta, \delta_\varphi, \delta_{\varphi\varphi}, \delta_{\varphi\varphi\varphi}, B, B_\varphi, N, N_\varphi); \quad B_\tau = f_2(\delta, \delta_\varphi, B, B_\varphi, N, N_\varphi); \quad \delta_\tau = f_3(\delta, \delta_\varphi, B, B_\varphi). \quad (26)$$

They are supplemented by initial conditions and periodicity boundary conditions for the angular coordinate.

Functions f_1, f_2, f_3 are obtained in the process of calculating the integrals and used for specific calculations. These values are complex and are not provided in this article. The equations of evolution of a not necessary thin layer in the case of an isothermal problem were obtained in [4]. They are:

$$\delta_\tau = H(\delta)B_\varphi + R(\delta, B)\delta_\varphi, \quad (27)$$

$$B_\tau = U(\delta, T)\delta_\varphi + V(\delta, T)B_\varphi - 60(\delta_\varphi + \delta_{\varphi\varphi\varphi} - 2\delta_\varphi\delta_{\varphi\varphi}(1 + \delta)^{-1})((1 + \delta)^2 E_0(\delta)We)^{-1} \\ + 30(\delta + 2)\cos(\varphi + \tau)(Fr E_0(\delta))^{-1} - 10B(6 - 3\delta - \delta^2)(Re\delta^2 E_0(\delta))^{-1}, \quad (28)$$

where:

$$H(\delta) = \delta(5\delta + 8)(24(\delta + 1))^{-1}, \quad R(\delta, T) = B(5\delta + 4)(12(\delta + 1))^{-1},$$

$$E_0(\delta) = 20 + 25\delta + 9\delta^2, \quad E_1(\delta) = 20 + 50\delta + 27\delta^2,$$

$$U(\delta, T) = B^2 U_2(\delta) + B U_1(\delta) + U_0(\delta), \quad V(\delta, T) = B V_1(\delta) + V_0(\delta),$$

$$U_2(\delta) = \delta E_0^{-1}(\delta) \left(42^{-1}(336 + 553\delta + 38\delta^2) - (12(\delta + 1))^{-1}(5\delta + 4)(20 + 50\delta + 27\delta^2) \right),$$

$$U_1(\delta) = (40 + 50\delta)E_0^{-1}(\delta), \quad U_0(\delta) = -60(\delta + 1)E_0^{-1}(\delta), \quad V_0(\delta) = \delta(40 + 25\delta)E_0^{-1}(\delta),$$

$$V_1(\delta) = E_0^{-1}(\delta)((336 + 161\delta - 34\delta^2)21^{-1} - (5\delta + 8)(20 + 50\delta + 27\delta^2)(24(\delta + 1))^{-1}).$$

3. Numerical solution

The numerical method for solving the problem is based on the method of lines, when the flow region from 0 to 2π in the angular coordinate is divided into $N=360$; 720 or more rays. Partial derivatives with respect to φ are replaced by finite differences. Further, the system of $3N$ ordinary equations is integrated numerically remembering f_1, f_2, f_3 (26) on the previous step.

We study the evolution of the shape of the free surface, that has initial constant thickness δ_0 or a sinusoidal form $\delta_0(\varphi) = \delta_0 + a \sin(k\varphi)$. The numeric solution of the problem was carried out under

the following conditions: liquid is aqueous solutions of glycerol with a density of 1260 kg/m^3 , surface tension coefficient of 0.07 N/m , ambient and cylinder temperature of $293 \text{ }^\circ\text{K}$, the initial temperature of the fluid is $323 \text{ }^\circ\text{K}$; the cylinder radius is 2.5 cm . The kinematic viscosity of glycerol was approximated by a quadratic function depending on a temperature using tabular data:

$$\mu(T) = 365.41 T^2 - 2.314 T + 0.0037.$$

Let's show an example of solving the isothermal problem (27), (28). If the initial profile of the free surface of a layer is in a sinusoidal form, then at the first the main initial perturbations are developed as shown in Figure 1. Then the secondary perturbation remains small, and the instability is growing due to one of the main maxima of the free surface, which is shown in Figure 2 when $\delta_0(\varphi) = 0.1 + 0.01 \sin(4\varphi)$.

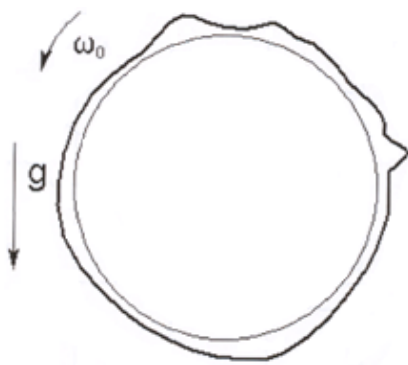


Figure 1. Shape of the free surface of the viscous layer: $\tau = \pi$, $Re = 31.8$, $Fr = 8.15$, $We = 899.4$.

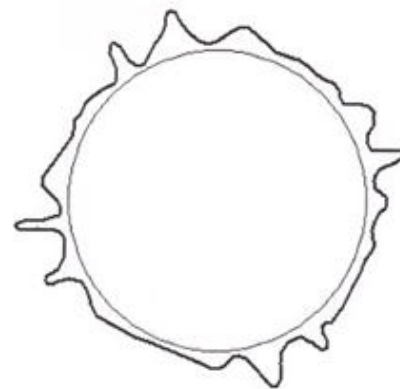


Figure 2. Shape of the free surface of the viscous layer: $\tau = 1.5\pi$, $Re = 31.8$, $Fr = 8.15$, $We = 899.4$.

Figure 3 shows non-isothermal and isothermal solutions. In the non-isothermal case, the layer is cooled, so the viscosity of the liquid increases, and the disturbances grow more slowly. We study the development of perturbations of a layer whose thickness is constant at the initial moment of time. First, the development of disturbances is determined by the influence of gravity. One maximum and one surface minimum appear. Over time, under the action of inertial forces and nonlinear effects, the number of local extrema increases, and other disturbances arise, as shown in Figure 4.

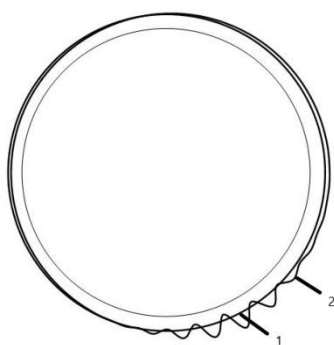


Figure 3. The shapes of the free surface of viscous layer for 1 – non-isothermal problem, 2 – isothermal problem: $\delta_0 = 0.1$, $\tau = 2.5$, $Re = 17.7$, $Fr = 2.5$, $We = 279.1$, $Eck = 8.8 \cdot 10^{-7}$, $Pe = 32275$, $Nu_1 = 8.13$, $Nu_2 = 25.11$, $T_0 = 0.1024$.

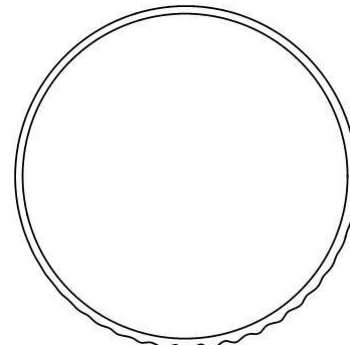


Figure 4. Shape of the free surface of the viscous layer: $\delta_0 = 0.05$, $\tau = 27$, $Re = 90.1$, $Fr = 1.02$, $We = 1800$, $Eck = 1.4 \cdot 10^{-6}$, $Pe = 163934$, $Nu_1 = 18.33$, $Nu_2 = 56.58$, $T_0 = 0.1024$.

The distribution of temperature along the angle φ is not uniform too as we can see on Figure 5. We can see that due to influence of dissipative forces and greater heat transfer coefficient the temperature on the free surface is higher than on the surface of the cylinder.

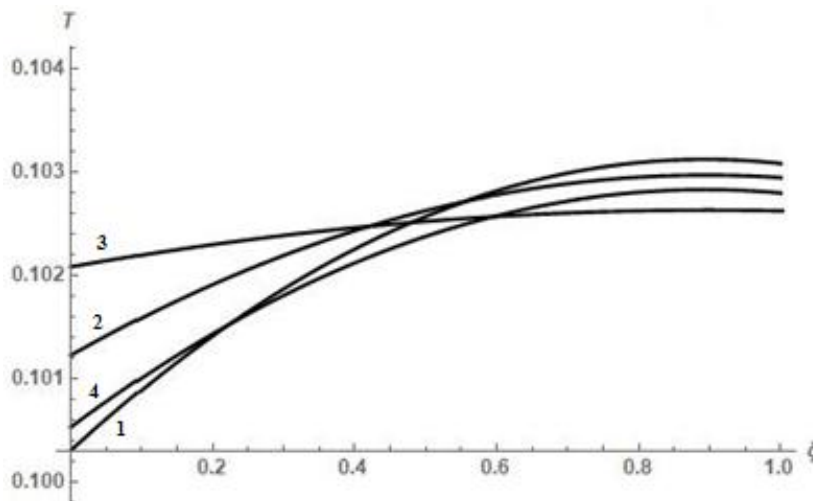


Figure 5. The distribution of temperature for different angles φ : 1 – $\varphi = 0$, 2 – $\varphi = 0.5\pi$, 3 – $\varphi = \pi$, 4 – $\varphi = 1.5\pi$, $\delta_0 = 0.05$, $\tau = 16$, $Re = 90.1$, $Fr = 1.02$, $We = 1800$, $Eck = 1.4 \cdot 10^{-6}$, $Pe = 163934$, $Nu_1 = 18.33$, $Nu_2 = 56.58$, $T_0 = 0.1024$.

4. Conclusions

The plane motion of a viscous fluid layer on the outer surface of a horizontally placed rotating cylinder was investigated in the fields of surface tension, gravity and inertia forces with the varying temperature. The energy equation takes into account convective and dissipative terms.

In the case of a sufficiently fast rotation of the cylinder, the equations of the first approximation were obtained, similar to the equations of the boundary layer. The system of partial differential equations for determining the evolution of the free surface was obtained by a direct method. It was numerically solved and analyzed. The obtained system allows to study the motion of thin and not thin layers with arbitrary Reynolds numbers. The shapes of the free surface were investigated, and a comparison was made with the results of isothermal solution.

Acknowledgments

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