Lowest order QED radiative effects in polarized SIDIS

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The explicit exact analytical expressions for the lowest order radiative corrections to the semi-inclusive deep inelastic scattering of the polarized particles are obtained in the most compact, covariant form convenient for the numerical analysis. The infrared divergence from the real photon emission is extracted and canceled using the Bardin-Shumeiko approach. The contribution of the exclusive radiative tail is presented. The analytic results obtained within the ultrarelativistic approximation are also shown.

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I. INTRODUCTION

Nowadays the polarized semi-inclusive deep-inelastic scattering (SIDIS) plays a crucial role in our understanding of the internal spin structure of the nucleons. Information on the three-dimensional structure of the polarized proton and neutron can be obtained by extracting the quark transverse momentum distributions from the various single spin asymmetries measured in SIDIS with polarized particles. Specifically, the Sivers and Collins contributions can be selected [1] from the present data on transversely polarized targets $\vec{p}(e,e'\pi)x$ in HERMES [2], $\vec{D}(\mu,\mu'\pi)x$ in COMPASS [3], and ${}^3\vec{\mathrm{He}}(e,e'\pi)x$ in JLab [4] which show a strong flavor dependence of transverse momentum distributions. Moreover in the near future, highly accurate experiments are planned at 12 GeV Jlab [5] that will provide unique opportunities for the breakthrough in the investigation of the nucleon structure by carrying out multidimensional precision studies of longitudinal and transverse spin and momentum degrees of freedom from SIDIS experiments with high luminosity in combination with large acceptance detectors.

It is well known that one of the important sources of the systematical uncertainties in SIDIS experiments with and without polarization of initial particles are the QED radiative corrections (RC). RC to the threefold differential cross section ($d\sigma/dxdydz$, where x and y are the standard Bjorken variables and the z is the fraction of the virtual-photon energy transferred to the detected hadron) can be calculated using the patch SIRAD of FORTRAN code POLRAD [6] created based on the original calculations in Refs. [7,8] for unpolarized and polarized particles. The calculation of RC to the fivefold differential cross section of unpolarized particles ($d\sigma/dxdydzdp_t^2d\phi_h$, where p_t is the detected hadron transverse momentum and ϕ_h is the azimuthal angle between the lepton scattering and hadron production planes) was performed in Ref. [9]. These calculations did not contain the radiative tail from the exclusive reactions as a separate contribution involving the exclusive structure functions (SF). This limitation was addressed in Ref. [10] in which the authors explicitly calculated the exclusive radiative tail and implemented the exclusive SF using the approach of MAID [11].

In the present paper we consider the general task of RC calculation when the initial nucleon can be arbitrarily polarized. The analytical expressions for RC to SIDIS are obtained for the sixfold cross section with the longitudinally polarized lepton and arbitrarily polarized target, $d\sigma/dxdydzdp_t^2d\phi_hd\phi$, where the azimuthal angle ϕ between the lepton scattering and ground planes is introduced to appropriately account for the transverse target polarization. The contribution of the exclusive radiative tail to the total RC is also presented. Similar to the previous analyses we calculated RC in the model-independent way. These corrections are induced by the unobservable real photon emission from the lepton leg, leptonic vertex correction, and vacuum polarization. The model-independent correction is proportional to the leading logarithm $\log(Q^2/m^2)$, which is large because of high transferring momentum squared Q^2 (>1 GeV^2) and small electron mass m. What is not accounted for in this approach

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is the correction due to the real and additional virtual photon emission by hadrons including the two-photon exchange and QED hadronic vertex correction. However, this correction should not be accounted for in the majority of cases, e.g., when the used model for SF was extracted from the experiment in which emission by hadrons had not been applied in the RC procedure of experimental data.

The Bardin-Shumeiko approach [12] is used for extraction and cancellation of the infrared divergence coming from the real and virtual photon emission. In contrast to the widely used Mo-Tsai approach [13,14] the final expression for RC within the Bardin-Shumeiko approach does not depend on an artificial parameter that is introduced in [13,14] for separation of the photon emission on the hard and soft parts.

In this paper we apply an approach for decomposition of the initial nucleon and virtual photon polarization as well as the real photon four-momentum over the respective bases (the Appendix A). The polarization decomposition is used for the hadronic tensor representation in a covariant form. The momentum decomposition is used to simplify integration over the momentum of the unobserved photon. Specifically, this allows us to essentially reduce the number of pseudoscalars occurring after the convolution of the leptonic tensors of radiative effects with the hadronic tensor and present the final expressions for RC in a compact, covariant form convenient for the numerical analysis. All calculations have been performed in an exact way keeping the lepton mass at all stages of the calculation. The dependence of certain terms in the exact final expressions for RC on the electron mass is quite tricky, and therefore, we analyze respective contributions in the ultrarelativistic approximation allowing for extraction of the electron mass dependence explicitly and classifying all terms in RC as leading (i.e., containing the leading logarithms), next-to-leading (i.e., independent of the electron mass), and other potentially negligible terms (i.e., the terms vanishing in the approximation of $m \to 0$). Thus the results obtained in the paper contain both exact formulas for RC and expressions in ultrarelativistic approximations allowing us to explicitly control the dependence on the electron mass. Thus, the analytic expressions for RC are valid for experiments with muons (e.g., COMPASS [3]) in which the approximation of the zero lepton mass could not be appropriate.

The rest of the article is organized as follows. The hadronic tensor, different sets for the SF used in the literature, as well as the lowest order (Born) contribution to the SIDIS process are discussed in Sec. II. The calculation of the lowest order QED RC to the observables in SIDIS as well as the explicit results for both the semi-inclusive final hadronic state and exclusive radiative tail contributions are presented in Sec. III. The infrared divergence in these calculations are extracted from the real photon emission with the semi-inclusive final hadronic state by the Bardin-Shumeiko approach [12] and then canceled with the corresponding term from the leptonic vertex correction in such a way that the obtained results are free from an intermediate parameter $\bar{k}_0$. For the parametrization of the infrared and ultraviolet divergences the dimension regularization is used. The results of analyses of the exact expressions in ultrarelativistic approximation are given in Sec. IV. Particularly we show that the double leading logarithms coming from the terms with the soft photon emission and the leptonic vertex correction cancel in their sum. A brief discussion and conclusion are presented in Sec. V. Technical details and the most cumbersome parts of the RC are presented in four Appendixes. The bases for the decomposition of the initial target and virtual photon polarization as well as the real photon momentum are presented in Appendix A. The explicit expressions for the real photon emission quantities are presented in Appendix B. The details of the approach for the infrared divergence extraction and cancellation are given in Appendix C. The detailed calculations of the additional virtual particle contributions are presented in Appendix D.

II. HADRONIC TENSOR AND BORN CONTRIBUTION

The sixfold differential cross section of SIDIS with polarized particles

$$e(k_1,\xi) + n(p,\eta) \to e(k_2) + h(p_h) + x(p_x) \qquad (1)$$

($k_1^2 = k_2^2 = m^2,\ p^2 = M^2,\ p_h^2 = m_h^2$) where ξ (η) is the initial lepton (nucleon) polarized vector can be described by the following set of variables:

$$x = -\frac{q^2}{2qp}, \qquad y = \frac{qp}{k_1 p}, \qquad z = \frac{p_h p}{pq},$$
$$t = (q - p_h)^2, \qquad \phi_h, \qquad \phi. \qquad (2)$$

Here $q = k_1 - k_2$, ϕ_h is the angle between $(\mathbf{k_1}, \mathbf{k_2})$ and $(\mathbf{q}, \mathbf{p_h})$ planes, and ϕ is the angle between $(\mathbf{k_1}, \mathbf{k_2})$ and the ground planes in the target rest frame reference system ($\mathbf{p} = 0$).

Also we use the following set of invariants:

$$S = 2pk_1, \qquad Q^2 = -q^2, \qquad Q_m^2 = Q^2 + 2m^2,$$
$$X = 2pk_2, \qquad S_x = S - X, \qquad S_p = S + X,$$
$$V_{1,2} = 2k_{1,2}p_h, \qquad V_+ = \frac{1}{2}(V_1 + V_2),$$
$$V_- = \frac{1}{2}(V_1 - V_2) = \frac{1}{2}(m_h^2 - Q^2 - t),$$
$$S' = 2k_1(p + q - p_h) = S - Q^2 - V_1,$$
$$X' = 2k_2(p + q - p_h) = X + Q^2 - V_2,$$
$$p_x^2 = (p + q - p_h)^2 = M^2 + t + (1 - z)S_x.$$
$$\lambda_S = S^2 - 4M^2m^2, \qquad \lambda_Y = S_x^2 + 4M^2Q^2,$$
$$\lambda_1 = Q^2(SX - M^2Q^2) - m^2\lambda_Y, \qquad \lambda_m = Q^2(Q^2 + 4m^2),$$
$$\lambda'_S = S'^2 - 4m^2p_x^2, \qquad \lambda'_X = X'^2 - 4m^2p_x^2. \qquad (3)$$

Noninvariant variables including the energy p_{h0}, longitudinal p_l, and transverse p_t (k_t) three-momenta of the detected hadron (the incoming or scattering lepton) with respect to the virtual photon direction in the target rest frame are expressed in terms of the above invariants:

$$p_{h0} = \frac{zS_x}{2M},$$
$$p_l = \frac{zS_x^2 - 4M^2V_-}{2M\sqrt{\lambda_Y}} = \frac{zS_x^2 + 2M^2(t + Q^2 - m_h^2)}{2M\sqrt{\lambda_Y}},$$
$$p_t = \sqrt{p_{h0}^2 - p_l^2 - m_h^2}, \qquad k_t = \sqrt{\frac{\lambda_1}{\lambda_Y}}. \tag{4}$$

As a result the quantities $V_{1,2}$ can be written through $\cos\phi_h$ and other variables defined in Eqs. (2)–(4) as

$$V_1 = p_{h0}\frac{S}{M} - \frac{p_l(SS_x + 2M^2Q^2)}{M\sqrt{\lambda_Y}} - 2p_tk_t\cos\phi_h,$$
$$V_2 = p_{h0}\frac{X}{M} - \frac{p_l(XS_x - 2M^2Q^2)}{M\sqrt{\lambda_Y}} - 2p_tk_t\cos\phi_h. \tag{5}$$

The sine of ϕ_h is expressed as

$$\sin\phi_h = -\frac{2\varepsilon_\perp p_h}{p_t\sqrt{\lambda_1}}, \tag{6}$$

where

$$\varepsilon_\perp^\mu = \varepsilon^{\mu\nu\rho\sigma}p_\nu k_{1\rho}q_\sigma \tag{7}$$

is the pseudovector with a normal direction to the scattering plane $(\mathbf{k_1}, \mathbf{k_2})$. Our definitions of ϕ_h and other kinematic variables are in agreement with the common convention introduced in [15].

The lowest order QED (Born) contribution to SIDIS is presented by the Feynman graph in Fig. 1(a). The cross section for this process reads

$$d\sigma_B = \frac{(4\pi\alpha)^2}{2\sqrt{\lambda_S}Q^4}W_{\mu\nu}L_B^{\mu\nu}d\Gamma_B, \tag{8}$$

where the phase space is parametrized as

$$d\Gamma_B = (2\pi)^4\frac{d^3k_2}{(2\pi)^3 2k_{20}}\frac{d^3p_h}{(2\pi)^3 2p_{h0}}$$
$$= \frac{1}{4(2\pi)^2}\frac{SS_xdxdyd\phi}{2\sqrt{\lambda_S}}\frac{S_xdzdp_t^2d\phi_h}{4Mp_l}. \tag{9}$$

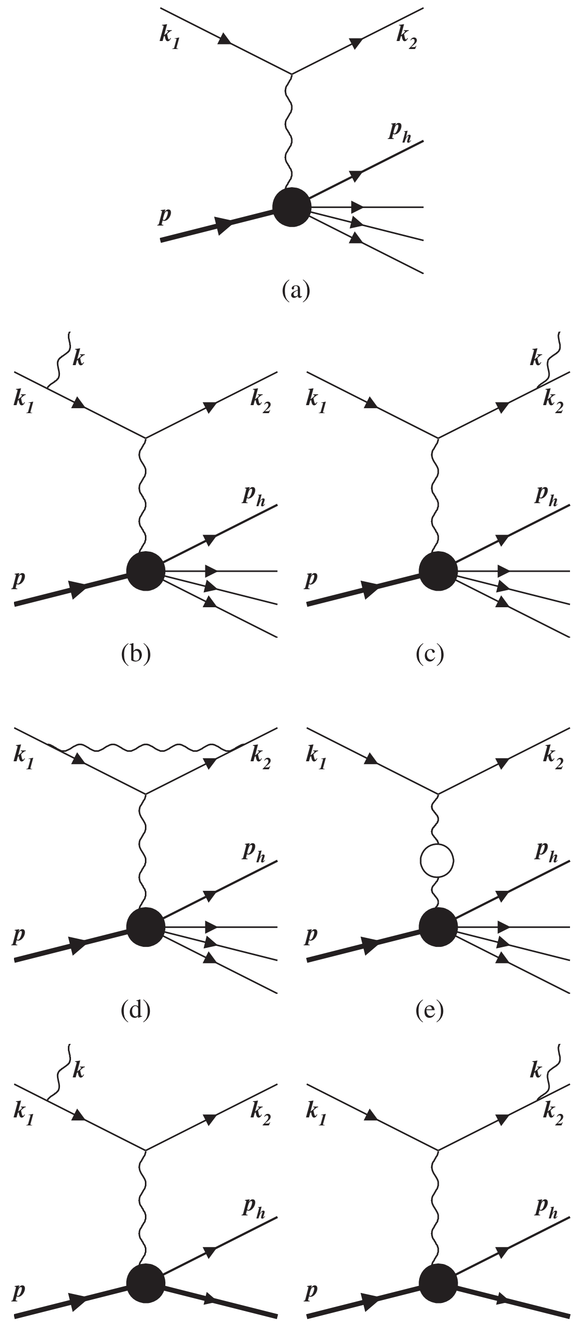

FIG. 1. Feynman graphs for (a) the lowest order, (b)–(e) SIDIS, and (f), (g) exclusive radiative tail contributions to the lowest order RC for SIDIS scattering.

Since the initial lepton is considered to be longitudinally polarized, its polarization vector has the form [16]

$$\xi = \frac{\lambda_e S}{m\sqrt{\lambda_S}}k_1 - \frac{2\lambda_e m}{\sqrt{\lambda_S}}p = \xi_0 + \xi_1. \tag{10}$$

As a result the leptonic tensor is

$$L_B^{\mu\nu} = \frac{1}{2}\mathrm{Tr}[(\hat{k}_2+m)\gamma_\mu(\hat{k}_1+m)(1+\gamma_5\hat{\xi})\gamma_\nu] = 2\left[k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - \frac{Q^2}{2}g^{\mu\nu} + \frac{i\lambda_e}{\sqrt{\lambda_S}}\varepsilon^{\mu\nu\rho\sigma}(Sk_{2\rho}k_{1\sigma}+2m^2q_\rho p_\sigma)\right]. \quad (11)$$

According to [17] the hadronic tensor for the SIDIS process $\gamma^* + n \to h + X$ can be decomposed in the terms of the scalar spin-independent $H_{ab}^{(0)}$ and spin-dependent $H_{abi}^{(S)}$ structures functions

$$W_{\mu\nu} = \sum_{a,b=0}^{3} e_\mu^{\gamma(a)} e_\nu^{\gamma(b)}\left(H_{ab}^{(0)} + \sum_{\rho,i=0}^{3}\eta^\rho e_\rho^{h(i)} H_{abi}^{(S)}\right), \quad (12)$$

where $e_\mu^{\gamma(a)}$ (or $e_\nu^{\gamma(b)}$) and $e_\rho^{h(i)}$ are the complete set of the basis vectors for the polarization four-vectors of the virtual photon and nucleon in the target rest frame. These vectors can be represented in a covariant form [18] using (A1) and (A2).

Due to the parity and current conservation, hermiticity as well as $p\eta \equiv 0$, only the following set of independent SF $H_{ab}^{(0)}$ and $H_{abi}^{(S)}$ in (12) survives [17]: 5 spin-independent $H_{00}^{(0)}$, $H_{11}^{(0)}$, $H_{22}^{(0)}$, $\mathrm{Re}H_{01}^{(0)}$, $\mathrm{Im}H_{01}^{(0)}$ and 13 spin-dependent $H_{002}^{(S)}$, $\mathrm{Re}H_{012}^{(S)}$, $\mathrm{Im}H_{012}^{(S)}$, $\mathrm{Re}H_{021}^{(S)}$, $\mathrm{Im}H_{021}^{(S)}$, $\mathrm{Re}H_{023}^{(S)}$, $\mathrm{Im}H_{023}^{(S)}$, $H_{112}^{(S)}$, $\mathrm{Re}H_{121}^{(S)}$, $\mathrm{Im}H_{121}^{(S)}$, $\mathrm{Re}H_{123}^{(S)}$, $\mathrm{Im}H_{123}^{(S)}$, $H_{222}^{(S)}$. All the rest of the SF have to be set to zero [17].

The hadronic tensor in terms of these SF can be obtained by substitution (A1) and (A2) into (12) resulting in

$$\begin{aligned} W_{\mu\nu} = \sum_{i=1}^{9} w_{\mu\nu}^i \mathcal{H}_i = & -g_{\mu\nu}^\perp \mathcal{H}_1 + p_\mu^\perp p_\nu^\perp \mathcal{H}_2 + p_{h\mu}^\perp p_{h\nu}^\perp \mathcal{H}_3 \\ & + (p_\mu^\perp p_{h\nu}^\perp + p_{h\mu}^\perp p_\nu^\perp)\mathcal{H}_4 + i(p_\mu^\perp p_{h\nu}^\perp - p_{h\mu}^\perp p_\nu^\perp)\mathcal{H}_5 \\ & + (p_\mu^\perp n_\nu + n_\mu p_\nu^\perp)\mathcal{H}_6 + i(p_\mu^\perp n_\nu - n_\mu p_\nu^\perp)\mathcal{H}_7 \\ & + (p_{h\mu}^\perp n_\nu + n_\mu p_{h\nu}^\perp)\mathcal{H}_8 \\ & + i(p_{h\mu}^\perp n_\nu - n_\mu p_{h\nu}^\perp)\mathcal{H}_9. \end{aligned} \quad (13)$$

Here $g_{\mu\nu}^\perp = g_{\mu\nu} - q_\mu q_\nu/q^2$ and $n^\mu = \varepsilon^{\mu\nu\rho\sigma} q_\nu p_\rho p_{h\sigma}$.

The generalized SF $\mathcal{H}_i$ can be expressed via $H_{ab}^{(0)}$ and $H_{abi}^{(S)}$ using the decomposition of the nucleon polarized three-vector $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3)$ over the basis (A2) in the following way:

$$\begin{aligned} \mathcal{H}_1 &= H_{22}^{(0)} - \eta_2 H_{222}^{(S)}, \\ \mathcal{H}_2 &= \frac{4}{\lambda_Y^2 p_t^2}[\lambda_Y p_t^2 Q^2 (H_{00}^{(0)} - \eta_2 H_{002}^{(S)}) + \lambda_3^2 S_x^2 (H_{11}^{(0)} \\ &\quad - \eta_2 H_{112}^{(S)}) - \lambda_2\lambda_Y (H_{22}^{(0)} - \eta_2 H_{222}^{(S)}) \\ &\quad - 2S_x\lambda_3 p_t Q\sqrt{\lambda_Y}(\mathrm{Re}H_{01}^{(0)} - \eta_2 \mathrm{Re}H_{012}^{(S)})], \\ \mathcal{H}_3 &= \frac{1}{p_t^2}(H_{11}^{(0)} - H_{22}^{(0)} + \eta_2(H_{222}^{(S)} - H_{112}^{(S)})), \\ \mathcal{H}_4 &= \frac{2}{\lambda_Y p_t^2}[\lambda_3 S_x (H_{22}^{(0)} - H_{11}^{(0)} + \eta_2(H_{112}^{(0)} - H_{222}^{(S)})) \\ &\quad + p_t Q\sqrt{\lambda_Y}(\mathrm{Re}H_{01}^{(0)} - \eta_2 \mathrm{Re}H_{012}^{(S)})], \\ \mathcal{H}_5 &= \frac{2Q}{p_t\sqrt{\lambda_Y}}(\mathrm{Im}H_{01}^{(0)} - \eta_2 \mathrm{Im}H_{012}^{(S)}), \\ \mathcal{H}_6 &= \frac{4M}{\lambda_Y^{3/2} p_t^2}[Q p_t \sqrt{\lambda_Y}(\eta_1 \mathrm{Re}H_{021}^{(S)} + \eta_3 \mathrm{Re}H_{023}^{(S)}) \\ &\quad - \lambda_3 S_x (\eta_1 \mathrm{Re}H_{121}^{(S)} + \eta_3 \mathrm{Re}H_{123}^{(S)})], \\ \mathcal{H}_7 &= \frac{4M}{\lambda_Y^{3/2} p_t^2}[Q p_t \sqrt{\lambda_Y}(\eta_1 \mathrm{Im}H_{021}^{(S)} + \eta_3 \mathrm{Im}H_{023}^{(S)}) \\ &\quad - \lambda_3 S_x (\eta_1 \mathrm{Im}H_{121}^{(S)} + \eta_3 \mathrm{Im}H_{123}^{(S)})], \\ \mathcal{H}_8 &= \frac{2M}{\sqrt{\lambda_Y} p_t^2}(\eta_1 \mathrm{Re}H_{121}^{(S)} + \eta_3 \mathrm{Re}H_{123}^{(S)}), \\ \mathcal{H}_9 &= \frac{2M}{\sqrt{\lambda_Y} p_t^2}(\eta_1 \mathrm{Im}H_{121}^{(S)} + \eta_3 \mathrm{Im}H_{123}^{(S)}). \end{aligned} \quad (14)$$

Here $\lambda_2 = V_-^2 + m_h^2 Q^2$, $\lambda_3 = V_- + zQ^2$, and V_- is defined in Eqs. (3).

Finally we find the Born contribution in the form

$$\sigma^B \equiv \frac{d\sigma^B}{dxdydzdp_t^2 d\phi_h d\phi} = \frac{\alpha^2 S S_x^2}{8MQ^4 p_l \lambda_S}\sum_{i=1}^{9}\theta_i^B \mathcal{H}_i, \quad (15)$$

where $\theta_i^B = L^{\mu\nu} w_{\mu\nu}^i/2$,

$$\begin{aligned} &\theta_1^B = Q^2 - 2m^2, \quad \theta_2^B = (SX - M^2Q^2)/2, \\ &\theta_3^B = (V_1V_2 - m_h^2Q^2)/2, \quad \theta_4^B = (SV_2 + XV_1 - zQ^2S_x)/2, \\ &\theta_5^B = \frac{2\lambda_e S\varepsilon_\perp p_h}{\sqrt{\lambda_S}}, \quad \theta_6^B = -S_p\varepsilon_\perp p_h, \\ &\theta_7^B = \frac{\lambda_e S}{4\sqrt{\lambda_S}}[\lambda_Y V_+ - S_p S_x(zQ^2 + V_-)], \quad \theta_8^B = -2V_+\varepsilon_\perp p_h, \\ &\theta_9^B = \frac{\lambda_e}{2\sqrt{\lambda_S}}[S(Q^2(zS_xV_+ - m_h^2S_p) + V_-(SV_2 - XV_1)) \\ &\qquad + 2m^2(4M^2V_-^2 + \lambda_Y m_h^2 - zS_x^2(zQ^2 + 2V_-))]. \end{aligned} \quad (16)$$

The quantities $H_{ab}^{(0)}$ and $H_{abi}^{(S)}$ can be expressed through another set of the SF presented in [19]. Taking into account that $\eta_1 = \cos(\phi_s - \phi_h)S_\perp$, $\eta_2 = \sin(\phi_s - \phi_h)S_\perp$, and $\eta_3 = S_{||}$ we find that

$$
\begin{aligned}
H_{00}^{(0)} &= C_1 F_{UU,L},\\
H_{01}^{(0)} &= -C_1(F_{UU}^{\cos\phi_h} + iF_{LU}^{\sin\phi_h}),\\
H_{11}^{(0)} &= C_1(F_{UU}^{\cos 2\phi_h} + F_{UU,T}),\\
H_{22}^{(0)} &= C_1(F_{UU,T} - F_{UU}^{\cos 2\phi_h}),\\
H_{002}^{(S)} &= C_1 F_{UT,L}^{\sin(\phi_h-\phi_s)},\\
H_{012}^{(S)} &= C_1(F_{UT}^{\sin\phi_s} - F_{UT}^{\sin(2\phi_h-\phi_s)} - i(F_{LT}^{\cos\phi_s} - F_{LT}^{\cos(2\phi_h-\phi_s)})),\\
H_{021}^{(S)} &= C_1(F_{UT}^{\sin(2\phi_h-\phi_s)} + F_{UT}^{\sin\phi_s} - i(F_{LT}^{\cos(2\phi_h-\phi_s)} + F_{LT}^{\cos\phi_s})),\\
H_{023}^{(S)} &= C_1(F_{UL}^{\sin\phi_h} - iF_{LL}^{\cos\phi_h}),\\
H_{121}^{(S)} &= C_1(-F_{UT}^{\sin(3\phi_h-\phi_s)} - F_{UT}^{\sin(\phi_h+\phi_s)} + iF_{LT}^{\cos(\phi_h-\phi_s)}),\\
H_{123}^{(S)} &= C_1(-F_{UL}^{\sin 2\phi_h} + iF_{LL}),\\
H_{112}^{(S)} &= C_1(F_{UT}^{\sin(3\phi_h-\phi_s)} + F_{UT,T}^{\sin(\phi_h-\phi_s)} - F_{UT}^{\sin(\phi_h+\phi_s)}),\\
H_{222}^{(S)} &= C_1(F_{UT}^{\sin(\phi_h+\phi_s)} + F_{UT,T}^{\sin(\phi_h-\phi_s)} - F_{UT}^{\sin(3\phi_h-\phi_s)}),
\end{aligned}
\tag{17}
$$

where

$$C_1 = \frac{4Mp_l(Q^2 + 2xM^2)}{Q^4}. \tag{18}$$

III. LOWEST ORDER RADIATIVE CORRECTIONS

The six matrix elements shown in Figs. 1(b)–1(g) contribute to the lowest order QED RC to the cross section of the base SIDIS process [Fig. 1(a)]. A critical difference in the graphs 1(a)–1(e) compared to the graphs 1(f) and 1(g) is the distinct final unobserved hadronic state: continuum of particles in the former case and a single hadron in the latter case. The underlying processes are semi-inclusive and exclusive hadron leptoproduction, respectively. At the level of RC, both of them include the unobservable real photon emission from the lepton leg as presented in Figs. 1(b) and 1(c) as well as 1(f) and 1(g). The contribution to RC from the semi-inclusive process contains also the leptonic vertex correction and vacuum polarization [Figs. 1(d) and 1(e)]. Thus these two separate contributions to the total RC to the SIDIS cross section are considered in two separate subsections below.

A. Semi-inclusive contribution

The real photon emission in the semi-inclusive process,

$$e(k_1,\xi) + n(p,\eta) \to e(k_2) + h(p_h) + x(\tilde{p}_x) + \gamma(k), \tag{19}$$

where k is a real photon four-momentum depicted in Figs. 1(b) and 1(c) is described by the set variables presented in (2) and three additional quantities,

$$R = 2kp, \qquad \tau = \frac{kq}{kp}, \qquad \phi_k, \tag{20}$$

where ϕ_k is an angle between $(\mathbf{k}_1, \mathbf{k}_2)$ and $(\mathbf{k}, \mathbf{q})$ planes. Its sine in the covariant form is

$$\sin\phi_k = \frac{2\varepsilon_\perp k\sqrt{\lambda_Y}}{R\sqrt{\lambda_1(Q^2 + \tau(S_x - \tau M^2))}}. \tag{21}$$

The contribution of real photon emission from the leptonic leg is

$$d\sigma_R = \frac{(4\pi\alpha)^3}{2\sqrt{\lambda_S}\tilde{Q}^4}\tilde{W}_{\mu\nu}L_R^{\mu\nu}d\Gamma_R. \tag{22}$$

Here the "tilde" symbol denotes that the arguments of the hadronic tensor such as Q^2, W^2, z, t, and ϕ_h are defined through the shifted $q \to q - k$, i.e., $\tilde{Q}^2 = -(q-k)^2 = Q^2 + R\tau$. The phase space of the considered process has the form

$$d\Gamma_R = (2\pi)^4 \frac{d^3k}{(2\pi)^3 2k_0}\frac{d^3k_2}{(2\pi)^3 2k_{20}}\frac{d^3p_h}{(2\pi)^3 2p_{h0}}, \tag{23}$$

where

$$\frac{d^3k}{k_0} = \frac{RdRd\tau d\phi_k}{2\sqrt{\lambda_Y}}. \tag{24}$$

For the representation of explicit results in the simplest way the leptonic tensor $L_R^{\mu\nu}$ in (22) is separated into two parts:

$$L_R^{\mu\nu} = L_{R0}^{\mu\nu} + L_{R1}^{\mu\nu}. \tag{25}$$

The first term includes the part of the leptonic tensor that contains spin-independent terms and terms containing ξ_0, i.e., the part of the polarization vector (10),

$$L_{R0}^{\mu\nu} = -\frac{1}{2}\mathrm{Tr}[(\hat{k}_2 + m)\Gamma_R^{\mu\alpha}(\hat{k}_1 + m)(1 + \gamma_5\hat{\xi}_0)\bar{\Gamma}_{R\alpha}^{\nu}], \tag{26}$$

where

$$
\begin{aligned}
\Gamma_R^{\mu\alpha} &= \left(\frac{k_1^\alpha}{kk_1} - \frac{k_2^\alpha}{kk_2}\right)\gamma^\mu - \frac{\gamma^\mu\hat{k}\gamma^\alpha}{2kk_1} - \frac{\gamma^\alpha\hat{k}\gamma^\mu}{2kk_2},\\
\bar{\Gamma}_{R\alpha}^{\nu} &= \gamma_0\Gamma_{R\alpha}^{\nu\dagger}\gamma_0\\
&= \left(\frac{k_{1\alpha}}{kk_1} - \frac{k_{2\alpha}}{kk_2}\right)\gamma^\nu - \frac{\gamma^\nu\hat{k}\gamma_\alpha}{2kk_2} - \frac{\gamma_\alpha\hat{k}\gamma^\nu}{2kk_1}.
\end{aligned}
\tag{27}
$$

The second term in (25) is proportional only to the residual part ξ_1 of the polarization vector ξ,

$$L_{R1}^{\mu\nu} = -\frac{1}{2}\mathrm{Tr}[(\hat{k}_2 + m)\Gamma_R^{\mu\alpha}(\hat{k}_1 + m)\gamma_5\hat{\xi}_1\bar{\Gamma}_{R\alpha}^{\nu}]. \tag{28}$$

As shown below this part of the leptonic tensor gives a nonvanishing contribution to RC in the ultrarelativistic approximation for both the semi-inclusive (72) and the exclusive (74) final hadronic states.

The convolution of the leptonic tensors $L^{\mu\nu}_{R0}$ and $L^{\mu\nu}_{R1}$ with the shifted hadronic tensor can be presented as

$$\tilde{W}_{\mu\nu}L^{\mu\nu}_{R0}=\sum_{i=1}^{9}\tilde{w}^{i}_{\mu\nu}\tilde{\mathcal{H}}_i L^{\mu\nu}_{R0}=-2\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta^0_{ij}R^{j-3},$$

$$\tilde{W}_{\mu\nu}L^{\mu\nu}_{R1}=\sum_{i=5,7,9}\tilde{w}^{i}_{\mu\nu}\tilde{\mathcal{H}}_i L^{\mu\nu}_{R1}=-2\sum_{i=5,7,9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta^1_{ij}R^{j-3}, \qquad (29)$$

where i enumerates the contributions of respective SF in (13). The sum over j represents the decomposition of the leptonic (26) and (28) and hadronic tensor convolutions over R. In this decomposition quantities $\theta^{0,1}_{ij}$ do not depend on R. Their explicit expressions are presented in Appendix B. The number of terms is different for different SF: $k_i=\{3,3,3,3,3,4,4,4,4\}$.

The lowest order SIDIS process (1) is described by the four independent four-momenta such us p, k_1, q, and p_h. Therefore, the Born cross section contains only one pseudoscalar $\varepsilon^{\mu\nu\rho\sigma}p_{h\mu}p_\nu k_{1\rho}q_\sigma$. This pseudoscalar contributes to $\theta^B_{5,6,8}$ as it was shown in Eqs. (16) and, according to Eqs. (6) and (7), can be expressed in terms of the variables (2)–(4) as $\varepsilon^{\mu\nu\rho\sigma}p_{h\mu}p_\nu k_{1\rho}q_\sigma=\varepsilon_\perp p_h=-p_t\sqrt{\lambda_1}\sin\phi_h/2$. When we deal with real photon emission, the additional independent four-momentum k appears. As a result the number of pseudoscalar quantities that can exist in the expressions for the cross section grows up to five. They are not independent, and their number can be reduced to two, namely $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$, using the decomposition of the photonic four-momentum over the basis introduced in Appendix A by Eqs. (A5). As shown in Eqs. (A9) the remaining three pseudoscalars are expressed through the linear combination of $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$. The explicit expression for $\varepsilon_\perp k$ follows from (21):

$$\varepsilon_\perp k=\frac{\sin\phi_k R\sqrt{\lambda_1(Q^2+\tau(S_x-\tau M^2))}}{2\sqrt{\lambda_Y}}. \qquad (30)$$

After substituting (29) into (22)

$$d\sigma_R=-\frac{\alpha^3}{4\pi^2\tilde{Q}^4\sqrt{\lambda_S}}\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}R^{j-3}\frac{d^3k}{k_0}\frac{d^3k_2}{k_{20}}\frac{d^3p_h}{p_{h0}}$$
$$=-\frac{\alpha^3SS_x^2dxdydzdp_td\phi_hd\phi d\tau d\phi_kdR}{64\pi^2Mp_l\lambda_S\sqrt{\lambda_Y}\tilde{Q}^4}\times\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}R^{j-2}, \qquad (31)$$

where $\theta_{ij}=\theta^0_{ij}$ for $i=1$–4,6,8 and $\theta_{ij}=\theta^0_{ij}+\theta^1_{ij}$ for $i=5$, 7, 9, we found that the term with $j=1$ in (31) contains the infrared divergence at $R\to0$ that does not allow one to perform the straightforward integration of $d\sigma_R$ over the photonic variable R. For the correct extraction and cancellation of the infrared divergence the Bardin-Shumeiko approach [12] is used. Following this method the identical transformation,

$$d\sigma_R=d\sigma_R-d\sigma_R^{\rm IR}+d\sigma_R^{\rm IR}=d\sigma_R^F+d\sigma_R^{\rm IR}, \qquad (32)$$

is performed. Here $d\sigma_R^F$ is the infrared free contribution and $d\sigma_R^{\rm IR}$ contains only the $j=1$ term in which arguments of SF are taken for $k=0$,

$$d\sigma_R^{\rm IR}=-\frac{\alpha^3}{4\pi^2Q^4\sqrt{\lambda_S}}\sum_{i=1}^{9}\frac{\mathcal{H}_i\theta_{i1}}{R^2}\frac{d^3k}{k_0}\frac{d^3k_2}{k_{20}}\frac{d^3p_h}{p_{h0}}. \qquad (33)$$

This decomposition allows us to perform the treatment of the infrared divergence analytically since the arguments of the SF in (33) do not depend on photonic variables. Due to $\theta_{i1}=4F_{\rm IR}\theta^B_i$ one can find that this contribution can be factorized in front of the Born cross section

$$d\sigma_R^{\rm IR}=-\frac{\alpha}{\pi^2}d\sigma^B\frac{F_{\rm IR}}{R^2}\frac{d^3k}{k_0}, \qquad (34)$$

where

$$F_{\rm IR}=\left(\frac{k_1}{z_1}-\frac{k_2}{z_2}\right)^2, \qquad (35)$$

$z_{1,2}=kk_{1,2}/kp$, and the explicit expressions of these quantities are given in Appendix B [see (B4)].

The term (34) is then separated into the soft δ_S and hard δ_H parts,

$$\sigma_R^{\rm IR}=\frac{\alpha}{\pi}(\delta_S+\delta_H)\sigma^B, \qquad (36)$$

by the introduction of the infinitesimal photonic energy $\bar{k}_0\to0$ that is defined in the system $\mathbf{p}+\mathbf{q}-\mathbf{p_h}=0$:

$$\delta_S=-\frac{1}{\pi}\int\frac{d^3k}{k_0}\frac{F_{\rm IR}}{R^2}\theta(\bar{k}_0-k_0),$$
$$\delta_H=-\frac{1}{\pi}\int\frac{d^3k}{k_0}\frac{F_{\rm IR}}{R^2}\theta(k_0-\bar{k}_0). \qquad (37)$$

The additional regularization with the parameter $\bar{k}_0$ allows us to calculate δ_H for $n=4$ and to simplify the integration for δ_S in the dimensional regularization by choosing the individual reference systems for each invariant variables z_1 and z_2 to make them independent of the azimuthal angle ϕ_k.

The explicit integration, details of which are described in Appendix C, results in the final explicit expressions for these two contributions in the form

$$\delta_S = 2(Q_m^2 L_m - 1)\left(P_{\rm IR} + \log\frac{2\bar{k}_0}{\nu}\right) + \frac{1}{2}S' L_{S'} + \frac{1}{2}X' L_{X'} + S_\phi,$$
$$\delta_H = 2(Q_m^2 L_m - 1)\log\frac{p_x^2 - M_{th}^2}{2\bar{k}_0\sqrt{p_x^2}}. \quad (38)$$

Here M_{th} is the minimum value of the invariant mass of the undetected hadrons p_x for the SIDIS process, e.g., $M_{th} = M + m_\pi$ when the detected hadron is the pion. The symbols L_m, $L_{S'}$, and $L_{X'}$ are defined in Eq. (C10).

The sum of δ_S and δ_H does not depend on the separated photonic energy $\bar{k}_0$ but includes the term representing the infrared divergence

$$P_{\rm IR} = \frac{1}{n-4} + \frac{1}{2}\gamma_E + \log\frac{1}{2\sqrt{\pi}} \quad (39)$$

as well as the arbitrary parameter ν, the mass scale of dimensional regularization. These two quantities should be canceled by summing the infrared divergent part with the contribution from the leptonic vertex correction that is considered below.

The term S_ϕ has the form

$$S_\phi = -\frac{Q_m^2}{2\sqrt{\lambda_m}}\left\{\log\frac{X' - \sqrt{\lambda'_X}}{X' + \sqrt{\lambda'_X}}\log\frac{(z-z_1)(z-z_3)}{(z-z_2)(z-z_4)} + \sum_{i,j}^{4} S_j(-1)^{i+1}\left(\frac{1}{2}\delta_{ij}\log^2(|z-z_i|) + (1-\delta_{ij})\left[\log(|z-z_i|)\log(|z_i-z_j|) - {\rm Li}_2\left(\frac{z-z_i}{z_j-z_i}\right)\right]\right)\right\}\Bigg|_{z=z_d}^{z=z_u}, \quad (40)$$

where

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\log|1-y|}{y}dy \quad (41)$$

is Spence's dilogarithm and

$$z_{1,2} = \frac{1}{\sqrt{\lambda'_X}}\left(X' - S' + \frac{2p_x^2(Q^2 \mp \sqrt{\lambda_m})}{X' - \sqrt{\lambda'_X}}\right),$$
$$z_{3,4} = \frac{1}{\sqrt{\lambda'_X}}\left(S' - X' - \frac{2p_x^2(Q^2 \pm \sqrt{\lambda_m})}{X' + \sqrt{\lambda'_X}}\right),$$
$$z_u = \sqrt{\frac{\lambda'_S}{\lambda'_X}} - 1, \qquad z_d = \frac{X'(S'-X') - 2p_x^2 Q^2}{\lambda'_X},$$
$$S_j = \{1, 1, -1, -1\}. \quad (42)$$

The infrared free contribution $d\sigma_R^F$ from (32) integrated over the three photonic variables reads

$$\sigma_R^F = -\frac{\alpha^3 S S_x^2}{64\pi^2 M p_l \lambda_S\sqrt{\lambda_Y}}\int_{\tau_{\rm min}}^{\tau_{\rm max}} d\tau \int_0^{2\pi} d\phi_k \int_0^{R_{\rm max}} dR \times \sum_{i=1}^{9}\left[\frac{\theta_{i1}}{R}\left(\frac{\tilde{\mathcal{H}}_i}{\tilde{Q}^4} - \frac{\mathcal{H}_i}{Q^4}\right) + \sum_{j=2}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}\frac{R^{j-2}}{\tilde{Q}^4}\right], \quad (43)$$

where the limits of integration are

$$R_{\rm max} = \frac{p_x^2 - M_{th}^2}{1+\tau-\mu},$$
$$\tau_{\rm max/min} = \frac{S_x \pm \sqrt{\lambda_Y}}{2M^2} \quad (44)$$

and the quantity μ is defined in Eq. (B3).

The additional virtual particle contributions consist of the leptonic vertex correction [Fig. 1(d)] and vacuum polarization by leptons and hadrons [Fig. 1(e)]. These contributions are given by Eq. (8) with the replacement of the leptonic tensor $L_B^{\mu\nu}$ by

$$L_V^{\mu\nu} = \frac{1}{2}{\rm Tr}[(\hat{k}_2 + m)\Gamma_V^\mu(\hat{k}_1 + m)(1+\gamma_5\hat{\xi})\gamma^\nu] + \frac{1}{2}{\rm Tr}[(\hat{k}_2 + m)\gamma^\mu(\hat{k}_1 + m)(1+\gamma_5\hat{\xi})\bar{\Gamma}_V^\nu], \quad (45)$$

where

$$\Gamma_V^\mu = \Lambda^\mu + \Pi_\alpha^{l\mu}\gamma^\alpha + \frac{\alpha}{2\pi}\delta_{\rm vac}^h\gamma^\mu \quad (46)$$

and $\bar{\Gamma}_V^\nu = \gamma_0\Gamma_V^{\nu\dagger}\gamma_0$.

The first two terms corresponding to the leptonic vertex correction Λ_μ and vacuum polarization by leptons $\Pi_\alpha^{l\mu}$ are calculated analytically using Feynman rules while the fit for the vacuum polarization by hadrons $\delta_{\rm vac}^h$ can be taken from the experimental data [20].

Since Λ_μ and $\Pi_\alpha^{l\mu}$ contain the ultraviolet divergence while Λ_μ also includes the infrared divergent term, the dimensional regularization is used for the calculation of the loop integrals:

$$\Lambda_\mu = -ie^2\int\frac{d^n l}{(2\pi)^n\nu^{n-4}} \times \frac{\gamma_\alpha(\hat{k}_2 - \hat{l} + m)\gamma_\mu(\hat{k}_1 - \hat{l} + m)\gamma^\alpha}{l^2(l^2 - 2lk_2)(l^2 - 2lk_1)},$$
$$\Pi_{\alpha\mu}^l = -\frac{ie^2}{Q^2}\int\frac{d^n l}{(2\pi)^n\nu^{n-4}} \times \left\{\sum_{i=e,\mu,\tau}\frac{{\rm Tr}[(\hat{l} + m_i)\gamma_\alpha(\hat{l} - \hat{q} + m_i)\gamma_\mu]}{(l^2 - m_i^2)((l-q)^2 - m_i^2)}\right\}. \quad (47)$$

Details of the calculations are presented in Appendix D; Λ_μ and $\Pi_{\alpha\mu}^i$ have the following structure:

$$\Lambda_\mu = \frac{\alpha}{2\pi}\left(\delta^{\rm UV}_{\rm vert}(Q^2)\gamma_\mu - \frac{1}{2}mL_m[\hat{q},\gamma_\mu]\right),$$
$$\Pi^l_{\alpha\mu} = \sum_{i=e,\mu,\tau}\frac{\alpha}{2\pi}\delta^{i\,\rm UV}_{\rm vac}(Q^2)g^\perp_{\alpha\mu}, \qquad (48)$$

where the second term in Λ_μ is the anomalous magnetic moment. To remove the ultraviolet divergence the standard on the mass-shell renormalization procedure is used: $\delta^{\rm UV}_{\rm vert}(Q^2)$ and $\delta^{i\,\rm UV}_{\rm vac}(Q^2)$ are substituted by the difference of these quantities and their values at $Q^2 = 0$:

$$\delta_{\rm vert} = \delta^{\rm UV}_{\rm vert}(Q^2) - \delta^{\rm UV}_{\rm vert}(0),$$
$$\delta^i_{\rm vac} = \delta^{i\,\rm UV}_{\rm vac}(Q^2) - \delta^{i\,\rm UV}_{\rm vac}(0). \qquad (49)$$

Here $\delta^{\rm UV}_{\rm vert}(0) = 2 - P_{\rm UV} - 2P_{\rm IR} - 3\log(m/\nu)$, $\delta^{i\,\rm UV}_{\rm vac}(0) = 4(P_{\rm UV} + \log(m_i/\nu))/3$, $P_{\rm UV} = P_{\rm IR}$, and the ultraviolet free terms have the form

$$\delta_{\rm vert} = -2(Q^2_mL_m - 1)\left(P_{\rm IR} + \log\frac{m}{\nu}\right) - 2 + \left(\frac{3}{2}Q^2 + 4m^2\right)L_m - \frac{Q^2_m}{\sqrt{\lambda_m}}\left(\frac{1}{2}\lambda_mL^2_m + 2{\rm Li}_2\left(\frac{2\sqrt{\lambda_m}}{Q^2+\sqrt{\lambda_m}}\right) - \frac{\pi^2}{2}\right),$$
$$\delta^l_{\rm vac} = \sum_{i=e,\mu,\tau}\delta^i_{\rm vac} = \sum_{i=e,\mu,\tau}\left[\frac{2}{3}(Q^2+2m^2_i)L^i_m - \frac{10}{9} + \frac{8m^2_i}{3Q^2}\left(1 - 2m^2_iL^i_m\right)\right]. \qquad (50)$$

The quantity L_m is defined in (C10) while the expressions for λ^i_m and L^i_m are defined by Eqs. (D3).

Finally the contribution of the inelastic tail to the sixfold SIDIS cross section reads

$$\sigma^{\rm in} = \frac{\alpha}{\pi}(\delta_{VR} + \delta^l_{\rm vac} + \delta^h_{\rm vac})\sigma^B + \sigma^F_R + \sigma^{\rm AMM}, \qquad (51)$$

where the sum of the infrared divergent terms,

$$\delta_{VR} = \delta_S + \delta_H + \delta_{\rm vert} = 2(Q^2_mL_m - 1)\log\frac{p^2_x - M^2_{th}}{m\sqrt{p^2_x}} + \frac{1}{2}S'L_{S'} + \frac{1}{2}X'L_{X'} + S_\phi - 2 + \left(\frac{3}{2}Q^2 + 4m^2\right)L_m - \frac{Q^2_m}{\sqrt{\lambda_m}}\left(\frac{1}{2}\lambda_mL^2_m + 2{\rm Li}_2\left(\frac{2\sqrt{\lambda_m}}{Q^2+\sqrt{\lambda_m}}\right) - \frac{\pi^2}{2}\right), \qquad (52)$$

is free both from the infrared divergent term $P_{\rm IR}$ appearing in δ_S and $\delta_{\rm vert}$ that are defined by Eqs. (38) and (50) and the arbitrary parameter ν. The infrared free contribution σ^F_R is defined by Eq. (43).

At last the contribution of the anomalous magnetic moment coming from the second term in Λ_μ given by Eqs. (48) has the form

$$\sigma^{\rm AMM} = \frac{\alpha^3m^2SS^2_x}{16\pi MQ^2p_l\lambda_S}L_m\sum_{i=1}^{9}\theta^{\rm AMM}_i\mathcal{H}_i, \qquad (53)$$

with

$$\theta^{\rm AMM}_1 = 6, \quad \theta^{\rm AMM}_2 = -\frac{\lambda_Y}{2Q^2},$$
$$\theta^{\rm AMM}_3 = -2m^2_h - 2\frac{V^2_-}{Q^2}, \quad \theta^{\rm AMM}_4 = -2S_x\left(z + \frac{V_-}{Q^2}\right),$$
$$\theta^{\rm AMM}_5 = \frac{2\lambda_e(2S+S_x)\varepsilon_\perp p_h}{\sqrt{\lambda_S}Q^2},$$
$$\theta^{\rm AMM}_7 = \frac{\lambda_e(2S+S_x)}{4\sqrt{\lambda_S}Q^2}(S_x(SV_2 - XV_1 - zS_pQ^2) + 4M^2Q^2V_+),$$
$$\theta^{\rm AMM}_9 = \frac{\lambda_e}{2\sqrt{\lambda_S}Q^2}(S^2_x(4m^2(m^2_h - z(zQ^2+2V_-)) + V_1V_-) - 4(M^2(Q^2-4m^2)+S^2)(m^2_hQ^2+V^2_-) + zQ^2S_x(S_x(zQ^2+V_1+V_-)+2SV_+)+2SS_xV_-V_+),$$
$$\theta^{\rm AMM}_6 = \theta^{\rm AMM}_8 = 0. \qquad (54)$$

B. Exclusive radiative tail

The exclusive radiative tail is the process

$$e(k_1,\xi) + n(p,\eta) \to e(k_2) + h(p_h) + u(p_u) + \gamma(k), \qquad (55)$$

where p_u is the four-momentum of a single undetected hadron ($p^2_u = m^2_u$) shown in Figs. 1(f) and 1(g). The final unobserved state contains the photon radiated from the lepton line and a hadron produced in an exclusive reaction of γ^* and p. The process (55) gives a contribution to the RC in SIDIS because two observed particles in the final state can have the same momenta as the unobserved particles in the SIDIS process (1). The square of the invariant mass of the unobserved state $p^2_x = (p+q-p_h)^2 = 2k(p+q-p_h) + m^2_u$ depends on the photonic variables. Emission of the soft photons would result in $p^2_x = m^2_u$. This is beyond the kinematic region of SIDIS. Therefore, the process (55) being the contribution to RC to the SIDIS cross section does not contain the infrared divergence [10].

A description of the exclusive process without the radiated photon requires only five of the six variables of SIDIS presented in Eqs. (2): x, y, t, ϕ_h, and ϕ. The process with the radiated photon is additionally described by the three photonic variables R, τ, and ϕ_k introduced above by Eq. (20). In this case the sixth SIDIS variable z is expressed through other SIDIS and photonic variables:

$$z = \frac{M^2 - m_u^2 + t - R(1+\tau-\mu)}{S_x} + 1, \tag{56}$$

where μ is defined by Eq. (B3). Since we calculate RC to SIDIS we need to keep z and use this equation in order to express R in terms of z and two remaining photonic variables:

$$R_{\rm ex} = \frac{p_x^2 - m_u^2}{1+\tau-\mu}, \tag{57}$$

and therefore to reduce the integration over the photon momentum to the two-dimensional integral with respect to variables τ and ϕ_k.

The contribution of the exclusive radiative tail in the form similar to (22) reads

$$d\sigma_R^{\rm ex} = \frac{(4\pi\alpha)^3}{2\sqrt{\lambda_S}\tilde{Q}^4}\tilde{W}_{\rm ex}^{\mu\nu}L_{\mu\nu}^R d\Gamma_R^{\rm ex}, \tag{58}$$

where the hadronic tensor $W_{\rm ex}^{\mu\nu}$ describes the exclusive process $\gamma^* + n \to h + u$ and has the same structure as the hadronic tensor in Eq. (13) but with the SF dependent only on Q^2, W^2, and t variables. The leptonic tensor $L_{\mu\nu}^R$ and its convolution with the hadronic structures $\tilde{w}_i^{\mu\nu}$ are the same as in Eqs. (26)–(29).

The phase space of this process is

$$\begin{aligned} d\Gamma_R^{\rm ex} &= \frac{1}{(2\pi)^8}\frac{d^3k_2}{2k_{20}}\frac{d^3k}{2k_0}\frac{d^3p_h}{2p_{h0}}\frac{d^3p_u}{2p_{u0}} \\ &\quad\times\delta^4(k_1+p-k_2-p_h-p_u-k) \\ &= \frac{2R_{\rm ex}SS_x^2dxdyd\phi dzd\phi_h dp_t^2 d\tau d\phi_k}{(4\pi)^8(1+\tau-\mu)Mp_l\sqrt{\lambda_S\lambda_Y}}. \end{aligned} \tag{59}$$

The use of the phase space (59) and convolution of leptonic and hadronic tensors (29) with replacement $\tilde{\mathcal{H}}_i \to \tilde{\mathcal{H}}_i^{\rm ex}$ in (58) and the subsequent integration of the obtained expression over two photonic variables results in the contribution of the exclusive radiative tail to the SIDIS process in the form

$$\begin{aligned} \sigma_R^{\rm ex} &= -\frac{\alpha^3 SS_x^2}{2^9\pi^5 Mp_l\lambda_S\sqrt{\lambda_Y}}\int_{\tau_{\min}}^{\tau_{\max}} d\tau\int_0^{2\pi} d\phi_k \\ &\quad\times\sum_{i=1}^{9}\sum_{j=1}^{k_i}\frac{\tilde{\mathcal{H}}_i^{\rm ex}\theta_{ij}R_{\rm ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}. \end{aligned} \tag{60}$$

IV. ULTRARELATIVISTIC APPROXIMATION

In Sec. III all contributions to the lowest order RC are presented by exact formulas. Some of them have a rather complicated analytical structure. However, due to the smallness of the leptonic mass compared to other quantities that describe kinematics of the process it is rather useful to obtain RC in the ultrarelativistic approximation keeping the leptonic mass m only as an argument of the logarithmic function. This allows us to simplify the analytical expressions essentially as well as clarify the leading log behavior of the obtained results. In other words, the lowest order QED RC in this approximation has the form

$$\sigma_{\rm RC} = \frac{\alpha}{\pi}\left[Al_m + B + \mathcal{O}\left(\frac{m^2}{Q^2}\right)\right], \tag{61}$$

where $l_m = \log Q^2/m^2$ and the terms A and B are independent of the leptonic mass and represent the lowest order leading and next-to-leading contributions to the RC to the cross section, respectively.

The terms in (51) that are factorized in front of the Born contribution are essentially simplified, resulting in a more transparent structure after applying the ultrarelativistic approximation, e.g., the terms (36)

$$\begin{aligned} \sigma_R^{\rm IR} = \frac{\alpha}{\pi}\bigg[&(l_m-1)\left(2P_{\rm IR} + 2\log\frac{m}{\nu} + \log\frac{(p_x^2-M_{th}^2)^2}{S'X'}\right) \\ &+\frac{1}{2}l_m^2 - \frac{1}{2}\log^2\frac{S'}{X'} + {\rm Li}_2\left\{1-\frac{Q^2p_x^2}{S'X'}\right\} - \frac{\pi^2}{3}\bigg]\sigma_0 \end{aligned} \tag{62}$$

contain both l_m and l_m^2. The latter comes from the soft photon emission whose contribution cancels in the sum with the leptonic vertex correction:

$$\begin{aligned} \delta_{VR} &= (l_m-1)\log\frac{(p_x^2-M_{th}^2)^2}{S'X'} + \frac{3}{2}l_m \\ &\quad -\frac{1}{2}\log^2\frac{S'}{X'} + {\rm Li}_2\left\{1-\frac{Q^2p_x^2}{S'X'}\right\} - \frac{\pi^2}{6} - 2. \end{aligned} \tag{63}$$

The vacuum polarization by lepton i ($i = e, \mu, \tau$) in the limit $Q^2 \gg m_i^2$ reads

$$\delta_{\rm vac}^i = \frac{2}{3}\log\frac{Q^2}{m_i^2} - \frac{10}{9}. \tag{64}$$

The ultrarelativistic approximation for the hard photon emission contribution (43), (60) requires additional care because of the integration over photonic variables and the nontrivial dependence of the integrand on the leptonic mass. Specifically, the integrand contains the terms $1/z_1$ and $1/z_1^2$:

$$\begin{aligned} \int_0^{2\pi}\frac{d\phi_k}{z_1} &= \frac{2\pi\sqrt{\lambda_Y}}{\sqrt{(Q^2+\tau S)^2 + 4m^2(\tau(S_x-\tau M^2)+Q^2)}}, \\ \int_0^{2\pi}\frac{d\phi_k}{z_1^2} &= \frac{2\pi(Q^2S_p + \tau(SS_x+2M^2Q^2))\sqrt{\lambda_Y}}{((Q^2+\tau S)^2+4m^2(\tau(S_x-\tau M^2)+Q^2))^{3/2}}. \end{aligned} \tag{65}$$

These have a sharp peaking behavior in the region $\tau \to \tau_s \equiv -Q^2/S$ due to the smallness of the lepton mass. The integration of the expressions (65) over ϕ_k and τ gives

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} \frac{d\phi_k}{z_1} = 2\pi\sqrt{\frac{\lambda_Y}{\lambda_S}} \log\frac{S+\sqrt{\lambda_S}}{S-\sqrt{\lambda_S}},$$
$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} \frac{d\phi_k}{z_1^2} = \frac{2\pi\sqrt{\lambda_Y}}{m^2}. \tag{66}$$

Since

$$\lim_{m\to 0} \log\frac{S+\sqrt{\lambda_S}}{S-\sqrt{\lambda_S}} = l_m + \log\frac{S^2}{Q^2M^2}, \tag{67}$$

the terms containing $1/z_1$ contribute to the leading and next-to-leading RC. The terms containing $1/z_1^2$ also contain m^2 in numerators and therefore contribute to the next-to-leading RC only (the only exception is $\hat{\theta}^0_{53}$ that is discussed below). The similar conclusions are true for the terms containing $1/z_2$ and $1/z_2^2$ terms.

Actually the integrand in (66) has to be multiplied by the SF according to (43) and (60). Therefore, we make the identical transformation for extraction of the leading and next-to-leading terms:

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \frac{\mathcal{G}(\tau,\phi_k)}{z_1} = 2\pi\sqrt{\frac{\lambda_Y}{\lambda_S}}\log\frac{\sqrt{\lambda_S}+S}{\sqrt{\lambda_S}-S}\mathcal{G}(\tau_s,0) + \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \frac{\mathcal{G}(\tau,\phi_k)-\mathcal{G}(\tau_s,0)}{z_1},$$
$$m^2\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \frac{\mathcal{G}(\tau,\phi_k)}{z_1^2} = 2\pi\sqrt{\lambda_Y}\mathcal{G}(\tau_s,0) + \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k m^2 \frac{\mathcal{G}(\tau,\phi_k)-\mathcal{G}(\tau_s,0)}{z_1^2}, \tag{68}$$

where $\mathcal{G}(\tau,\phi_k)$ is a regular function of τ and ϕ_k. The second term in the right-hand side of the first transformation does not include the leading terms, and the second term in the second equality is proportional to m^2 and vanishes in the ultrarelativistic approximation.

The approach of extraction of the leading and next-to-leading contributions can be illustrated by considering the terms originated from the convolution of the leptonic tensor (28) with the hadronic structures $\tilde{w}^i_{\mu\nu}$. Summing up the terms $\theta^1_{ij}R^{j-3}$ in the last expression of Eq. (29) and keeping the leptonic mass only in the term m^2/z_1^2 (in θ^1_{ij} the term $1/z_2^2$ is proportional to m^4) results in

$$\tilde{w}^i_{\mu\nu}L^{\mu\nu}_{R1} = -2\sum_{j=1}^{k_i}\theta^1_{ij}R^{j-3} = \frac{m^2}{z_1^2}\theta^1_i(R,\tau,\phi_k) \tag{69}$$

with the quantities $\theta^1_i(R,\tau_s,0)$ expressed through (16) as

$$\theta^1_i(R,\tau_s,0) = \frac{4R}{S(S-R)}\theta^B_i\left(k_1 \to \left(1-\frac{R}{S}\right)k_1\right). \tag{70}$$

The replacement in the brackets is applied for any kinematic variable defined through k_1, e.g., $S \to S-R$, $Q^2 \to (1-S/R)Q^2$, and $\varepsilon_\perp p_h \to (1-S/R)\varepsilon_\perp p_h$. Note that $R = R_{\text{ex}}$ has to be used for the exclusive radiative tail.

The resulting equation for the $\sigma^{\xi_1}_R$ is obtained using the second equation of (68) with the regular function $\mathcal{G}(\tau,\phi_k)$,

$$\mathcal{G}(\tau,\phi_k) = \int_0^{R_{\max}} \frac{RdR}{(Q^2+\tau R)^2}\sum_{i=5,7,9}\theta^1_i(R,\tau,\phi_k)\tilde{\mathcal{H}}_i. \tag{71}$$

Therefore, the contribution from the second part ξ_1 of the lepton polarized vector (10) reads

$$\sigma^{\xi_1}_R = -\frac{\alpha S_x^2}{\pi M p_l S^2}\int_0^{R^s_{\max}} \frac{p^s_l R dR}{(S_x-R)^2}\tilde{\sigma}^B_{pl}, \tag{72}$$

where

$$p^s_l = \frac{zSS_x(S_x-R)+2M^2(RV_1-2SV_-)}{2M\sqrt{S(4M^2Q^2(S-R)+S(S_x-R)^2)}},$$
$$R^s_{\max} = S(p_x^2-M_{th}^2)/S', \tag{73}$$

and $\tilde{\sigma}^B_{pl}$ is proportional to the λ_e part of the Born contribution with the following replacement: $m\to 0$, $S\to S-R$, $Q^2\to Q^2(1-R/S)$, $V_1\to V_1(1-R/S)$, and $z\to zS_x/(S_x-R)$.

A similar calculation of the exclusive radiative tail results in

$$\sigma^{\text{ex}\,\xi_1}_R = -\frac{\alpha S_x^2 R^s_{\text{ex}} p^{s\,\text{ex}}_l}{\pi M p_l SS'(S_x-R^s_{\text{ex}})}\frac{d\tilde{\sigma}^{\text{ex}\,B}_{pl}}{d\tilde{x}d\tilde{y}d\tilde{p}_t d\phi_h d\phi}, \tag{74}$$

where

$$p^{s\,\text{ex}}_l = \frac{1}{2M\sqrt{S(4M^2Q^2(S-R^s_{\text{ex}})+S(S_x-R^s_{\text{ex}})^2)}} \times [(S_x-R^s_{\text{ex}})(S(S_x-2V_-+m_h^2-m_u^2) - R^s_{\text{ex}}(S-V_1)) - Q^2(S-R^s_{\text{ex}})(S_x-R^s_{\text{ex}}) + M^2(S(S_x-4V_-)-R^s_{\text{ex}}(S-2V_1))], \tag{75}$$

$R^s_{\rm ex} = S(p_x^2 - m_u^2)/S'$, and the exclusive Born cross section reads

$$\frac{d\sigma^{\mathrm{ex}\,B}_{pl}}{dxdydp_t d\phi_h d\phi} = \frac{\alpha^2 S S_x}{64\pi^3 Q^4 M p_l \lambda_S} \times \sum_{i=5,7,9} \mathcal{H}^{\mathrm{ex}}_i \theta^B_i \left(z \to \frac{t + M^2 - m_u^2}{S_x} + 1 \right). \tag{76}$$

Finally, we consider the extraction of the leading and next-to-leading terms in the quantity $\hat\theta^0_{53}$ given in Appendix B. In contrast to other $\hat\theta^0_{ij}$, the quantity $\hat\theta^0_{53}$ includes terms $1/z_1^2$ without factors proportional to m^2 and therefore can potentially result in electron mass singularity $\sim m^{-2}$ after integration (66). This is, however, not the case because $\hat\theta^0_{53} = 0$ at the peak point, i.e., for $\tau = \tau_s = -Q^2/S$ (and $\mu = V_1/S$). Explicit integration in the limit $m^2 \to 0$,

$$\int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \hat\theta^0_{53} = -\frac{2\lambda_e \pi p_t \sin\phi_h \sqrt{\lambda_Y}}{M^2 S^2 \sqrt{Q^2(SX - M^2Q^2)}} \times \left[4M^2Q^2(SX - M^2Q^2)\left(l_m + \log\frac{S^2}{Q^2M^2} - 3 \right) + S^2\lambda_Y \right] \tag{77}$$

shows that $\hat\theta^0_{53}$ has a standard form $A\log(Q^2/m^2) + B$.

The final result for the observed cross section in the ultrarelativistic approximation is obtained by the following substitutions in Eqs. (51) and (60): (i) $\sigma^{\mathrm{AMM}} = 0$; (ii) Eqs. (63) and (64) for δ_{VR} and $\delta^i_{\rm vac}$; (iii) $m = 0$ in the Born cross section [Eq. (15)]; and (iv) $\sigma^F_R = \sigma^{F1}_R + \sigma^{\xi_1}_R$ and $\sigma^{\rm ex}_R = \sigma^{\mathrm{ex}\,1}_R + \sigma^{\mathrm{ex}\,\xi_1}_R$, where $\sigma^{\xi_1}_R$ and $\sigma^{\mathrm{ex}\,\xi_1}_R$ are given by (72) and (74), respectively, and σ^{F1}_R and $\sigma^{\mathrm{ex}\,1}_R$ are given by Eqs. (43) and (60) with $\theta^1_{ij} = 0$ and the leptonic mass keeping only in the coefficients at F_{21} and F_{22}.

V. CONCLUSION

Newly achieved accuracies in modern SIDIS experiments in TJNAF and CERN require renewed attention to RC calculations and their implementation in data analysis software. In this paper we obtained the exact analytical expressions for the lowest order model-independent part of QED RC to the SIDIS cross section with the longitudinally polarized initial lepton and arbitrarily polarized target and demonstrated how the leading and next-to-leading contributions can be extracted. The model-independent RC includes (i) the contributions of radiated SIDIS processes and loop diagrams (51) and (ii) the contribution of the exclusive radiative tail (60). The methodology developed in this paper is the extension of the covariant approach for the RC calculations developed earlier: (i) the method of covariant extraction and cancellation of the infrared divergence suggested by Bardin and Shumeiko [12]; (ii) the set of integration variables used in RC calculation to DIS [16]; (iii) RC to unpolarized and polarized SIDIS in the quark-parton model [6–8], (iv) RC for SIDIS of unpolarized particles [9], and (v) the calculation of the exclusive radiative tail for unpolarized SIDIS [10]. The calculations of RC in SIDIS measurements were performed by the model-independent way that involves constructing and using the SIDIS (and exclusive) hadronic tensor containing the 18 SIDIS and exclusive SF. We obtained the explicit form of the hadronic tensor using the approaches of [17,18] and demonstrated that the Born cross section exactly coincides with that given by [19]. The next step in the RC calculation includes coding of the formulas and numeric evaluation of the effects of the RC. However, this requires models of the SIDIS/exclusive SF that are not known now. Therefore, a broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SF as well as SF in the resonance region and exclusive SF. Further development will include development of (i) the iteration procedure with fitting of measured SF and joining with models beyond SIDIS measurements at each iteration step, and (ii) tools for generation of the radiated photon. Such a generator can be constructed based on a code for RC in SIDIS in the same way RADGEN [21] is constructed based on POLRAD 2.0. Generation of semi-inclusive processes based on DIS Monte Carlo generators can provide only approximate cross sections, because a part of the SIDIS cross section involving pure semi-inclusive SF and respective convolutions of the leptonic and hadronic tensors are not presented in such DIS Monte Carlo generators.

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APPENDIX A: BASES IN THE FOUR-DIMENSIONAL SPACE

In this Appendix three bases in the four-dimensional space that are used in our analyses are presented. The first two are used for the decomposition of the initial target and virtual photon polarization in the hadronic tensor defined by (12). The latter allows us to decompose the real photon momentum in such a way that all five pseudoscalar quantities appearing in processes (19) and (55) reduce down to two: $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$.

For the decomposition of the hadronic tensor over the SF it is convenient to introduce the reference system $(\mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ in the target rest frame where the two polar axes are defined as follows: $\mathbf{z}_h$ is chosen in the virtual photon three-momentum direction $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$, and the $\mathbf{x}_h$ along the part of the registrated hadronic momentum that is transverse to the $\mathbf{z}_h$ axis. The direction of the rest axial $\mathbf{y}_h$ axis is defined as $\mathbf{y}_h = \mathbf{z}_h \times \mathbf{x}_h$. In this system the complete basis for polarization vectors can be presented in covariant form [18] for both the virtual photon

$$
\begin{aligned}
e_\mu^{\gamma(0)} &= \frac{2Q}{\sqrt{\lambda_Y}} p_\mu^\perp, \\
e_\mu^{\gamma(1)} &= \frac{1}{p_t}\left[p_{h\mu}^\perp - \frac{S_x(m_h^2 + (2z-1)Q^2 - t)}{\lambda_Y} p_\mu^\perp\right], \\
e_\mu^{\gamma(2)} &= 2\frac{\varepsilon^{\mu\nu\rho\sigma} p_\nu q_\rho p_{h\sigma}}{p_t\sqrt{\lambda_Y}}, \\
e_\mu^{\gamma(3)} &= \frac{q_\mu}{Q}
\end{aligned}
\tag{A1}
$$

and the nucleon

$$
\begin{aligned}
e_\mu^{h(0)} &= \frac{p_\mu}{M}, \\
e_\mu^{h(1)} &= \frac{1}{p_t}\left[p_{h\mu}^\perp - \frac{S_x(m_h^2 + (2z-1)Q^2 - t)}{\lambda_Y} p_\mu^\perp\right], \\
e_\mu^{h(2)} &= 2\frac{\varepsilon^{\mu\nu\rho\sigma} p_\nu q_\rho p_{h\sigma}}{p_t\sqrt{\lambda_Y}}, \\
e_\mu^{h(3)} &= \frac{2M^2 q_\mu - S_x p_\mu}{M\sqrt{\lambda_Y}},
\end{aligned}
\tag{A2}
$$

where $Q = \sqrt{Q^2}$ as well as for any four-vector $a_\mu^\perp = a_\mu + aq q_\mu/Q^2$. Note that the direction of $e^{h(2)}$ (and $e^{\gamma(2)}$ as well) is chosen in such a way that the projection of $\mathbf{k}_{1,2}$ on $\mathbf{y}_h$ reads $\mathbf{y}_h \cdot \mathbf{k}_1 = \mathbf{y}_h \cdot \mathbf{k}_2 = -e^{h(2)} k_1 = -e^{h(2)} k_2 = k_t \sin(\phi_h)$.

The components of these two bases in the reference system $(\mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ read

$$
\begin{aligned}
e_\mu^{\gamma(0)} &= \frac{1}{2MQ}(\sqrt{\lambda_Y}, 0, 0, S_x), & e_\mu^{h(0)} &= (1,0,0,0), \\
e_\mu^{\gamma(1)} &= (0,1,0,0), & e_\mu^{h(1)} &= (0,1,0,0), \\
e_\mu^{\gamma(2)} &= (0,0,1,0), & e_\mu^{h(2)} &= (0,0,1,0), \\
e_\mu^{\gamma(3)} &= \frac{1}{2MQ}(S_x, 0, 0, \sqrt{\lambda_Y}), & e_\mu^{h(3)} &= (0,0,0,1).
\end{aligned}
\tag{A3}
$$

In the rest frame system the virtual photon longitudinal and transverse polarizations correspond to $e^{\gamma(0)}$ and $e^{\gamma(1,2)}$, respectively, and the left and right circular polarizations are defined as

$$
e^{\gamma(\pm)} = \mp\frac{1}{\sqrt{2}}(e^{\gamma(1)} \pm i e^{\gamma(2)}).
\tag{A4}
$$

To decompose the photonic four-momentum the other reference system $(\mathbf{x}_l, \mathbf{y}_l, \mathbf{z}_l)$ in the rest target frame has to be introduced. In this system the polar $\mathbf{z}_l$ axis has the same direction as the three-vector $\mathbf{q}$, the other polar $\mathbf{x}_l$-axis is chosen along the incoming or outgoing lepton part that is transverse to $\mathbf{q}$, and the axial $\mathbf{y}_l$ axis is defined as $\mathbf{y}_l = \mathbf{z}_l \times \mathbf{x}_l$. As a result $(\mathbf{x}_l, \mathbf{y}_l)$ is the scattering plane. In the covariant form this basis reads as

$$
\begin{aligned}
e_\mu^{l(0)} &= \frac{p_\mu}{M}, \\
e_\mu^{l(1)} &= \sqrt{\frac{\lambda_Y}{\lambda_1}}\left[\frac{1}{2}(k_{1\mu} + k_{2\mu}) - \frac{S_p Q^2}{\lambda_Y} p_\mu^\perp\right], \\
e_\mu^{l(2)} &= -\frac{2\varepsilon_{\perp\mu}}{\sqrt{\lambda_1}}, \\
e_\mu^{l(3)} &= \frac{2M^2 q_\mu - S_x p_\mu}{M\sqrt{\lambda_Y}}.
\end{aligned}
\tag{A5}
$$

Note that the direction of $\mathbf{y}_l$ is chosen in such a way that the projection of $\mathbf{p}_h$ on $\mathbf{y}_l$ is $\mathbf{y}_l \cdot \mathbf{p}_h = -e^{l(2)} p_h = -p_t \sin(\phi_h)$. The two reference systems $(\mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ and $(\mathbf{x}_l, \mathbf{y}_l, \mathbf{z}_l)$ can be expressed through each other in the following way:

$$
\begin{aligned}
\mathbf{x}_h &= \mathbf{x}_l \cos(\phi_h) - \mathbf{y}_l \sin(\phi_h), \\
\mathbf{y}_h &= \mathbf{x}_l \sin(\phi_h) + \mathbf{y}_l \cos(\phi_h), \\
\mathbf{z}_h &= \mathbf{z}_l,
\end{aligned}
\tag{A6}
$$

where $\cos(\phi_h)$ and $\sin(\phi_h)$ are defined by Eqs. (5) and (6), respectively.

It should also be noted that for $i = \gamma, h, l$

$$
\begin{aligned}
e_\mu^{i(a)} e_\nu^{i(b)} g^{\mu\nu} &= g^{ab}, \\
e_\mu^{i(a)} e_\nu^{i(b)} g_{ab} &= g_{\mu\nu}.
\end{aligned}
\tag{A7}
$$

The photonic four-momentum can be decomposed into the following way, $k = k_{(a)} e^{(a)}$, where

$$
\begin{aligned}
k_{(0)} &= k e^{l(0)} = \frac{R}{2M}, \\
k_{(1)} &= -k e^{l(1)} = \frac{R(Q^2 S_p + \tau(S S_x + 2M^2 Q^2) - z_1 \lambda_Y)}{2\sqrt{\lambda_1 \lambda_Y}}, \\
k_{(2)} &= -k e^{l(2)} = \frac{2\varepsilon_\perp k}{\sqrt{\lambda_1}}, \\
k_{(3)} &= -k e^{l(3)} = \frac{R(S_x - 2\tau M)}{2M\sqrt{\lambda_1}}.
\end{aligned}
\tag{A8}
$$

This decomposition for the four-momentum of the real unobservable photon allows us to express all pseudoscalars through the linear combinations of two of them, $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$:

$$\varepsilon^{\mu\nu\rho\sigma}k_\mu p_{h\nu}k_{1\rho}q_\sigma = \frac{1}{2\lambda_1}(R\varepsilon_\perp p_h(\tau(Q^2S+2m^2S_x)+Q^2(4m^2+Q^2-z_1S_p)) + \varepsilon_\perp k(Q^2(SV_2+XV_1-zQ^2S_x)-4m^2S_x(zQ^2+V_-))),$$
$$\varepsilon^{\mu\nu\rho\sigma}k_\mu p_\nu p_{h\rho}q_\sigma = \frac{1}{2\lambda_1}(R\varepsilon_\perp p_h(z_1\lambda_Y-Q^2S_p-\tau(SS_x+2M^2Q^2)) + \varepsilon_\perp k(S_x(zQ^2S_p-SV_2+XV_1)-4V_+M^2Q^2)),$$
$$\varepsilon^{\mu\nu\rho\sigma}k_\mu p_\nu k_{1\rho}p_{h\sigma} = \frac{1}{2\lambda_1}(R\varepsilon_\perp p_h(\tau\lambda_S+2m^2S_x+Q^2S-z_1(SS_x+2M^2Q^2)) + \varepsilon_\perp k(2m^2(4V_-M^2-zS_x^2)+S(SV_2-XV_1-zQ^2S_x)+2V_1M^2Q^2)). \quad (A9)$$

APPENDIX B: EXPLICIT EXPRESSION FOR θ_{ij}

For all $i = 1$–8, the quantities $\theta^0_{i1} = 4F_{\rm IR}\theta^B_i$ and $F_{\rm IR}$ are defined by (B5). The other θ^0_{ij} read

$$\theta^0_{12} = 4\tau F_{\rm IR},$$
$$\theta^0_{13} = -4-2F_d\tau^2,$$
$$2\theta^0_{22} = S_xS_pF_{1+}+2m^2S_pF_{2-}+2(S_x-2\tau M^2)F_{\rm IR}-\tau S_p^2F_d,$$
$$2\theta^0_{23} = (4m^2+\tau(2\tau M^2-S_x))F_d-S_pF_{1+}+4M^2,$$
$$\theta^0_{32} = 2((\mu V_- -\tau m_h^2)F_{\rm IR}+V_+(\mu m^2F_{2-}+V_-F_{1+}-\tau V_+F_d)),$$
$$\theta^0_{33} = (2\mu^2m^2+\tau(\tau m_h^2-\mu V_-))F_d-\mu V_+F_{1+}+2m_h^2,$$
$$\theta^0_{42} = (SV_1-XV_2)F_{1+}+m^2(\mu S_p+2V_+)F_{2-}-2\tau S_pV_+F_d+((\mu-2\tau z)S_x+2V_-)F_{\rm IR},$$
$$2\theta^0_{43} = (8\mu m^2+\tau((2\tau z-\mu)S_x-2V_-))F_d-(\mu S_p+2V_+)F_{1+}+4zS_x,$$
$$\theta^0_{52} = \frac{\lambda_eS}{\lambda_1\sqrt{\lambda_S}}\Big[\varepsilon_\perp p_h(2(S_x(Q^2+4m^2)+2\tau(SX-2M^2(Q^2+2m^2)))F_{\rm IR}+Q^2(S_p(S_xF_{1+}+2m^2F_{2-}) - \tau(4SX+S_x^2)F_d))+2\frac{\varepsilon_\perp k}{R}(m^2(S_x(SV_2-XV_1-zQ^2S_p)+4M^2Q^2V_+)F_{2-} + ((Q^2+4m^2)(4M^2V_- -zS_x^2)+S_p(SV_2-XV_1))F_{\rm IR})\Big],$$
$$\theta^0_{53} = \hat\theta^0_{53}+\frac{\lambda_eS}{\lambda_1\sqrt{\lambda_S}}\Big[\varepsilon_\perp p_h(8m^2(\tau(\tau M^2-S_x)-Q^2)F_{21}+(Q^2(4\tau M^2+S_p)+2\tau SS_x)F_{1+}+\tau(4m^2(2\tau M^2-S_x) + Q^2(S_x-4S)-2\tau S^2)F_d)+2\frac{\varepsilon_\perp k}{R}(2m^2(S_x(2zQ^2+2V_-+(\tau z-\mu)S_x)-4M^2(\mu Q^2+\tau V_-))F_{21} + \tau(2m^2(zS_x^2-4M^2V_-)-2M^2Q^2V_1+S(zS_xQ^2-SV_2+XV_1))F_d)\Big],$$
$$\hat\theta^0{}_{53} = \frac{2\lambda_eS}{\lambda_1\sqrt{\lambda_S}}F_{21}\Big[\frac{\varepsilon_\perp k}{R}(2(\mu Q^2+\tau V_1)(SX-M^2Q^2)+(Q^2+\tau S)(zQ^2S_x-SV_2-XV_1))-\varepsilon_\perp p_h(Q^2+\tau S)^2\Big],$$
$$\theta^0_{62} = \frac{1}{2\lambda_1}\Big[\varepsilon_\perp p_h((4M^2Q^2(Q^2+4m^2)-S_x^2(Q^2-4m^2)-8Q^2SX)(S_xF_{1+}+2m^2F_{2-}-\tau S_pF_d) + 2S_p(2\tau(2M^2(Q^2+2m^2)-SX)-S_x(Q^2+4m^2))F_{\rm IR})+2S_p\frac{\varepsilon_\perp k}{R}(m^2(S_x(zS_pQ^2-SV_2+V_1X) - 4M^2Q^2V_+)F_{2-}+((Q^2+4m^2)(zS_x^2-4M^2V_-)+S_p(XV_1-SV_2))F_{\rm IR})\Big],$$

$$\theta^0_{63} = \frac{1}{2\lambda_1}\Big[2\varepsilon_\perp p_h((2Q^2(SX-2M^2Q^2) - \tau S_x(S_x^2+3SX-4m^2M^2) - (Q^2+2m^2)S_x^2)F_{1+}$$
$$+ m^2(2\tau(2M^2(Q^2+2m^2) - SX) - S_x(Q^2+4m^2))F_{2-} - Q^2S_pF_{\rm IR} + S_p(\tau^2(S_x^2+2SX$$
$$- 2M^2(Q^2+4m^2)) + 2\tau S_x(Q^2+2m^2) - 4m^2Q^2)F_d + S_pS_x^2) + \frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(zS_x^2-4M^2V_-)$$
$$+ S_p(XV_1 - SV_2))(S_xF_{1+}+2m^2F_{2-}) + 2m^2(S_x(zQ^2S_p - SV_2 + XV_1) - 4M^2Q^2V_+)F_{2+}$$
$$+ (4\tau(M^2Q^2(4SV_- + S_x(V_2-V_-)) + 2SX(SV_2-XV_1) + 2m^2S_p(4M^2V_- - zSx^2))$$
$$+ (3\tau S_x + 2(Q^2-2m^2))(SV_2 - XV_1 - zQ^2S_p)S_x + 8(Q^2-2m^2)M^2Q^2V_+)F_d)\Big],$$

$$\theta^0_{64} = \frac{1}{2\lambda_1}\Big[\varepsilon_\perp p_h(((Q^2+4m^2)S_x + 2\tau(SX - 2M^2(Q^2+2m^2)))F_{1+} + S_p(\tau Q^2F_d - 2S_x))$$
$$+ \frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(4M^2V_- - zS_x^2) + S_p(SV_2 - XV_1))F_{1+} + \tau(4M^2Q^2V_+ + S_x(SV_2 - XV_1 - zQ^2S_p))F_d)\Big],$$

$$\theta^0_{72} = \frac{\lambda_e S}{2\sqrt{\lambda_S}}[Q^2(4M^2V_- - zS_x^2)F_{1+} + m^2(\mu\lambda_Y - 2S_x(zQ^2+V_-))F_{2-} + (2(4\tau M^2 - S_x)V_+$$
$$+ (\mu - 2\tau z)S_pS_x - 2SV_2 + 2XV_1)F_{\rm IR} + \tau(Q^2(zS_xS_p - 4M^2V_+) + S_x(XV_1 - SV_2))F_d],$$

$$\theta^0_{73} = \frac{\lambda_e S}{4\sqrt{\lambda_S}}[(S_x(4zQ^2 + 2V_- - \mu S_x) - 8\,\mu M^2Q^2)F_{1+} + 2m^2(4\mu\tau M^2 + 2V_- - (\mu+2\tau z)S_x)F_{2-}$$
$$+ 2(2V_+ - \mu S_p)F_{\rm IR} + \tau(4(S_x - 2\tau M^2)V_+ + S_p((2\tau z - \mu)S_x - 2V_-))F_d],$$

$$\theta^0_{74} = \frac{\lambda_e S}{4\sqrt{\lambda_S}}[((\mu+2\tau z)S_x - 2V_- - 4\mu\tau M^2)F_{1+} + \tau(\mu S_p - 2V_+)F_d],$$

$$\theta^0_{82} = \frac{1}{\lambda_1}\Big[\varepsilon_\perp p_h((Q^2S_x(S_xV_+ - 2SV_2) - 2V_-(2\lambda_1 + Q^2SS_x))F_{1+} - 2m^2(2\mu\lambda_1 + Q^2S_pV_+)F_{2-}$$
$$+ V_+(2m^2(2\tau(2(Q^2+2m^2)M^2 - SX) - (Q^2+4m^2)S_x)F_{2+} + (4m^2((3Q^2+4m^2)S_x$$
$$+ \tau(2SX - 4(3Q^2+2m^2)M^2 - S_x^2)) + Q^2(\tau(12SX + S_x^2) + 2Q^2(S_x - 6\tau M^2)))F_d))$$
$$+ 2V_+\frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(zS_x^2 - 4M^2V_-) + S_p(XV_1 - SV_2))F_{\rm IR} + m^2(S_x(XV_1 - SV_2 + zS_pQ^2)$$
$$- 4Q^2V_+M^2)F_{2-})\Big],$$

$$\theta^0_{83} = \frac{1}{2\lambda_1}\Big[\varepsilon_\perp p_h((2\mu(Q^2-2m^2)Q^2S_p + \tau(2(Q^2+8m^2)S_xV_+ + Q^2(\mu S_xS_p + 2SV_1 - 2XV_2))$$
$$- 2\tau^2(4(Q^2+4m^2)V_+M^2 - S_p(SV_1 + XV_2)))F_d + 2\,\mu m^2(2\tau(2(Q^2+2m^2)M^2 - SX) - (Q^2+4m^2)S_x)F_{2-}$$
$$+ 4S_pS_xV_- - 2\,\mu m^2Q^2S_pF_{2+} + (2\tau(X^2V_2 - S^2V_1) + 8m^2V_-(2\tau M^2 - S_x) + Q^2(4\mu(SX - 2Q^2M^2)$$
$$- S_x(2V_- + \mu S_x)))F_{1+}) + 2\frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(zS_x^2 - 4M^2V_-) + S_p(XV_1 - SV_2))(V_-F_{1+} + \mu m^2F_{2-})$$
$$+ \mu m^2(S_x(zS_pQ^2 + XV_1 - SV_2) - 4Q^2V_+M^2)F_{2+} + (\mu(Q^2-2m^2)(4Q^2V_+M^2 + S_x(SV_2 - XV_1$$
$$- zS_pQ^2)) + \tau(S_xS_pV_2(V_1+V_+) + 2V_-V_+((S_x - 4S)X + 2(3Q^2+8m^2)M^2) + zS_x(Q^2(XV_2 - SV_1)$$
$$- (Q^2+8m^2)S_xV_+)))F_d)\Big],$$

$$\theta^0_{84} = \frac{\mu}{2\lambda_1}\bigg[\varepsilon_\perp p_h(((Q^2+4m^2)S_x + 2\tau(SX - 2(Q^2+2m^2)M^2))F_{1+} + S_p(\tau Q^2 F_d - 2S_x)) + \frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(4M^2V_- - zS_x^2) + S_p(SV_2 - XV_1))F_{1+} + \tau(4Q^2V_+M^2 + S_x(SV_2 - XV_1 - zS_pQ^2))F_d)\bigg],$$

$$\theta^0_{91} = \frac{2\lambda_e S}{\sqrt{\lambda_S}}(Q^2(zS_xV_+ - m_h^2 S_p) + V_-(SV_2 - XV_1))F_{\rm IR},$$

$$\theta^0_{92} = \frac{\lambda_e S}{\sqrt{\lambda_S}}(Q^2S_x(zV_- - m_h^2)F_{1+} + m^2(Q^2(\mu z S_x - 2m_h^2) + V_-(\mu S_x - 2V_-))F_{2-} + \tau(Q^2(m_h^2S_p - zS_xV_+) + V_-(XV_1 - SV_2))F_d + (2V_-(2\mu S - V_+) + 2\tau(zS_xV_+ - m_h^2S_p) - \mu(V_1 + V_-)S_x)F_{\rm IR}),$$

$$\theta^0_{93} = \frac{\lambda_e S}{2\sqrt{\lambda_S}}((2(2m_h^2Q^2 + V_-^2) - \mu S_x(2zQ^2 + V_-))F_{1+} + m^2(\mu((2\tau z - \mu)S_x + 2V_-) - 4\tau m_h^2)F_{2-} + \mu(2V_+ - \mu S_p)F_{\rm IR} + \tau(2\tau(m_h^2S_p - zS_xV_+) + \mu S_x(V_- + V_1) + 2V_-(V_+ - 2\mu S))F_d),$$

$$\theta^0_{94} = \frac{\lambda_e S}{4\sqrt{\lambda_S}}((2\tau(2m_h^2 - \mu z S_x) + \mu(\mu S_x - 2V_-))F_{1+} + \mu\tau(\mu S_p - 2V_+)F_d). \qquad \text{(B1)}$$

The quantities θ^1_{ij} have the form

$$\theta^1_{51} = 0,$$

$$\theta^1_{52} = \frac{2m^2\lambda_e}{\lambda_1\sqrt{\lambda_S}}\bigg[\varepsilon_\perp p_h(2(2m^2\lambda_Y + (Q^2+\tau S)(2M^2Q^2 + SS_x))F_{21} - S_x\lambda_Y F_{1+} + (2Q^2XS_x + \tau S_x(2S^2 - S_p^2) + 4M^2Q^2(\tau S - Q^2) - 4m^2\lambda_Y)F_d) + 2\frac{\varepsilon_\perp k}{R}(S_x(zS_pQ^2 + XV_1 - SV_2) - 4M^2Q^2V_p)(XF_d - SF_{21})\bigg],$$

$$\theta^1_{53} = \frac{2m^2\lambda_e}{\lambda_1\sqrt{\lambda_S}}\bigg[\varepsilon_\perp p_h(2((Q^2+2m^2)(2\tau M^2 + X) - (\tau X + 2m^2)S)F_{21} - \lambda_Y F_{1+} + (4m^2(S_x - 2\tau M^2) + 2Q^2S + \tau(S^2 + X^2))F_d) + 2\frac{\varepsilon_\perp k}{R}((2m^2(zS_x^2 - 4M^2V_-) + 2M^2Q^2V_2 + X(XV_1 - SV_2 - zS_xQ^2))F_{21} + (2m^2(4M^2V_- - zS_x^2) + 2M^2Q^2V_1 + S(SV_2 - XV_1 - zQ^2S_x))F_d)\bigg],$$

$$\theta^1_{71} = 0,$$

$$\theta^1_{72} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}((4M^2(\tau SV_- - Q^2V_+) - S_x^2(\tau z S + zQ^2 + V_1) + \mu\lambda_Y S)F_{21} + (4M^2(Q^2V_+ - \tau XV_-) + S_x^2(\tau z X + V_2 - zQ^2) - \mu\lambda_Y X)F_d),$$

$$\theta^1_{73} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[(2M^2(\mu(Q^2+\tau S) - 2\tau V_+) + S_x((\tau z - 2\mu)S + 2V_+ - zQ^2) + (\mu - \tau z)S_x^2)F_{21} + (2M^2(\mu(Q^2 - \tau X) + 2\tau V_+) + S_x((2\mu - \tau z)S - zQ^2 - 2V_+) - \mu S_x^2)F_d],$$

$$\theta^1_{74} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[(2\mu\tau M^2 + \mu X - \tau z S_x - V_2)F_{21} + (2\mu\tau M^2 + V_1 - \mu S - \tau z S_x)F_d],$$

$$\theta^1_{91} = \frac{4\lambda_e m^2(m_h^2\lambda_Y + 4M^2V_-^2 - zS_x^2(zQ^2 + 2V_-))}{\sqrt{\lambda_S}}F_{\rm IR},$$

$$\begin{aligned}
\theta^1_{92} &= \frac{2m^2\lambda_e}{\sqrt{\lambda_S}}[2m^2(2m_h^2(2\tau M^2 - S_x) + 2(zS_x - 2\,\mu M^2)V_- + z(\mu - \tau z)S_x^2)F_{2+} \\
&\quad + S_x(zQ^2(\mu S - V_+) + V_-((\mu + \tau z)S - V_1) - m_h^2(Q^2 + \tau S))F_{21} \\
&\quad + (S_x((m_h^2 - zV_-)(\tau X + 3Q^2 + 8m^2) + (V_- + zQ^2)(V_2 - \mu X)) \\
&\quad + 2(4M^2(\mu V_- - \tau m_h^2) + (z\tau - \mu)zS_x^2)(Q^2 + 2m^2))F_d], \\
\theta^1_{93} &= \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[4m^2(m_h^2 + \mu^2 M^2 - \mu z S_x)F_{2+} + (2m_h^2(\tau X - Q^2) + S_x(\mu(\tau z - \mu)S - 2\tau z V_+ + \mu z Q^2 \\
&\quad + \mu V_1) + 2V_-(V_2 - \mu X))F_{21} + (\mu((Q^2 + 2m^2)(5zS_x - 4\,\mu M^2) + (\mu - \tau z)XS_x) \\
&\quad - 2m_h^2(\tau S + 3Q^2 + 4m^2) + S_x(2\tau z V_+ - \mu V_2 - 2\mu z m^2) + 2V_-(\mu S - V_1))F_d], \\
\theta^1_{94} &= \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[(\mu(\tau z S_x + \mu X - V_2) - 2\tau m_h^2)F_{21} + (\mu(\tau z S_x + V_1 - \mu S) - 2\tau m_h^2))F_d]. \qquad \text{(B2)}
\end{aligned}$$

The variable μ is defined as

$$\mu = \frac{kp_h}{kp} = \frac{p_{h0}}{M} + \frac{p_l(2\tau M^2 - S_x)}{M\sqrt{\lambda_Y}} - 2Mp_t\cos(\phi_h + \phi_k)\sqrt{\frac{(\tau_{\max} - \tau)(\tau - \tau_{\min})}{\lambda_Y}}. \qquad \text{(B3)}$$

The quantities F_i ($i = d$, $1+$, $2+$, $2-$, IR) are expressed through

$$\begin{aligned}
z_1 &= \frac{k_1 k}{pk} \\
&= \frac{Q^2 S_p + \tau(SS_x + 2M^2Q^2) - 2M\sqrt{\lambda_z}\cos\phi_k}{\lambda_Y}, \\
z_2 &= \frac{k_1 k}{pk} \\
&= \frac{Q^2 S_p + \tau(XS_x - 2M^2Q^2) - 2M\sqrt{\lambda_z}\cos\phi_k}{\lambda_Y}, \\
\lambda_z &= (\tau_{\max} - \tau)(\tau - \tau_{\min})\lambda_1 \qquad \text{(B4)}
\end{aligned}$$

in the following way:

$$\begin{aligned}
F_{2\pm} &= F_{22} \pm F_{21} = \frac{1}{z_2^2} \pm \frac{1}{z_1^2}, \\
F_d &= \frac{1}{z_1 z_2}, \\
F_{1+} &= \frac{1}{z_1} + \frac{1}{z_2}, \\
F_{\mathrm{IR}} &= m^2 F_{2+} - (Q^2 + 2m^2)F_d. \qquad \text{(B5)}
\end{aligned}$$

APPENDIX C: CALCULATION OF δ_S AND δ_H

The dimensional regularization is used for the calculation of δ_S in (37),

$$\frac{d^3k'}{k'_0} \to \frac{d^{n-1}k'}{(2\pi\nu)^{n-4}k'_0} = \frac{2\pi^{n/2-1}k_0'^{n-3}dk'_0(1 - x^2)^{n/2-2}dx}{(2\pi\nu)^{n-4}\Gamma(n/2 - 1)}, \qquad \text{(C1)}$$

where $x = \cos\theta$ [θ is defined as the spatial angle between the photon three-momentum and $\mathbf{k}'_i$ ($i = 1$–3) that are introduced below] and ν is an arbitrary parameter of the dimension of a mass. The Feynman parametrization of propagators in F_{IR},

$$F_{\mathrm{IR}} = \frac{R^2}{4k_0'^2}\int_0^1 dy\mathcal{F}(x, y), \qquad \text{(C2)}$$

where y is the Feynman parameter and

$$\mathcal{F}(x, y) = \frac{m^2}{k_{10}'^2(1 - x\beta_1)^2} + \frac{m^2}{k_{20}'^2(1 - x\beta_2)^2} - \frac{Q_m^2}{k_{30}'^2(1 - x\beta_3)^2}. \qquad \text{(C3)}$$

The energies of the real photon (k'_0), initial (k'_{10}), and final (k'_{20}) leptons are defined in the system $\mathbf{p} + \mathbf{q} - \mathbf{p}_h = 0$ while $k'_{30} = yk'_{10} + (1 - y)k'_{20}$ and $\beta_i = |\mathbf{k}'_i|/k'_{i0}$.

Then, the substitution of Eqs. (C1) and (C3) into the definitions of δ_S by Eq. (37), the integration over k'_0, and the expansion of the obtained result into the Laurent series around $n = 4$ result in

$$\delta_S = \delta_S^{\mathrm{IR}} + \delta_S^1, \qquad \text{(C4)}$$

where

$$\delta_S^{\rm IR} = -\frac{1}{2}\left[P_{\rm IR} + \log\frac{\bar{k}_0}{\nu}\right]\int_0^1 dy\int_{-1}^1 dx\mathcal{F}(x,y) \qquad \text{(C5)}$$

and

$$\delta_S^1 = -\frac{1}{4}\int_0^1 dy\int_{-1}^1 dx\,\log(1-x^2)\mathcal{F}(x,y). \qquad \text{(C6)}$$

Here $P_{\rm IR}$ is the infrared divergent term defined by Eq. (39). Since $k_{30}'^2 - |\mathbf{k}_3'|^2 = m^2 + y(1-y)Q^2$, the integration over x and y variables in $\delta_S^{\rm IR}$ is performed explicitly:

$$\delta_S^{\rm IR} = 2(Q_m^2 L_m - 1)\left[P_{\rm IR} + \log\frac{\bar{k}_0}{\nu}\right]. \qquad \text{(C7)}$$

For the covariant analytical integration in δ_S^1 we express the initial and final lepton energies through the invariants:

$$k_{10}' = \frac{S'}{2\sqrt{p_x^2}}, \qquad k_{20}' = \frac{X'}{2\sqrt{p_x^2}}. \qquad \text{(C8)}$$

As a result,

$$\delta_S^1 = 2(Q_m^2 L_m - 1)\log(2) + \frac{1}{2}S'L_{S'} + \frac{1}{2}X'L_{X'} + S_\phi, \qquad \text{(C9)}$$

where the quantities L_m, $L_{S'}$, and $L_{X'}$ are

$$L_m = \frac{1}{\sqrt{\lambda_m}}\log\frac{\sqrt{\lambda_m}+Q^2}{\sqrt{\lambda_m}-Q^2},$$
$$L_{S'} = \frac{1}{\sqrt{\lambda_S'}}\log\frac{S'+\sqrt{\lambda_S'}}{S'-\sqrt{\lambda_S'}},$$
$$L_{X'} = \frac{1}{\sqrt{\lambda_X'}}\log\frac{X'+\sqrt{\lambda_X'}}{X'-\sqrt{\lambda_X'}}, \qquad \text{(C10)}$$

and

$$S_\phi = \frac{1}{2}Q_m^2\int_0^1 \frac{dy}{\beta_3(m^2 + y(1-y)Q^2)}\log\frac{1-\beta_3}{1+\beta_3}. \qquad \text{(C11)}$$

The explicit expression for S_ϕ after integration over y is given in Eq. (40).

For the calculation of δ_H we carry out the integration in the same reference system $\mathbf{p} + \mathbf{q} - \mathbf{p}_h = 0$,

$$\delta_H = -\frac{1}{\pi}\int_{\bar{k}_0}^{k_0^{\max\prime}} k_0' dk_0' \int_0^\pi \sin(\theta_k')d\theta_k' \int_0^{2\pi} d\phi_k' \frac{F_{\rm IR}}{R^2}, \qquad \text{(C12)}$$

where θ_k' is the angle between $\mathbf{k}$ and $\mathbf{q}$ three momenta, and ϕ_k' is the angle between $(\mathbf{k}_1, \mathbf{k}_2)$ and $(\mathbf{k}, \mathbf{q})$ planes.

In this system

$$z_1 = \frac{2k_0'}{R}(k_{10}' - k_t'\cos\phi_k'\sin\theta_k' - k_{13}'\cos\theta_k'),$$
$$z_2 = \frac{2k_0'}{R}(k_{20}' - k_t'\cos\phi_k'\sin\theta_k' - k_{23}'\cos\theta_k'), \qquad \text{(C13)}$$

which allows us to take the first integration in respect to ϕ_k:

$$\delta_H = \int_{\bar{k}_0}^{k_0^{\max\prime}} \frac{dk_0'}{2k_0'}\int_0^\pi \sin(\theta_k')d\theta_k'\left[\frac{Q_m^2}{B_1 - B_2}\left(\frac{1}{\sqrt{C_2}} - \frac{1}{\sqrt{C_1}}\right)\right.$$
$$\left. - \frac{m^2 B_1}{C_1^{3/2}} - \frac{m^2 B_2}{C_2^{3/2}}\right]. \qquad \text{(C14)}$$

Here

$$B_i = k_{i0}' - \cos(\theta_k')k_{i3}', \qquad C_i = B_i^2 - \sin^2(\theta_k')k_t'^2 \qquad \text{(C15)}$$

for $i = 1, 2$.

After the integration with respect to θ_k' and the use of the following replacements:

$$k_t' = \sqrt{k_{10}'^2 - k_{13}'^2 - m^2} = \sqrt{k_{20}'^2 - k_{23}'^2 - m^2},$$
$$k_{13}' = \frac{2k_{10}'q_0' + Q^2}{2\sqrt{Q^2 + q_0'^2}}, \qquad k_{23}' = \frac{2k_{20}'q_0' - Q^2}{2\sqrt{Q^2 + q_0'^2}} \qquad \text{(C16)}$$

with $q_0' = k_{10}' - k_{20}'$, the hard contribution δ_H is expressed in the form

$$\delta_H = 2\int_{\bar{k}_0}^{k_0^{\max\prime}} \frac{dk_0}{k_0}(Q_m^2 L_m - 1). \qquad \text{(C17)}$$

Since $k_0^{\max\prime} = (p_x^2 - M_{th}^2)/2\sqrt{p_x^2}$, the integration for δ_H is finally presented in the form of (38).

APPENDIX D: CALCULATION OF Λ_μ AND $\Pi^l_{\alpha\mu}$

The γ-matrix recombination, convolution over α indexes in Eq. (47) for Λ_μ, and calculation of the traces for $\Pi^l_{\alpha\mu}$ in n-dimensional space result in

$$\Lambda_\mu = \frac{\alpha}{4\pi}\{\gamma_\mu[(n-2)J_\delta^\delta - 4J^\delta(k_{1\delta} + k_{2\delta}) + 2Q_m^2 J]$$
$$+ 2\gamma_\delta[2J^\delta(k_{1\mu} + k_{2\mu}) - (n-2)J_\mu^\delta] - 4mJ_\mu\},$$
$$\Pi^l_{\alpha\mu} = \frac{\alpha}{\pi}\frac{1}{Q^2}\left(\sum_{i=e,\mu,\tau}\{g_{\alpha\mu}(q_\delta J_i^\delta + m_i^2 J_i - J_{i\delta}^\delta)\right.$$
$$\left. + 2J_{i\alpha\mu} - q_\alpha J_{i\mu} - q_\mu J_{i\alpha}\}\right), \qquad \text{(D1)}$$

where

$$J = \frac{1}{i\pi^2}\lim_{n\to 4}\int \frac{(2\pi\nu)^{4-n}d^n l}{l^2(l^2-2lk_2)(l^2-2lk_1)} = -2L_m\left(P_{\rm IR}+\log\frac{m}{\nu}\right) - \frac{1}{2}\sqrt{\lambda_m}L_m^2 + \frac{1}{2\sqrt{\lambda_m}}\left(\pi^2 - 4{\rm Li}_2\frac{2\sqrt{\lambda_m}}{\sqrt{\lambda_m}+Q^2}\right),$$

$$J_\delta = \frac{1}{i\pi^2}\lim_{n\to 4}\int \frac{l_\delta(2\pi\nu)^{4-n}d^n l}{l^2(l^2-2lk_2)(l^2-2lk_1)} = -L_m(k_{1\delta}+k_{2\delta}),$$

$$J_{\delta\rho} = \frac{1}{i\pi^2}\lim_{n\to 4}\int \frac{l_\delta l_\rho(2\pi\nu)^{4-n}d^n l}{l^2(l^2-2lk_2)(l^2-2lk_1)} = \frac{1}{4}\left\{g_{\delta\rho}\left(3-2P_{\rm UV}-2\log\frac{m}{\nu}-\frac{\lambda_m}{Q^2}L_m\right)+q_\delta q_\rho\frac{2Q^2-\lambda_m L_m}{Q^4}\right.$$
$$\left. - L_m(k_{1\delta}+k_{2\delta})(k_{1\rho}+k_{2\rho})\right\},$$

$$J_i = \frac{1}{i\pi^2}\lim_{n\to 4}\int \frac{(2\pi\nu)^{4-n}d^n l}{(l^2-m_i^2)((l-q)^2-m_i^2)} = 2-2P_{\rm UV}-2\log\frac{m_i}{\nu}-\frac{\lambda_m^i}{Q^2}L_m^i,$$

$$J_{i\delta} = \frac{1}{i\pi^2}\lim_{n\to 4}\int \frac{l_\delta(2\pi\nu)^{4-n}d^n l}{(l^2-m_i^2)((l-q)^2-m_i^2)} = \frac{1}{2}q_\delta J_i,$$

$$J_{i\delta\rho} = \frac{1}{i\pi^2}\lim_{n\to 4}\int \frac{l_\delta l_\rho(2\pi\nu)^{4-n}d^n l}{(l^2-m_i^2)((l-q)^2-m_i^2)} = \frac{1}{72}\left\{g_{\delta\rho}\left(6\left[Q^2-\frac{3\lambda_m^i}{Q^2}\right]\left(P_{\rm UV}+\log\frac{m_i}{\nu}\right)+\left[21-\frac{6\lambda_m^i}{Q^2}L_m^i\right]\frac{\lambda_m^i}{Q^2}-5Q^2\right)\right.$$
$$\left.+q_\delta q_\rho\left(40-48P_{\rm UV}-48\log\frac{m_i}{\nu}+12\frac{\lambda_m^i}{Q^4}-6\frac{\lambda_m^i}{Q^2}\left[3+\frac{\lambda_m^i}{Q^4}\right]L_m\right)\right\}. \tag{D2}$$

The infrared divergent $P_{\rm IR}$ term is defined by Eq. (39) while the ultraviolet divergent term has the same structure $P_{\rm UV} = P_{\rm IR}$ and

$$L_m^i = \frac{1}{\sqrt{\lambda_m^i}}\log\frac{\sqrt{\lambda_m^i}+Q^2}{\sqrt{\lambda_m^i}-Q^2}, \qquad \lambda_m^i = Q^2(Q^2+4m_i^2). \tag{D3}$$

After substituting (D2) into (D1) and using $nP_{\rm UV} = 4P_{\rm UV}+1+\mathcal{O}(n-4)$ we find the final expressions for Λ_μ and $\Pi_{\alpha\mu}^l$ (48).

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