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The effect of VCSEL intrinsic dynamics on polarization bistability

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ABSTRACT

This work is devoted to a theoretical study that addresses the influence of the parameters of original laser system on the dynamics of formation of polarized radiation in VCSEL with his institution of symmetrical triangular pulses. The model is based on a sequential process of enhancing radiation in the resonator, taking into account anisotropy of gain and loss. It is shown that all features of polarization hysteresis can be associated with ultimate speed formation characteristics of reinforced radiation with the rapid build-up and decline in current, rather than with the process of competition between two independent polarization modes.

Introduction

Vertical cavity surface emitting lasers (VCSELs) have emerged as key components for versatile communication and sensing applications, including the data transmission in data centers [1–4], the optical heterodyne millimeter-wave generation [5], etc. Except of a lot of advantages VCSELs display the specific property of polarization switching (PS) that appears as a fast change of the initial linear polarization on the orthogonal one under a smooth injection current changes [6], and can play a dual role: it can be used in order to create different polarization devices (shatter, splitter, etc.) but it is extremely undesirable for optoelectronics circuits that are usually polarization sensitive. In any case it is very important to have the right understanding of the nature of this process but this is not so clear up to now [6].

The PS of VCSEL is not unique and can observe at standard semiconductor lasers, moreover PS is usually accompanied be polarization bistability (PB) [7,8]. This occurs due to the fact that with the increasing injection current the point of polarization switching (PS), for which switching between TM and TE modes takes place, is shifted to greater values, whereas with the decreasing current the PS point is shifted to lower values. As a result, for polarization modes [7,8] the ordinary hysteresis loop is formed and its width is significantly dependent on a rate of the current variations [9]. Actually, PB as well as ordinary bistability is associated with a positive feedback between the polarization modes exhibited as a difference in saturation factors of the modes with identical (self-saturation) and orthogonal (cross-saturation) polarizations [10]. For the effective control of such a feedback, it is usually realized by means of an external cavity and/or outer injection of optical radiation. This approach using PB underlies the development of a series of optoelectronic devices [11].

Since the discovery of the PS effect for vertical cavity surface emitting lasers (VCSEL) [12], the development of PB has been considered quite natural [13] and its study was based on the standard scheme for the feedback realization [13,14]. Much later [15–17] the PB phenomenon in VCSEL has been studied in greater detail within the scope of the widely accepted SFM model.

Based on this model [18,19], it is considered that in VCSEL two independent orthogonally polarized modes are formed, the concurrence of which may be accompanied with switching from one mode to the other at bifurcation points. Due to the rapidly varying injection current, the bifurcation point is shifted [15–17] to the region of great values when the current is growing or to lower values – when the current decreases, leading to the appearance of a polarization hysteresis loop. In other words, the hysteresis is interpreted as a dynamic effect in the bifurcation point behaviour observed at high rates of variations in the current [17]. At the present time this interpretation is most common.

Within the scope of the proposed approach [20–22], interpretation of PP is based on a mechanism of the polarization radiation formation process in VCSEL – nonpolarized (or weakly polarized) spontaneous radiation is sequentially amplified in an active layer under condition of gain and loss anisotropy. In the process, the growing intensity of amplified radiation results in changes of its polarization degree. When

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anisotropy of the amplification and loss factors is sufficiently high, the formation processes of stationary intensities and polarization degrees proceed in parallel, and hence the polarization degree tends to the limiting values ± 1 which is in agreement with a linear polarization.

VCSEL is characterized by a rather weak anisotropy of the gain and loss [22] but it is sufficient to disturb the initial axial symmetry of a system. In consequence, the rate of transition to the stationary polarization value is significantly lower than that for the total intensity [22], i.e., the total intensity reaches its stationary value rather fast and further evolution of radiation is due to redistribution of the intensity for the particular polarization components in favour of the components whose amplification conditions are more favourable. Then the effect of the finite photon lifetime in a cavity comes into force and transformation of polarization becomes too slow. Moreover, the amplified radiation is replenished to nonpolarized spontaneous radiation. As a result, some stationary balance is attained and at the output we have partially polarized radiation with a polarization degree of $|P| < 1$. At the PS point, when amplification anisotropy tends to zero, the polarization formation time becomes infinite and nonpolarized radiation at the output is obtained.

Within the scope of the proposed approach, the process of PS represents a deterministic transition from one limiting polarization state to the other through a sequence of partially polarized states, with successive variations of polarization degree for successive variations in the injection current. This is a distinguished feature of the proposed approach as compared to the polarization mode methods including the SFM method. Moreover, there is no reason to believe that the states having the limiting polarization degrees ± 1 are "pure" linear polarization states with a particular phase states. This is because adjustment of the coherent properties of laser radiation in a cavity may be considerably slower than the polarization formation process [23].

When the injection current is not stationary, then the characteristics time of its variation becomes of particular importance as follows: if that time is shorter than the time required for setting of the stationary polarization degree, then such a state is not attained and its output value corresponds to a lower value as the current grows. In fact, this is associated with displacement of the whole function $P(j)$ including the PS point to the region of higher currents. For the reverse process of lowering the current, displacement of $P(j)$ and of the PS point is realized to the region of lower currents. In this way, the PB phenomenon quite reasonably follows the mechanisms involved in the proposed approach.

It should be noted that inertia of the polarization formation process may be more advantageous for high rates of the current variations when the formation processes can be oscillatory in character. In this case variations in the polarization degree are monotonic [21], facilitating studies of rapid phenomena in a laser system. In conclusion, the occurrence of PB is a direct consequence of the polarization radiation formation mechanisms in VCSEL. The main objective of this paper is to describe the features of PB observed experimentally and, possibly, to find other features of this phenomenon which have not been revealed by the present time.

Theoretical model

As the initial theoretical model, we assume the phenomenological (as regards definition of the relationship between the amplification factor and the injection current density) model considered in detail [22], though using a more general representation of the parameters [18]. Subsequently, a system of equations for the photon density orientational component $S(\psi)$ in a cavity and for the concentration of nonequilibrium charge carriers N are given as follows:

$$\frac{dS(\psi)}{dt} = \Gamma\beta\frac{N}{\pi\tau_N} - v(\Gamma G(\psi) - \rho - k_R)S(\psi),$$
$$\frac{dN}{dt} = \frac{j}{ed} - v\int_0^{\pi} G(\psi)S(\psi)d\psi - \frac{N}{\tau_N} \quad (1)$$

where $G(\psi)$ - orientational component of the amplification factor; ρ, k_R − internal loss and loss for cavity mirrors; v - speed of light in the active layer, Γ - optical limitation factor, d - active layer thickness, e - electron charge, j - injection current density, τ_N - lifetime of nonequilibrium carriers, β - spontaneous emission factor. The angle ψ determines the polarization vector orientation for the polarization component associated with the photon-number density $S(\psi)$ with respect to the chosen axis. According to this model [22], the orientational component of the amplification factor is given by

$$G(\psi) = g_0(N - N_{tr})\left(1 + k_0\left(1 - \frac{j}{j_{ps}}\right)\cos 2\psi\right) \quad (2)$$

Referring to the internal loss ρ and the loss for cavity mirrors k_R, the possible dependence on the angle ψ is disregarded because for ρ such a dependence may be included into the effective amplification factor, whereas the orientational dependence on k_R actually has no effect on character of polarization switching [24].

Since from the start PB is interpreted as a dynamic effect resulted from peculiarities in dynamics of the polarization forming of output radiation. This phenomenon is studied by applying the approach that involve triangular pulses with the same rates of rise and fall [15–17]. In this case the main characteristic of a pumping pulse is the current rise (fall) time T (usually half-period pulse). As a rule, such triangular pulses are superimposed on a certain stationary current which is somewhat lower than the threshold value. Though this condition is of importance, there is no limitation for selection of the stationary component. In the present paper the stationary component is considered zero. This enable one to study the amplified radiation formation dynamics in its pure form without any possible "aftereffects" associated with passage of a laser system to the stationary value as elaborately discussed in the following section.

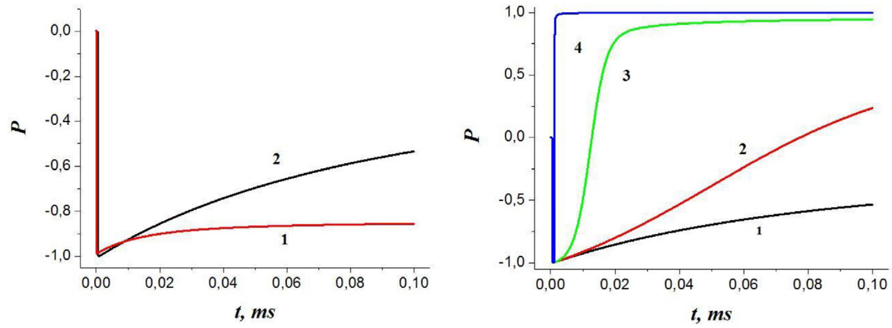
Numerical computations have been performed with parameters previously used [22], for which the threshold current has limiting values in the region of 1.15 mA. J_{ps} varies within the limits from 1.21 to 4.0 mA. Because of this, the pulse amplitude of 5.0 mA has been selected for all half-periods T varying within the range 10^{-3}–10^{-8} s. Sometimes, to demonstrate the attained effects more clearly, other parameters in the computations were introduced.

Results of numerical simulation

First, the stationary injection current superimposed by a triangular pulse is selected. According to the proposed approach formulated elsewhere [15–17], this current should be significantly lower than the threshold value.

Transition to the stationary value always includes some transient process that usually is very short in time. This situation was observed only in our case for the total intensity. As regards to a degree of polarization, the situation is different especially at the PS point. The dynamics of the transition to the stationary polarization degrees for this region were calculated when the current is linearly growing from zero to the stationary value t_p as demonstrated in Fig. 1. As apparent from the curves in Fig. 1a, the degree of the stationary polarization P at the PS point is dependent on the rate of the current rise, e.g. zero value of P (PS point by definition) is attainable only when $t_p \to \infty$.

Such significant deceleration in the forming-up process of the degree of polarization P is usually interpreted as a critical slowing-down close to the bifurcation point [25], taking PS point into consideration [15–17]. On the other hand, within the proposed approach, the point of PS is simply a characteristic point for zero degree of the polarization P rather than a bifurcation point. In the stationary approximation, this point should correspond to zero anisotropy of the amplification factor $G(\psi)$. However, the initial stage of current switching results in shifting of this point to the region of greater current values [22]. Fig. 1b shows that the current range where this effect is considerable, is fairly small



a b

Fig. 1. Evolution of a degree of the polarization ***P*** on passage to the stationary value close to the PS point (J_{ps}= 2.10000 mA). a) J_{st}= 2.10000 мА, t_p= 10^{-7} (1), 10^{-6} (2). b) t_p= 10^{-6}, J_{st}= 2.10000 (1), 2.10001 (2), 2.10010 (3), 2.11000 (4) mA.

yet significance of its presence is still essential. Because of this, the stationary component value is assumed zero so as to exclude its effect on the injection current.

Let us consider the results obtained during studies of the polarization hysteresis loop itself. It should be noted that the numerical results agree well with the data reported earlier [15–17], which are completely being in the direction of the proposed approach. Specifically, shorter pulse length $2T$ leads to broadening of a hysteresis loop and to the rising slope as demonstrated by the curves in Fig. 2. The two graphs have different scales for the current because the width of the hysteresis loop grows rapidly with a corresponding decrease in T. This characteristic of PB hysteresis loop shows that the difference in the injection currents ΔJ_H, for which a degree of polarization approaches zero with rise and fall of the current, is nonlinear in manner where an increase of ΔJ_H is associated with a decrease in T. Similar characteristic was also reported by other researchers [17]. According to them, ΔJ_H is defined as

$$\Delta J_H = \Delta J_H^0 + bT^{-\alpha} \tag{3}$$

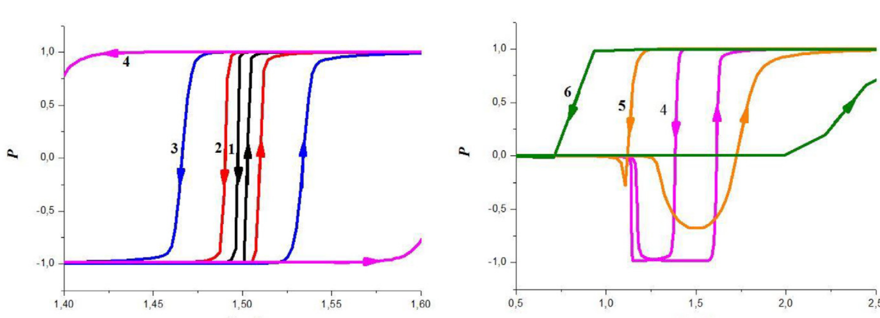
where ΔJ_H^0, is a half-width of the polarization hysteresis loop in the "stationary approximation", pointing to sufficiently slow variations of the current.

Now, the question arises of how slow the current variations should be when $T \to \infty$ as there should be no hysteresis due to its formation logic, irrespective of the approach used for the description of this phenomenon.

By assuming $\Delta J_H^0 = 0$ in relation (3), it is hardly to find such a value of the parameter α so that the relation can be satisfied with a sufficient accuracy. At the same time, relation (3) is satisfied rather well if we take $T = 10^{-3}$ s as a "stationary" reference point. For this condition the value of α lies approximately within the interval 0.55–0.88 (depending on the parameters of VCSEL, with first priority given to k_0 and J_{ps}), but in any case it is a rather rough estimation (for better coincidence it is necessary to suppose that the parameter b is also slightly dependent on the parameters of VCSEL). Therefore, it is hard considering relation (3) as the universal one other than the reflection of nonlinearity of the hysteresis loop forming.

On the other hand, for sufficiently rapid variations in the injection current, one can observe nonlinear distortions of the polarization hysteresis loop associated with the parameters used to calculate the functions shown in Fig. 2, which are revealed at $T < 3{\cdot}10^{-7}$ s. Such a distortions appear due to the current changes become too fast that our laser system has no time to adapt and lag strongly with response. That leads not only to the PS point shift, but missing of some states that would be reached if the quasi stationary approximation is realized. This fact is illustrated by curve 5 in Fig. 2 where the polarization value -1 was not be reached during the current increasing. This value could be reached if we remove our PS point in the region of ~ 2 mA, but in any case the pattern of our curve will be some differ from "classical". At the same time the curve 6 in Fig. 2 shows that for the case of very fast current changes our laser system has the possibility to "rush" the region of the polarization negative value, that happens when "dynamical threshold current" becomes more then the PS current.

But more significant distortions of the curve $P(J)$ take place as the injection current falls. That is why when the curve $P(J)$ goes to the region below the threshold, then the lasing is quenched, P goes to zero and polarization hysteresis loop is no longer symmetric as illustrated by curve 5 in Fig. 2. Moreover, this effect will be approximately the same with further rise in the variation rate of the injection current. It should be noted, however, that to some extent, a relatively symmetric shape of the polarization hysteresis loop for $T = 10^{-8}$ s, exhibited by curve 6 in



a b

Fig. 2. Evolution of the polarization hysteresis loop on excitation by a triangular pulse with the half-period T. J_{ps}= 1.5 мА, k_0= 10^{-2}. T = 10^{-3} (1), 10^{-4} (2), 10^{-5} (3), 10^{-6} (4), 10^{-7} (5), 10^{-8} (6) s.

Fig. 2, is a random factor. Our calculations showed that a hysteresis loop distortions may be very intricate in form, though the function $P(J)$ remains fairly smooth because changing of the polarization degree in the region of PS is a rather slow process.

But we would like to point that the results on the high current changes have more qualitative rather than quantitative character because our model is not orientated on investigation of dynamical effects because it does not consider the possible delay of the induced anisotropy effect on the speed of current change. We just want to underline that the pattern of hysteresis loop can be greatly dependent on the process's parameters that creates such kind of response.

As demonstrated by the computations, the domain of T associated with distortions of polarization hysteresis is greatly dependent on the ratio between J_{ps} and the threshold current J_{th}: as J_{ps} is growing, the polarization hysteresis boundary is shifted drastically to the region of lower T. For example, when J_{ps} increases from 1.5 to 2.1 mA, this boundary is shifted approximately from 3×10^{-7} to 4.5×10^{-8} s. In other words, the higher the value of J_{ps}, the greater the range of T, where the polarization hysteresis loop symmetry is retained. Moreover, with a growth in J_{ps}, the symmetric form of polarization hysteresis can also be retained. Here, the point J_{ps} is lying practically at the centre of the length ΔJ_H as demonstrated by the curves in Fig. 3. This effect may be considered for the determination of J_{ps} representing an internal parameter of a laser system.

Note that the above-enumerated effects and also an increase in the width of the polarization hysteresis loop ΔJ_H with growing J_{ps} is in line with the proposed interpretation of the amplified radiation formed in VCSEL. Nonetheless, a natural conclusion about the need to increase J_{ps} with respect to J_{th} in order to realize better characteristics of this system may be questioned due to the problem associated with transition of lasing to the multimode regime [20], and therefore it should be studied further.

As regards to the broadening of polarization hysteresis loop with a corresponding increase in J_{ps}, it is worth noting that at a relatively low variation rate of the current, interpreted earlier as a "stationary approximation" [15–17], ΔJ_H varies slightly in a linear fashion (curves 2 and 3, Fig. 4), but grows drastically with a decrease in T(curve 1, Fig. 4).

It follows from the analysis of the functions in Figs. 2 and 3 that at a high rise rate of the current, a shift of the lasing threshold to the region of high injection currents is observed. This effect has been previously observed experimentally [17], but was interpreted as a dynamic shift of the bifurcation point [25]. Actually, the current shift is determined by the time delay associated with natural inertia of the amplified radiation formation process in a laser system. In this case the initiation of lasing is accompanied by fairly strong oscillations of the intensity. Fig. 5 shows the development of such oscillations - the lasing threshold is shifted with an increase in the current rising rate. In addition, these oscillations dies out and the process of lasing is monotonic for the falling current.

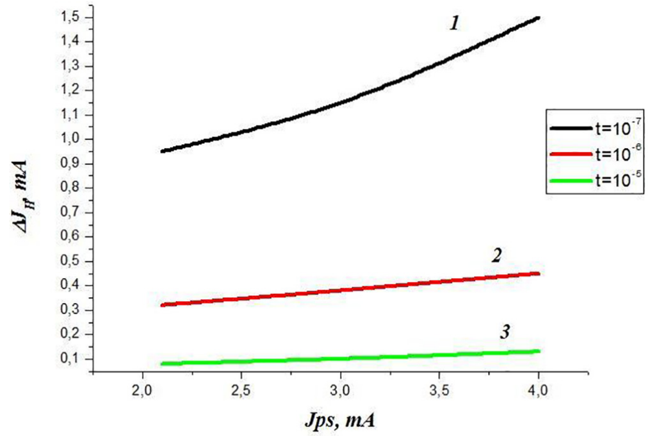


Fig. 4. Width of the polarization hysteresis loop ΔI_H as a function of the PS point position I_{ps} for different pulse half-periods T. $T = 10^{-7}$ (1); 10^{-6} (2); 10^{-5} (3) s.

However, at low values of T, as in the case of curve 6 in Fig. 2, the oscillations intensity doesn't die out for the whole pulse length. As it was pointed earlier we were not going to investigate dynamics of transient processes and so we don't pay a lot of attention to this phenomenon, just point the fact.

As seen from Fig. 4a, shifting the threshold value of the current appears to be independent of J_{ps}, which is attributed to low values of the parameter k_0 and weak dependence of the amplification factor $G(\psi)$ on J_{ps}. Besides, the effect of growing slope for $P(j)$ in PS region with lowering of pump pulse half-period T was well observed to be practically independent of J_{ps} value. Thus, it is concluded that the observed shift for the threshold value is pure dynamic in its character which is being a natural consequence of inertia during the formation process in a laser system.

The characteristic feature of this case shows that this shift is associated with oscillations intensity. When the current variation rate is not very high, these oscillations dies out, whereas when the current decreases, lasing goes to zero and at the threshold value the polarization hysteresis loop retains its characteristic form. As a result, the curve for the output intensity as a function of the injection current exhibits a loop that was previously interpreted as a hysteresis [15–17] due to dynamic displacement of the bifurcation point [25]. It is obvious that in this case the hysteresis effect is dynamic in character but it is unreasonable to attribute its occurrence to the bifurcation point behaviour because the existence of this point itself in this case is questionable. Indeed, for sufficiently low values of T, the oscillations intensity don't vanish when the current falls practically to zero. This leads to displacement of zero for the intensity and polarization to the region where the currents are

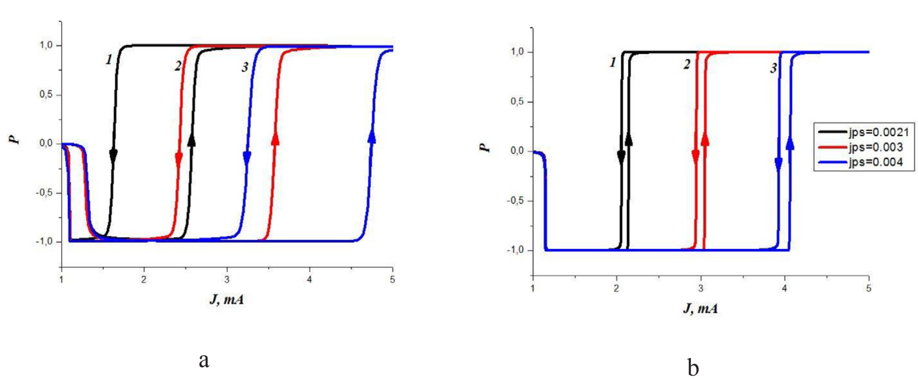


Fig. 3. Evolution of the polarization hysteresis loop on excitation by a triangular pulse with the half-period T when J_{ps} is varied. $T = 10^{-7}$ (a); 10^{-5} (b) s. J_{ps}= 2.1 (1); 3.0 (2); 4.0 (3) mA. k_0= 10^{-2}.

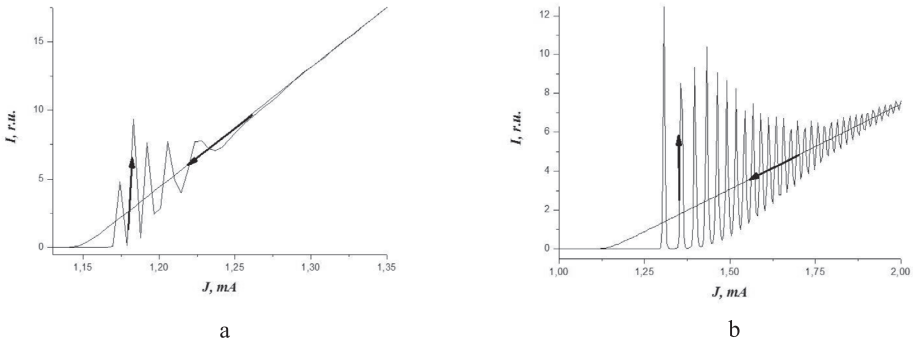


Fig. 5. Output intensity as a function of the injection current intensity on excitation by a triangular pulse with the half-period T. J_{ps}= 2.1 mA, k_0= 10^{-2}. T = 10^{-6} (a), 10^{-7} (b) s.

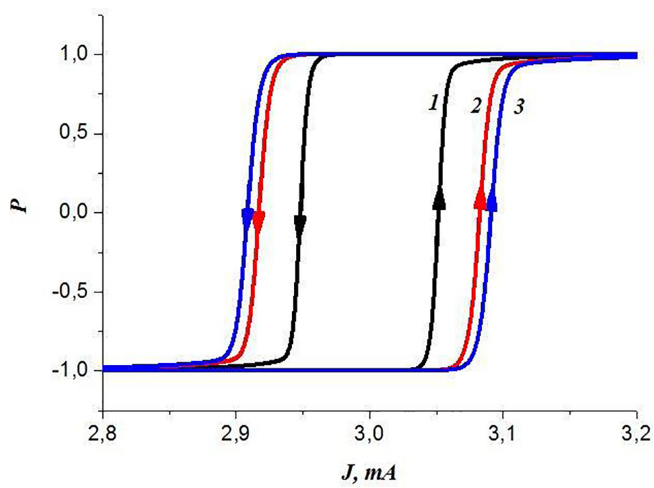


Fig. 6. Evolution of the polarization hysteresis loop as a function of I for different values of the reflection factor R of mirrors in a cavity. T = 10^{-5} s, R=0.995 (1); 0.999 (2); 0.9995 (3), J_{ps}=3 mA.

markedly lower than the threshold as clearly demonstrated by curves 5 and 6 in Fig. 2.

It has already been noted that the photon lifetime in a cavity is a characteristic time scale for consideration of dynamic effects [20–22]. When the current is growing for the period of time $T = 10^{-7}$ s, then the time of covering the PS region, where the polarization hysteresis loop is formed, comes to be $\sim 10^{-10}$s. Considering that usually the photon lifetime in a cavity of VCSEL is about several picoseconds [6], the quasi-stationarity condition for the intensity formation is fulfilled quite well over the whole range $T > 10^{-7}$ s. However, the inertia of the polarization formation process of the output-radiation in the region of PS is much greater [22], leading to the increased displacement and increased slope for $P(J)$ curve along with an increase in the current rise rate.

It becomes clear that the character of PB variations should be examined when the photon lifetime in a cavity is changed. This quantity is determined by the total cavity loss, i.e. by the Q-factor. In practice, the Q-factor of a cavity may be easily changed by variations in the reflection of mirrors R which is considered to be identical for simplicity reasons (It is noteworthy that in VCSEL, instead of mirrors, the Bragg reflectors are used, enabling variations in the reflection due to changes in number of the layers). Fig. 6 illustrates the numerical results which are in line with the mechanisms under study.

In the actual scenes, an increase in R leads to an increased in the lifetime of photons, i.e. a laser system becomes more inertial, the curve for $P(J)$ is shifted and its slope rises up. Nevertheless, this effect is deemed nonlinear due to the logarithmic dependence of the internal loss on the reflection factor R, for $R > 0.999$ being insignificant even when $T = 10^{-7}$ s.

Conclusions

In this research work, the phenomenon of the polarization hysteresis loop was interpreted as a result of internal inertia of the polarization formation process for the output radiation in the region of PS when the injection current varies sequentially. As a consequence, the curve of $P(J)$ shifted to the region of higher current values when the current has risen, and lower current values when the current has fallen down. In addition, in this process the shift and the slope angle of $P(J)$ have increased with increasing current-variation rate.

High rate of the injection current variations was associated with nonlinear distortions of the polarization hysteresis loop, however, as demonstrated by the results of simulation, the boundary of such nonlinear distortions has shifted due to the increment of PS current value. This fact is of particular importance for the development of optoelectronic systems based on the polarization hysteresis phenomenon.

Furthermore, the polarization hysteresis as a phenomenon and its behaviour due to variations in the parameters of a laser system or its operation modes were well described by the generalized model for the polarized radiation formation in VCSEL that was developed by previous researchers' work [20–22].

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