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Light dynamics in $\mathcal{PT}$-symmetric multilayers: Phase transition, nonreciprocity, and propagation direction locking

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Abstract. We report the numerical study of $\mathcal{PT}$-symmetry breaking in one-dimensional structures with resonantly absorbing and amplifying layers described by the Maxwell-Bloch equations. Temporal dynamics of light interacting with such a structure is governed by strong field amplification and subsequent saturation of loss and gain above the exceptional point resulting in the lasing-like regime with powerful pulses generated both in reflection and transmission. In this regime, we predict and investigate the uniqueness of phase transition due to saturation, transmission nonreciprocity, and the direction locking of the transmitted and reflected radiation.

Optical $\mathcal{PT}$-symmetric structures and the effects connected to $\mathcal{PT}$-symmetry breaking is one of the hot topics in modern photonics [1, 2, 3]. In one-dimensional (1D) geometry, optical $\mathcal{PT}$-symmetric structure can be realized as a photonic-crystal-like system composed of alternating loss and gain layers. Such a multilayer should have proper spatial variation of the complex permittivity satisfying the necessary condition $\varepsilon(z) = \varepsilon^*(-z)$, i.e., the real part of the permittivity is an even function of the coordinate, whereas the imaginary part is an odd function. There is a number of phenomena associated with violation of the $\mathcal{PT}$ symmetry, such as sharp change in polarization response of the system [4], enhanced sensitivity to external perturbations near exceptional point [5, 6], new effects of lasing [7, 8] and anti-lasing [9].

In this work, we consider a 1D structure in which both loss and gain materials are described as a two-level resonant medium. This allows us to describe self-consistently dynamics of field and material parameters and to shed new light on change of system's temporal response at the exceptional point where the phase transition of $\mathcal{PT}$-symmetry breaking occurs. The system considered in this work is a periodic planar structure composed of $2N$ alternating loss and gain layers. It is illuminated by normally incident monochromatic light of frequency ω. We describe both loss and gain in a similar manner, using the model of a homogeneously-broadened two-level

medium. Then light interaction with the structure is given by the Maxwell-Bloch equations for microscopic polarization ρ, population difference w, and electric field amplitude A [10],

$$\frac{d\rho}{d\tau} = il\Omega w + i\rho\delta - \gamma_2\rho, \tag{1}$$

$$\frac{dw}{d\tau} = 2i(l^*\Omega^*\rho - \rho^*l\Omega) - \gamma_1(w - w_{eq}), \tag{2}$$

$$\frac{\partial^2\Omega}{\partial\xi^2} - n_d^2\frac{\partial^2\Omega}{\partial\tau^2} + 2i\frac{\partial\Omega}{\partial\xi} + 2in_d^2\frac{\partial\Omega}{\partial\tau} + (n_d^2 - 1)\Omega$$
$$= 3\alpha l\left(\frac{\partial^2\rho}{\partial\tau^2} - 2i\frac{\partial\rho}{\partial\tau} - \rho\right), \tag{3}$$

where $\tau = \omega t$ and $\xi = kz$ are respectively the dimensionless time and distance, $\Omega = (\mu/\hbar\omega)A$ is the normalized Rabi frequency, $k = \omega/c$ is the wavenumber in vacuum, c is the speed of light, $\hbar$ is the reduced Planck constant, μ is the dipole moment of the quantum transition, $\alpha = \omega_L/\omega = 4\pi\mu^2 C/3\hbar\omega$ is the normalized Lorentz frequency, C is the concentration of active atoms, $\delta = (\omega_0 - \omega)/\omega$ is the normalized detuning of laser frequency from the atomic resonance, $\gamma_1 = 1/(\omega T_1)$ and polarization $\gamma_2 = 1/(\omega T_2)$ are the normalized relaxation rates expressed by means of the longitudinal T_1 and transverse T_2 relaxation times, $l = (n_d^2 + 2)/3$ is the local-field enhancement factor, and n_d is the real-valued refractive index of the background medium.

We describe both gain and loss materials with the same Maxwell-Bloch equations (1)-(3) using the equilibrium population difference w_{eq} as a key parameter governing the level of pumping. Indeed, in the stationary approximation the resonant medium can be characterized by the effective permittivity $\varepsilon_{eff} \approx n_d^2 + 3il^2\omega_L T_2 w_{eq}$ [11]. Since gain and loss correspond to negative and positive w_{eq}, it is straightforward to obtain a $\mathcal{PT}$-symmetric structure composed of alternating layers with balanced loss (ε_{eff+}) and gain (ε_{eff-}), where $\varepsilon_{eff\pm} \approx n_d^2 \pm 3il^2\omega_L T_2|w_{eq}|$. The necessary condition $\varepsilon(z) = \varepsilon^*(-z)$ is obviously fulfilled for such a structure, providing even (odd) function of z for the real (imaginary) part of the permittivity. It can be also shown that $\mathcal{PT}$ symmetry approximately follows directly from the Maxwell-Bloch equations, if the steady state is established.

Equations (1)–(3) are solved numerically using the FDTD approach developed earlier [12]. For calculations, we use semiconductor doped with quantum dots as an active material characterized by the following parameters: $n_d = 3.4$, $\omega_L = 10^{11}$ s^{-1}, $T_1 = 1$ ns, and $T_2 = 0.5$ ps. We also assume the exact resonance $\delta = 0$. The multilayer structure contains $N = 20$ unit cells with both loss and gain layers having the same thickness $d = 1$ μm.

We first explore the response of the system in $\mathcal{PT}$-symmetric phase, i.e., below the exceptional point. Our numerical simulations of monochromatic light propagation are in good agreement with the transfer-matrix calculations performed in the stationary approximation with the effective permittivities given above. This agreement is due to the rapid establishment of stationary transmission and reflection example of which is shown in Fig. 1(a). It is also seen that owing to reciprocity of the system, the transmission of oppositely propagating (forward and backward) waves is the same ($T_{LG} = T_{GL} = T$), but the reflection is different ($R_{LG} \neq R_{GL}$). In particular, we are able to reproduce the anisotropic transmission resonances characteristic for 1D $\mathcal{PT}$-symmetric structures when $T = 1$ and one of the reflection coefficients goes to zero [13].

$\mathcal{PT}$-symmetry breaking occurs when the pumping parameter reaches a certain threshold value, which can be determined in the stationary approximation as an exceptional point where eigenvalues of the scattering matrix cease to be unimodular, whereas eigenvectors coincide [13]. For the structure parameters given above, the position of this phase transition can be found within the transfer-matrix method as $w_{eq} \approx 0.222$. However, as the pumping parameter approaches the exceptional point, the stationary approximation becomes inapplicable: the
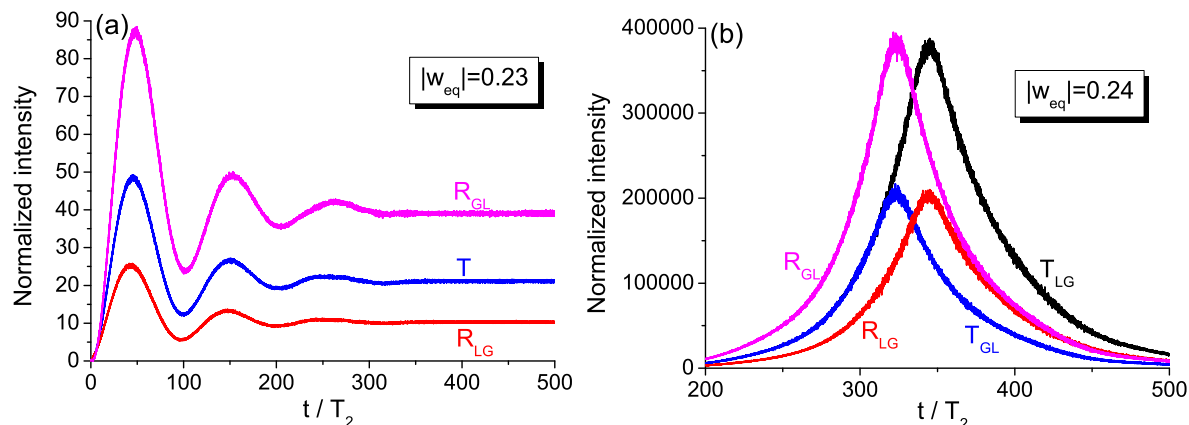
Figure 1. Temporal dynamics of the reflected (R) and transmitted (T) intensity for the pumping parameter (a) $|w_{eq}| = 0.23$ and (b) $|w_{eq}| = 0.24$. The two directions of wave propagation are denoted with subscripts LG and GL originating from the order of layers in the unit cell of the structure.

discrepancy between the results given by the transfer-matrix method and numerical simulations of the Maxwell-Bloch equations sharply increases. This means that near the phase transition point we should solve the differential equations in time domain in order to correctly predict the response of the system.

Above the exceptional point, a strong amplification of the electromagnetic field inside the structure occurs, since loss and gain are not balanced anymore. This enhancement is limited by saturation that develops in the resonant medium as the field becomes strong enough. Indeed, whereas population difference preserves its initial value $w(t) = w_{eq}$ in the $\mathcal{PT}$-symmetric phase below the exceptional point, it starts changing due to saturation above the exceptional point. The resulting temporal profiles of transmitted and reflected intensity have the form of powerful pulses due to the rapid release of energy stored in the gain layers [see Fig. 1(b)]. Therefore, the $\mathcal{PT}$-symmetry-broken phase can be called the lasing-like regime (in analogy with [11]), which can be treated as a dynamical feature of this phase state.

Saturation is also the reason for the irreversibility of $\mathcal{PT}$-symmetry breaking: since the necessary condition $\varepsilon(z) = \varepsilon^*(-z)$ is not fulfilled anymore due to changing population difference, the return of the system with a further increase of the pumping parameter back into the PT-symmetric state as predicted in the stationary approximation is now impossible.

In the lasing-like regime, the structure becomes nonreciprocal, since transmission is no more symmetric ($T_{LG} \neq T_{GL}$) as clearly seen in Fig. 1(b). Moreover, the intensities of the pulses escaping the system do not depend on the direction of the incident light: almost the same pulses are emitted from the gain and loss ends of multilayer after reversing the input light direction ($T_{LG} = R_{GL}$ and $T_{GL} = R_{LG}$). This novel unusual feature due to $\mathcal{PT}$-symmetry breaking can be called the propagation direction locking of radiation transmitted and reflected by the structure. Saturation is not enough to explain this effect as evidenced by calculations for relatively powerful incident wave when $\mathcal{PT}$ symmetry is broken at every value of the pumping parameter. Therefore, the $\mathcal{PT}$-symmetry breaking is necessary for the locking effect to occur. This phenomenon can be viewed as a possible basis for peculiar all-optical diodes and transistors.

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