

Article

A Model of Mechanothermodynamic Entropy in Tribology

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Academic Editor: Michael M. Khonsari

Received: 22 September 2016; Accepted: 6 March 2017; Published: 14 March 2017

Abstract: A brief analysis of entropy concepts in continuum mechanics and thermodynamics is presented. The methods of accounting for friction, wear and fatigue processes in the calculation of the thermodynamic entropy are described. It is shown that these and other damage processes of solids are more adequately described by tribo-fatigue entropy. It was established that mechanothermodynamic entropy calculated as the sum of interacting thermodynamic and tribo-fatigue entropy components has the most general character. Examples of usage (application) of tribo-fatigue and mechanothermodynamic entropies for practical analysis of wear and fatigue processes are given.

Keywords: entropy; mechanothermodynamics; tribo-fatigue; tribology; fatigue; damage

1. Thermodynamic Entropy

The following functions are used to describe the state of thermodynamic systems [1]:

$$U = U(T, V, N_k) \text{ or } S = S(T, V, N_k), \quad (1)$$

where temperature T , volume V , and number of moles of chemical components N_k are the macroscopic state variables.

In the general case of an open system, the change dU of internal energy U is presented [1] as:

$$dU = dQ + dA + dU_{sub} = TdS - pdV + \sum_1^n \mu_k dN_k \quad (2)$$

where dQ is the amount of heat; dA is the amount of mechanical energy; dU_{sub} is the amount of substance, which the system exchanged with the environment for a time interval dt ; p is the pressure; the μ_k 's are the chemical potentials. Planck especially emphasized that dU in Equation (2) is an infinitely small difference, whereas dQ , dA , and dU_{sub} are the infinitely small amounts.

It follows from (2) that entropy S change in a thermodynamic system (subscript T) is:

$$dS_T = \left(\frac{dU + pdV}{T} - \frac{1}{T} \sum_1^n \mu_k dN_k \right)_T \quad (3)$$

Entropy increment (3) can be presented [1] as the sum of its change $d_e S \geq 0$ due to *system exchange of energy and substance with the environment* and the change $d_i S \geq 0$ due to *irreversible* processes occurring inside the system:

$$dS_T = d_e S + d_i S \quad (4)$$

Thus, in thermodynamics, the entropy S is a measure of irreversible energy dissipation [2] that characterizes the system state from the viewpoint of its internal order or structure.

Equations (2) and (3) do not take into account many processes, for example, internal energy changes at damage of moving and deformable solids and tribo-fatigue systems [3–6]. Substance exchange [1] is considered only as a result of such processes as diffusion and chemical reactions, whereas substance exchange at surface wear and volume (e.g., fatigue) damage is taken into account. That is why, the problem of evaluating the entropy arises in relation to the mentioned and (other) conditions of the development of numerous phenomena of solids damageability.

We also note that in (2) and (3) as well as in a general analysis of changes in thermodynamic forces and fluxes [1] any critical states of the system are not considered. Meanwhile in the case for example of the processes of wear and fatigue of solids it is important to have a prediction if such a state is reached or not in predetermined operating conditions.

2. Thermomechanic Entropy

It is assumed in continuum mechanics [7,8] that the stress tensor can be decomposed into two parts:

$$\sigma_{ij} = \sigma_{ij}^{(c)} + \sigma_{ij}^{(d)} \quad (5)$$

where the superscript c denotes the *conservative* part for reversible processes (elastic deformation, liquid pressure) and superscript d denotes the *dissipative* part for irreversible processes (plastic deformation, liquid viscous stresses).

And then after the appropriate energy analysis the *thermomechanical entropy function* is obtained in the following form:

$$\frac{dS}{dt} = \frac{1}{T} \frac{dq}{dt} + \frac{1}{\rho T} \sigma_{ij}^{(d)} \dot{\epsilon}_{ij} \quad (6)$$

where dq/dt is the rate of heat flow to the environment per unit mass, $\frac{1}{\rho T} \sigma_{ij}^{(d)} \dot{\epsilon}_{ij}$ is the rate of energy dissipation per unit mass, ρ is the density of continuum.

Equation (6) is valid only for the continuum. If the continuity of a deformable solid is violated (for example due to origination and development of relevant damages) this equation cannot be used. Thus (6) does not describe physical damage of solids. Conditions of reaching critical state (for example fracture into separate parts) are also not addressed in this model. In addition, (6) considers entropy only for the case of quasi-static deformation while repeated and in particular cyclic deformation are of practical importance.

3. Entropy in Friction and Wear Processes

In a series of papers on tribology [9–21] and others the influence of friction and wear processes in the system of two solids on the change of the thermodynamic entropy S_i is studied. Its production is formulated as the amount of thermodynamic forces X_j^k and thermodynamic J_j^k in the form of:

$$\frac{d_i S}{dt} = \sum_j \sum_k \left(\frac{\partial_i S}{\partial p_j} \frac{\partial p_j}{\partial \sigma_j^k} \right) \frac{\partial \sigma_j^k}{\partial t} = \sum_j \sum_k X_j^k J_j^k \quad (7)$$

Bryant et al. [9] proposed the concept of friction pair degradation:

$$\frac{dw}{dt} = \sum_j \sum_k \left(\frac{\partial w}{\partial p_j} \frac{\partial p_j}{\partial \sigma_j^k} \right) \frac{\partial \sigma_j^k}{\partial t} = \sum_j \sum_k Y_j^k J_j^k \quad (8)$$

where the degradation parameter $w = \{p_j(\zeta_j^k)\}$ is considered as (thermodynamic Y_j^k) force of degradation.

Relation of friction and wear is constituted through common dissipative processes at sliding interfaces. Works [9,10] contain the review (represented by Equations (9)–(15)) of friction and wear aimed at identifying prevalent dissipative processes at tribological interfaces and estimating the irreversible entropies produced. These works discuss the following dissipative processes:

- (1) Adhesion of surfaces and films, associated with adhesive wear and adhesive friction, with entropy change:

$$\Delta S' = \frac{\gamma}{T_m} \Delta A_S \quad (9)$$

where interface surface energy γ is the work per unit area to create new surface area ΔA_S and T_m is the local temperature of the media.

- (2) Plastic deformation, associated with abrasive wear, friction ploughing, and/or cutting, with entropy change:

$$\Delta S' = \frac{U_c}{T_m} \Delta V \quad (10)$$

where U_c is work (plastic deformation or cutting) expended per unit volume ΔV affected, and T_m is temperature of the affected material media.

- (3) Fracture, associated with fatigue wear and surface damage, with entropy change:

$$\Delta S' = \frac{G - 2\gamma_o}{T_{cr}} da \quad (11)$$

where a is crack length, $G = -\partial U_S / \partial a$ is the energy release rate dependent on strain energy U_S , γ_o is surface energy, and T_{cr} is temperature of the cracked material at the crack tip.

- (4) Phase changes, associated with surface melting, and recrystallization of metals, with entropy change:

$$\Delta S' = \frac{\Delta H}{T_{phase}} \quad (12)$$

where ΔH is the change in enthalpy—the latent heat absorbed or shed during the phase change—and T_{phase} is the temperature associated with the phase change.

- (5) Chemical reactions, associated with chemical and oxidation wear, with entropy change:

$$\Delta S' = \frac{\bar{A}}{T_{cr}} d\xi, \quad \bar{A} = \sum_{\text{reactants}} \kappa_i \eta_i - \sum_{\text{products}} \kappa_i \eta_i \quad (13)$$

where chemical affinity A depends on the chemical potentials η_i and stoichiometric coefficients κ_i of the reaction. Since the stoichiometry of the reaction relates changes of molar masses dN'_i of reactants and products, extent of reaction ξ , has differential change $d\xi = \pm dN'_i / \kappa_i$ where minus pertains to reactions, and plus to products.

- (6) Diffusion, associated with gradient induced migration of material, with entropy change similar to that for chemical reactions.
- (7) Mixing, which for tribology involves mixing of third body [22,23] material entrained between surfaces, with molar entropy change:

$$\Delta S' = -R \sum_i^n \frac{N_i}{N} \ln \frac{N_i}{N}, \quad N = \sum_i^n N_i \quad (14)$$

Here R is the universal gas constant, N_i denotes molar masses of n species, and ratios N_i/N denote molar fractions.

- (8) Heat transfer, associated with diffusion of heat dQ from region of higher temperature T_h to lower temperature T_l , with entropy change:

$$\Delta S' = \left(\frac{1}{T_l} - \frac{1}{T_h} \right) dQ \quad (15)$$

In work [24] Kozyrev and Sedakova performed an experimental and theoretical analysis for the non-equilibrium thermodynamic statement in order to derive a dependence of wear rate on the load in stationary state and gave the concretization of Equation (8) for the determination of friction conditions. It was shown in [24] that the linear increase of wear rate due to load increase may be interrupted as a result of wear rate reduction. Nonlinear behavior of wear W is observed as a function of pressure p . Therefore, for the certain load range wear can decrease with the increase in pressure. This phenomenon is explained by formation of wear resistant oxide layers as the result of chemical reactions in tribological pair. Beside the friction process authors also take into account the diffusion of material into tribofilm. The diffusion process can be considered as an external element that lead to self-organization and wear reduction. For two dissipative processes: friction with forces $X_1 = -\nabla T/T^2$ and flows $J_1 = -k\nabla T = \mu Nu$; and diffusion with $X_2 = -\nabla \varphi/T$ and $J_2 = -\gamma_D \nabla \varphi$, Equation (7) results in the following expression for production of entropy:

$$\frac{d_i S}{dt} = \frac{(pu)^2 \mu^2 A^2}{kT^2} + \frac{\gamma_D (\nabla \varphi)^2}{T} \quad (16)$$

where p and A are pressure and nominal area of contact, φ is the chemical potential, $p = N/A$, γ_D is the transport coefficient, N is the normal load, μ is the coefficient of friction and u is the velocity.

It is assumed in [24] that in non-equilibrium stationary state the wear of the tribofilm is proportional to γ_D and the pu product is a characteristic of friction. Therefore, for the stationary operation the analysis is performed for the conditions of minimum γ_D depending on pu :

$$\left(\frac{d}{d(pu)} \right)_T \frac{d_i S}{dt} = \frac{2(pu)^2 \mu^2 A^2}{kT^2} + \frac{d\gamma_D (\nabla \varphi)^2}{T} \quad (17)$$

If (17) is set to zero and integrated, then:

$$\gamma_D = \gamma_{D0} - \frac{(pu)^2 \mu^2 A^2}{kT (\nabla \varphi)^2} \quad (18)$$

where γ_{D0} is the integration constant.

Equation (18) indicates that γ_D decreases at pu increases under stationary conditions for the assumption that γ_D is proportional to wear W . Therefore, under the stationary non-equilibrium conditions, Equation (18) offers a procedure for decreasing wear with increase in pu .

Note that the interdependence of normalized wear (determined experimentally) and normalized entropy (evaluated by calculation) is considered. This interdependence is presented (for different friction conditions) in the form of the straight lines (see Figure 7 of [13]). Thus, account of the friction and wear processes in the production of entropy is made by corresponding adjusting its traditional thermodynamic representation. Therefore, the obtained solutions have the same disadvantages that are inherent to the thermodynamics and thermodynamic entropy (see Sections 1 and 2). Also note that tribological analysis is carried out in terms of the parameter w of thermodynamic degradation of the friction pair mechanical meaning of which is not entirely clear.

4. Entropy in Fatigue Damage Processes

A damage evolution model based on thermodynamic theory was developed by Basaran et al. [25–34]. According to this model irreversible entropy production rate under loadings of different nature is

considered as a single universal damage term-damage metric. The model is based on Boltzmann thermodynamic concept of entropy in the form of relation between the entropy per unit mass s and the disorder parameter W [25]:

$$s = \frac{R}{m_s} \ln W \quad (19)$$

where m_s is the specific mass, R is the gas constant.

According to (19) the disorder function becomes the following:

$$W = e^{\frac{sm_s}{R}} \quad (20)$$

If an initial reference state of the continuous medium is characterized by disorder value W_0 , then the change in disorder at any arbitrary time with respect to the initial reference state is given by [27]:

$$\Delta W = W - W_0 = e^{\frac{sm_s}{R}} - e^{\frac{s_0 m_s}{R}} \quad (21)$$

where s_0 is the entropy for the initial reference state.

According to Basaran and Nie [28] the isotropic damage variable D is defined as the ratio of the change in disorder parameter (21) (or the change in entropy) to the current state disorder parameter with a proportional critical disorder coefficient D_{cr} :

$$D = D_{cr} \frac{\Delta W}{W} = D_{cr} \left[1 - e^{-\frac{m_s}{R}(s-s_0)} \right] \quad (22)$$

Coefficient D_{cr} allows correlating the value of entropy production based damage D with other material coordinates like degradation of material stiffness. This coefficient can be determined from experimental data. However, it can be different for different loading profiles such as cyclic and monotonous loading.

Experimental verification of Equation (22) is given in [27,28].

Processes of accumulation of fatigue damage are analyzed in a number of other works [35–41] by Khonsari, Amiri and Naderi basing on the traditional concept of entropy. One of these approaches presented in [35,36] is discussed below.

Damage accumulation can be formulated as a degradation process and it is possible to derive the following relationship [36] between the degradation caused by the cumulative damage and the entropy production using the degradation–entropy production theorem [21]:

$$D = A + B \ln \left(1 - \frac{s}{s_g} \right) \quad (23)$$

where A and B are material parameters, s represents the entropy production during fatigue life, and s_g is the total entropy production at the onset of fracture.

Thermal dissipation in solids with internal friction the entropy production due to plastic deformation according to the Clausius–Duhem inequality have the following relationship [35,36]:

$$\dot{s} = \frac{w_p}{T} - \mathbf{J}_q \cdot \frac{\mathbf{grad} T}{T^2} \quad (24)$$

where \dot{s} is entropy production rate ($\dot{s} \geq 0$), \mathbf{J}_q is heat flux, T is surface temperature, w_p is cyclic plastic energy per unit volume which can be calculated using Morrow's expression [42]:

$$w_p = AN_f^\alpha \quad (25)$$

where constant A and α are the material specifications (see [42]) and can be calculated from the following relationship:

$$A = 2^{2+b+c} \sigma'_f \epsilon'_f \left(\frac{c-b}{c+b} \right) \quad (26)$$

$$a = b + c \quad (27)$$

where σ'_f , b and ϵ'_f , c denote the fatigue strength and fatigue ductility coefficients, respectively.

Under low-cycle fatigue entropy production is mostly caused by plastic deformation while entropy production due to heat conduction is negligible [36]:

$$\dot{s} = \frac{w_p}{T} \quad (28)$$

The accumulation of the entropy production can be obtained by integrating (28):

$$s = \int_0^t \left(\frac{w_p}{T} \right) dt \quad (29)$$

The entropy production is calculated based on the temperature measurements and the cyclic energy approximation (28). Equation (23) is then used to determine fatigue damage evolution.

According to [35,37] entropy continuously increases simultaneously with the rise of degradation, towards the fatigue fracture entropy (FFE). Research shows that regardless of the type of the mechanical fatigue process (bending or torsion, tension–compression) FFE is constant unique to the material. Specifically for SS 304, $\text{FFE} \cong 60 \text{ MJm}^{-3} \cdot \text{K}^{-1}$.

Entropy production based on damage evolution is independent of load, frequency, and loading history and has no knee point under variable amplitude loading. (see Figure 8 of ref. [35]).

Entropy production based on damage evolution results shown in Figure 8 of ref. [35] is normalized with respect to the FFE value for SS 304. These results have two important implications [35,37]: (a) fatigue damage accumulation is independent of load, frequency, and geometry; and (b) damage behavior under variable loading has no knee point and independent of multistage loading and loading sequence. As damage progresses towards the final fracture, the accumulated entropy monotonically increases to the fracture fatigue entropy (FFE) which is unique for a given material [37]. Note that cumulative entropy production over individual amplitudes sums to unity: $\sum_{k=1}^n (s_k/s_g) = 1$. This idea can be used as a criterion for fatigue damage monitoring and preventing the system under cyclic loading before failure occurs.

Thus, the account of fatigue damage in entropy assessment is based on cyclic plastic deformation w_p . Therefore, only low-cycle fatigue is considered (see Figure 5 of ref. [35]). Processes of high-cycle fatigue, which are of great practical importance, are addressed in [43,44].

5. Tribo-Fatigue Entropy

Special attention is paid in Tribo-Fatigue to the analysis of physical and mechanical damageability of solids working in complex and diverse loading conditions [3–6]. According to the generalized concepts given in [4,45–47], the damage is the change in composition, construction, structure, size, shape, volume, continuity, mass and, hence, in the corresponding physical-chemical, mechanical and other properties of an object. Finally, the damage is related to the discontinuity and integrity of a solid up to its decomposition e.g., into atoms (see Table 1) [46,47]. Thus, the damageability is treated as a fundamental property (and duty) of moving and deformable systems [4,45–47], and the failure is considered as a specific type of damage, i.e., the corresponding discontinuity and integrity of objects including the ones of great practical importance [46–52].

Any damage is real because it can be seen and measured. It is therefore objective since it exists and develops independently of our consciousness, that is independently of what mechanisms of its formation and development are known to us.

It is shown in [3–5,47] that for tribo-fatigue systems the *irreversible damageability* ω_Σ (as well as Ψ_u^{eff}) is the function of the effective mechanical U_M^{eff} , thermal U_T^{eff} and electrochemical U_{ch}^{eff} energies. Here the mechanical energy due to changes in solid size (U_σ^{eff}) and due to changes in solid shape (U_τ^{eff}) is distinguished:

$$\omega_\Sigma = \omega_\Sigma(U_\sigma^{eff}, U_\tau^{eff}, U_T^{eff}, U_{ch}^{eff}) = \omega_\Sigma(\sigma_{ij}, \varepsilon_{ij}, T_\Sigma, \nu_{ch}(m_\nu), \Lambda_{\sigma \setminus p}, \Lambda_{T \setminus M}) = \omega_\Sigma(U_\Sigma^{eff}) \quad (30)$$

where Λ -functions characterize the interaction of damages due to different loads (volumetric and frictional contact denoted by the subscript σ/p ; thermal and mechanical—by the subscript T/M), T_Σ is the temperature caused by all heat sources, $U_\Sigma^{eff} = U_\Sigma^{eff}(\sigma_{ij}, \varepsilon_{ij}, T_\Sigma, \nu_{ch}(m_\nu), \Lambda_{\sigma \setminus p}, \Lambda_{T \setminus M})$ is the total effective energy [3]. Methods of Λ -functions determination are described for example in works [3,47,52,53].

Table 1. Characteristic of the states of objects.

Main Types of Systems States	Qualitative Characteristic of Damage	Parameter (Quantitative Characteristic) of Damage	Type of Evolution
A-state	Undamaged	$\omega_\Sigma = 0 = \omega_0$	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">↓</div> <div> <i>A</i>-evolution: characteristic system states in terms of damageability </div> </div>
B-state	Damaged	$0 < \omega_\Sigma < 1$	
C-state	Critical(limiting)	$\omega_\Sigma = 1 = \omega_c$	

In (30) the known interrelation of energy and corresponding force factors are taken (σ_{ij} and ε_{ij} are the stress and strain tensors, ν_{ch} is the rate of electrochemical processes with regard to the material properties (m_ν)). The energy directly spent for the formation and development of irreversible damages is called effective energy, i.e., U^{eff} is the absorbed part of the energy supplied to the system [4]. The methods of its determination are outlined in [3,4,47,52,53] and provide the formulas for estimation of ω_Σ under different operating conditions of tribo-fatigue systems. According to (30) the damage ω_Σ is thermomechanical function as it takes into account both force factors and temperature T_Σ due to all heat sources (including caused friction and volumetric deformation).

As a rule, irreversible damages are formed and accumulated not within the entire (geometric) volume of a deformable solid but only within its finite region with a critical state; this region is called the *dangerous volume* [4,52]. Model of a deformable solid with dangerous volume was developed in [53] and its generalization for the application to tribo-fatigue systems was given in [3–6]. According to [4] tribo-fatigue system is any mechanical system that receives and transmits repeated workload and in which friction processes in all their manifestations are realized simultaneously [3–6]. Complex wear-fatigue damage is characteristic for such a system. The regularities of such a damage are well known [3–6].

As internal irreversible damages of thermomechanical nature originate due to effective energy changes in the dangerous volume $V_{P\gamma}$ of the system, in the general case we have:

$$dU_\Sigma^{eff} = \omega_\Sigma dV_{P\gamma} \quad (31)$$

Techniques of calculating dangerous volumes with respect to various conditions of deformation of solids and tribo-fatigue systems are set forth in the works [3–6].

Then according to (2)–(4) it is possible to introduce the concept of *tribo-fatigue entropy*, whose change is:

$$(d_i S)_{TF} = \frac{\gamma_1^{(w)}}{T_\Sigma} \omega_\Sigma dV_{P\gamma} \quad (32)$$

where $\gamma_1^{(w)}$ is the stress (pressure) that causes the damage of a unit dangerous volume.

Thus, the tribo-fatigue entropy serves as a measure of irreversible absorption of the energy U_{Σ}^{eff} in the dangerous volume $V_{P\gamma}$ of the tribo-fatigue system. Let us now show the analogy between the concepts of tribo-fatigue and thermodynamic entropy. According to thermodynamics, in the general case, the irreversible change of the entropy $d_i S$ is associated with the flux of some quantity X (for example, heat or substance):

$$d_i S = F dX \quad (33)$$

where F is the thermodynamic force. So, at gas expansion in the piston engine the pressure of the gas volume (p_1) is always larger than the piston pressure (p_2). The difference ($p_1 - p_2$) then characterizes the pressure gradient and is the force per unit area that displaces the piston. At $T = \text{const}$, the irreversible entropy increment [1] is:

$$(d_i S)_T = \frac{p_1 - p_2}{T} dV > 0 \quad (34)$$

Here, $(p_1 - p_2)/T$ corresponds to the thermodynamic force (F) and dV characterizes the thermodynamic flux (dX) associated with it.

If ω_1 is the damage concentration at the point of a solid with the largest stress (p_1) and ω_2 is the damage concentration at any solid point, at which the stress (p_2) is smaller, then $\omega_2 < \omega_1$, i.e., there is a damage gradient characterized by the difference ($\omega_1 - \omega_2$) that is associated with the value of the dangerous volume $V_{P\gamma}$.

The irreversible tribo-fatigue entropy increment at $T_{\Sigma} = \text{constant}$ is:

$$(d_i S)_{TF} = \gamma_1^{(w)} \frac{\omega_1 - \omega_2}{T_{\Sigma}} dV_{P\gamma} > 0. \quad (35)$$

Thus, here $\gamma_1^{(w)}(\omega_1 - \omega_2)/T_{\Sigma}$ corresponds to the thermodynamic force F since $\omega_1 \sim p_1$ and $\omega_2 \sim p_2$, whereas $dV_{P\gamma}$ characterizes the thermodynamic flux X associated with this force.

Generalizing the aforesaid, we can write (32) in terms of the matrix determinant $L_{\omega_{\Sigma}}$ of the system damageability:

$$(d_i S)_{TF} = \gamma_1^{(w)} \frac{L_{\omega_{\Sigma}}}{T_{\Sigma}} dV_{P\gamma} \quad (36)$$

$$L_{\omega_{\Sigma}} = \begin{vmatrix} \omega_{11} & \omega_{12} & \dots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \dots & \omega_{2n} \\ \dots & \dots & \dots & \dots \\ \omega_{n1} & \omega_{n2} & \dots & \omega_{nn} \end{vmatrix} \quad (37)$$

It can be composed of such a number of the components (ω_{ij}) that corresponds to the number of stress tensors that affect the changes in the damageability state of the system. The concept of the *damage tensor* of type (37) was introduced in [4,46]. In the general case, according to (30) and (37), we have:

$$\omega_{ij} = \omega_{ij}(\sigma_{ij}, \sigma_*) \quad (38)$$

where σ_* is the characteristic limiting stress [4,46–52].

6. Mechano-thermodynamic Entropy

Now consider an open thermodynamic system containing a damageable solid; it is a mechano-thermodynamic system (MTD) [46,47,52–54]. Characteristic feature of the mechano-thermodynamic system in comparison with the thermodynamic one is that both tribo-fatigue and thermodynamic entropies are generated in it. The total change of entropy of such a system in contrast to the traditional notions of additive energies is determined on the basis of dialectical

interaction (function $\Lambda_{T\backslash TF}$) of damages of different nature caused by thermodynamic (subscript T) and tribo-fatigue (subscript TF) entropies:

$$(dS)_T + (d_iS)_{TF} = \left[\left(\frac{dU + \Delta p dV}{T} - \frac{1}{T} \sum_1^n \mu_k dN_k \right)_T + \left(\frac{\gamma_1^{(w)}}{T_\Sigma} \omega_\Sigma dW_{P_\gamma} \right)_{TF} \right] \Lambda_{T\backslash TF} \quad (39)$$

Here it is taken into account that $\Delta p dV = (p_M - p)dV$, $p_M dV$ is the mechanical energy supplied to the system from the environment. If $\omega_\Sigma = 0$, then (39) reduces to (2).

Equation (39) for the mechanothermodynamic state is *radically different* from Equation (3) for the thermodynamic state: the first permits any state of the system to be analyzed, including A -, B -, C -, D - and E -states of damage (see Table 1) since in the general case, $0 \leq \omega_\Sigma \leq \infty$ [4,46,47,52]. Hence, according to (39), exactly the growth of tribo-fatigue entropy production (32) due to the thermomechanical state of the system can cause both its damage and decomposition; thermodynamic Equation (3) does not take into account such states. But it must be admitted that the state of any system to a large extent depends, for example, whether some element present in the system will be destroyed or not. Three parameters shown in Table 2 can be used for the analysis of Equation (39) [47].

Table 2. Parameters describing the mechanothermodynamic state of different systems.

Parameter	Characteristic
$\text{div } F(\bullet) \gtrless 0$	Relative motion of physical points of matter or elements of a system (converging, diverging and other processes)
$\omega_j \gtrless 0$	Nature of irreversible damageability (hardening, softening, etc.)
$\Lambda \gtrless 1$	Direction and intensity of interaction of irreversible any-nature damages

For the definite parameter ratios presented in Table 2, Equation (39) predicts various and complex “trajectories” of entropy. A nature question that arises is: what is the difference between the mechanothermodynamic and thermodynamic processes?

An answer to this question is illustrated by Figure 1. Here the solid lines stand for the predicted behavior of the thermodynamic system, for which in (39) it is assumed that $S_{TF} = 0$ and $\Lambda_{T/TF} = 1$. Let the entropy S_T of such a system tend to some (for example, local) maximum. The behavior of the mechanothermodynamic system is shown by the dotted lines in Figure 1, assuming that $S_T \neq 0$ and $\Lambda_{T/TF} > 1$ in (39). The initial state of the both systems is assumed to be identical (points A_1, A_2). The fate of the system in the both cases is determined by the intensity of numerous irreversible internal processes caused by a diversity of reasons, but it will be fundamentally different for comparable systems.

On the other hand, the trajectory of the mechanothermodynamic state ($S_T + S_{TF}$) cannot coincide with that of the thermodynamic state (S_T) since in the first case, there appears a non-zero addition of tribo-fatigue entropy ($S_{TF} > 0$). This motivates quantitative differences in the trajectories of the systems to be compared. On the other hand, the principal difference is seen in their behavior: when the entropy of the thermodynamic system attains, for example, a local maximum (equilibrium state), the mechanothermodynamic system can have no such maximum—it will be in the non-equilibrium state. This is observed in the cases of the converging (Figure 1a) and diverging (Figure 1b) processes, as well as of hardening and softening of systems in time, in which new positive or negative information is generated. It should be noted the conservative mechanical system is in essence identical to the thermodynamic system since for it $S_{TF} = 0$. Some generalizations regarding a comparative behavior of thermodynamic and mechanothermodynamic systems are available in [47,52].

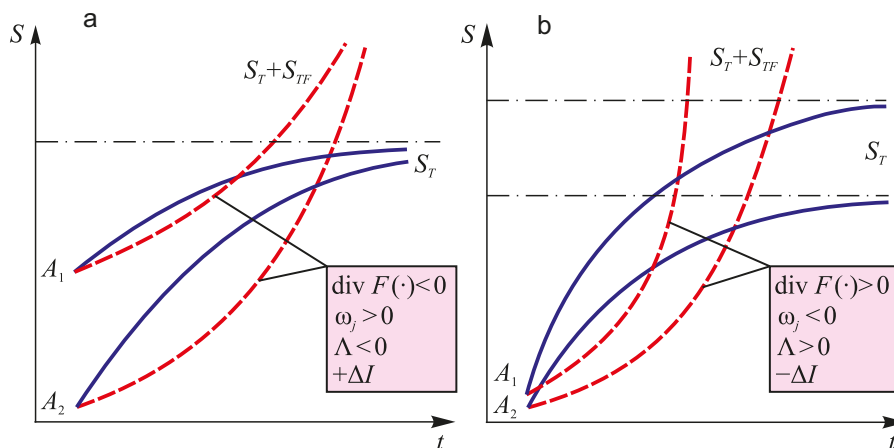


Figure 1. Evolution of the thermodynamic (S_T) or the mechanothermodynamic ($S_T + S_{TF}$) state of the system (A_1, A_2): (a) oscillatory and asymptotic converging processes; (b) oscillatory and asymptotic diverging processes.

Thus, it follows from the foregoing that the longevity (or fate) of a system in a general case is determined by the intensity of the processes of irreversible changes of entropy both thermodynamic and tribo-fatigue; production of internal mechanothermodynamic entropy is as eternal as the motion and damage.

Tribo-fatigue and mechanothermodynamics [4,52] study the *interaction between irreversible damages in the moving and deformable system*; the regularities of such interaction are due to a *field of stresses (strains) caused in it by appropriate internal force factors*. Let, for example, σ , p , T be such factors that generate appropriate fields in the MTD system: thermodynamic (T) and strain (stress σ_{ij} , pressure p). Sosnovskiy's generalized rule of interaction of damages ω_σ , ω_p , ω_T due to fatigue, friction and wear phenomena, changes in thermodynamic states is then given in the form presented by Figure 2.

$$F_\Lambda \left[(\omega_p \rightleftharpoons \omega_\sigma) \rightleftharpoons \omega_T \right] = \left[(\omega_p + \omega_\sigma) \Lambda_{\sigma \setminus p} + \omega_T \right] \Lambda_{M \setminus T} = \omega_\Sigma, \quad \Lambda \gtrless 1$$

analysis
synthesis

Figure 2. Sosnovskiy's generalized rule of interaction of damages.

In Figure 2 the Λ 's are the interaction parameters (functions) that can acquire three classes of values: $\Lambda > 1$, $\Lambda = 1$, $\Lambda < 1$ [46–52].

If $\Lambda > 1$ then a system is characterized by the destruction. If $\Lambda < 1$ then mainly hardening processes develop in the system. When $\Lambda = 1$ the system is stable.

According to Figure 2, the analysis of the system (the integral ω_Σ thought as multiple ω_σ , ω_p , ω_T) and its synthesis (many thought as the integer by the interaction functions of Λ) are possible. In this case, the analysis and the synthesis have a particular quantitative expression. This is the peculiarity and important advantage of the law in Figure 2. Its second important peculiarity is the following. Here we are talking not about external forces, as, for example, in *Newton's laws*, but about irreversible damages of the system due to fields of internal forces. And, at last, the main thing is: *Mechanothermodynamics should study and investigate not the mutual influence of factors but the interaction of phenomena*. Thereby, mechanics and thermodynamics are marked by a transition to the phenomenon analysis—in addition to the traditional factor analysis.

7. Discussion of the Research Results

Let us analyze further the generalized nature of the notion of MTD entropy in relation to the two practically important (specific) types of damage of solids: (1) under friction and wear and (2) under cyclic deformation in an environment with temperature T . Complex damage ω_Σ , contained in (31) and (32) is presented in [4,52] for the simple loading conditions as follows:

$$\omega_\Sigma = [(\omega_\sigma + \omega_\tau)\Lambda_{\sigma\backslash\tau} + \omega_T]\Lambda_{T\backslash M} = \left[\left(\frac{U_\sigma}{U_0} + \frac{U_\tau}{U_0}\right)\Lambda_{\sigma\backslash\tau} + \frac{U_T}{U_0}\right]\Lambda_{T\backslash M} = \left[\left(\frac{a_\sigma\sigma^2}{U_0} + \frac{a_\tau\tau_w^2}{U_0}\right)\Lambda_{\sigma\backslash\tau} + \frac{a_T T}{U_0}\right]\Lambda_{T\backslash M} \quad (40)$$

where ω_σ , ω_τ , ω_T are the damages under cyclic deformation (at the action of the normal stress σ), sliding friction with wear (at the influence of frictional τ_w stress) and thermal action (temperature T) respectively.

In the particular case of friction with wear Equation (40) (assuming $\omega_\sigma = 0$ and $U_\sigma = 0$) attains the form:

$$\omega_\Sigma = \omega_{\tau\backslash T} = \left(\frac{U_\tau}{U_0} + \frac{U_T}{U_0}\right)\Lambda_{T\backslash M} = \left(\frac{a_\tau\tau_w^2}{U_0} + \frac{a_T T}{U_0}\right)\Lambda_{T\backslash M} \quad (41)$$

With regard to Equation (40) the Sosnovskiy-Sherbakov rule of the interaction between thermodynamic and tribo-fatigue entropies has the form (Figure 3) which is similar to the rule in Figure 2.

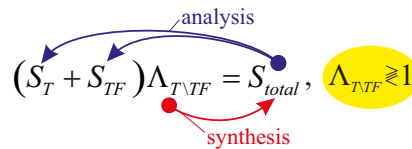


Figure 3. Sosnovskiy-Sherbakov rule of the interaction between thermodynamic and tribo-fatigue entropies.

Obviously the limiting state of the friction pair is reached under condition:

$$\omega_{\text{lim}} = \omega_{\tau_f\backslash T} = 1 \quad (42)$$

i.e.:

$$\left(\frac{a_\tau\tau_f^2}{U_0} + \frac{a_T T}{U_0}\right)\Lambda_{T\backslash M} = 1 \quad (43)$$

when τ_w reaches the value of sliding fatigue limit τ_f (at the given temperature T).

From (42) the following equation we can be easily obtained:

$$\lg \tau_f = \frac{1}{2} \lg C_T \quad (44)$$

$$C_T = \frac{U_0/\Lambda_{T\backslash M} - a_T T}{a_\tau} \quad (45)$$

According to which the dependence $\tau_f(C_T)$ in double logarithmic coordinates is the straight line with the slope equal to 1/2.

Damage analysis for the conditions of mechanical fatigue is similar. Thus from (40) and (41) for $\tau_w = 0$ and therefore $U_\tau = 0$ it follows that:

$$\omega_\Sigma = \omega_{\sigma\backslash T} = \left(\frac{U_\sigma}{U_0} + \frac{U_T}{U_0}\right)\Lambda_{T\backslash M} = \left(\frac{a_\sigma\sigma^2}{U_0} + \frac{a_T T}{U_0}\right)\Lambda_{T\backslash M} \quad (46)$$

that gives the condition of a limiting state (fatigue fracture):

$$\omega_{\text{lim}} = \omega_{\sigma_{-1}\backslash T} = 1 \quad (47)$$

i.e.:

$$\left(\frac{a_{\sigma} \sigma_{-1}^2}{U_0} + \frac{a_T T}{U_0} \right) \Lambda_{T \setminus M} = 1 \quad (48)$$

where σ_{-1} is the fatigue limit for cyclic loading and hence

$$\lg \sigma_{-1} = \frac{1}{2} \lg C_T \quad (49)$$

$$C_T = \frac{U_0 / \Lambda_{T \setminus M} - a_T T}{a_{\sigma}} \quad (50)$$

which is similar to (44) and (45).

The analysis of more complicated case of complex wear-fatigue damage can be made according to (40) when sliding friction ($\tau_w > 0$) and a cyclic deformation ($\sigma > 0$) are realized simultaneously in tribo-fatigue system:

$$\omega_{\Sigma} = \omega_{\sigma \setminus \tau} = \left(\frac{a_{\sigma} \sigma^2(T)}{U_0} + \frac{a_{\tau} \tau_w^2(T)}{U_0} \right) \Lambda_{\sigma \setminus \tau} \quad (51)$$

from which the limiting states equation is obtained similarly to the previous one:

$$\left(\frac{a_{\sigma} \sigma^2(T)}{U_0} + \frac{a_{\tau} \tau_w^2(T)}{U_0} \right) \Lambda_{\sigma \setminus \tau} = 1 \quad (52)$$

In the case of (52) analysis of limiting states is conducted by both criteria—either wear or fatigue fracture taking into account the mutual influence (interaction) both damaging phenomena [5], so if one studies the direct effect (the effect of friction processes and wear on the resistance to mechanical fatigue) [4] then it follows from (51) (for $\sigma = \sigma_{-1\tau}$) that:

$$\lg \sigma_{-1\tau} = \frac{1}{2} \lg C_T \quad (53)$$

$$C_T = \frac{U_0 / \Lambda_{\sigma \setminus \tau} - a_{\tau} \tau_w^2}{a_{\sigma}} \quad (54)$$

On the other hand if one studies the back effect (the effect of fatigue damage processes on the change of wear resistance) [4] then it similarly follows from (51) (for $\tau_w = \tau_{f\sigma}$) that:

$$\lg \tau_{f\sigma} = \frac{1}{2} \lg C_T \quad (55)$$

$$C_T = \frac{U_0 / \Lambda_{\sigma \setminus \tau} - a_{\sigma} \sigma^2}{a_{\tau}} \quad (56)$$

Generalizing the above stated analogous dependences can be obtained for the analysis of limiting states at static deformation (tension, bending, torsion).

8. Experimental Verification

Consider the cases of realization of processes of sliding friction with wear under sliding and mechano-sliding fatigue. The initial data for the analysis are shown in Table 3. The experimental verification of Equations (44) and (55) is made for three series of tests with amplitude cyclic bending load σ_a equal to 0.160 and 256 MPa.

According to (55) dependence of sliding fatigue limit $\tau_{f\sigma}$ on parameter C_T in the double logarithmic coordinates is a straight line with a slope of 0.5 (see Figure 4).

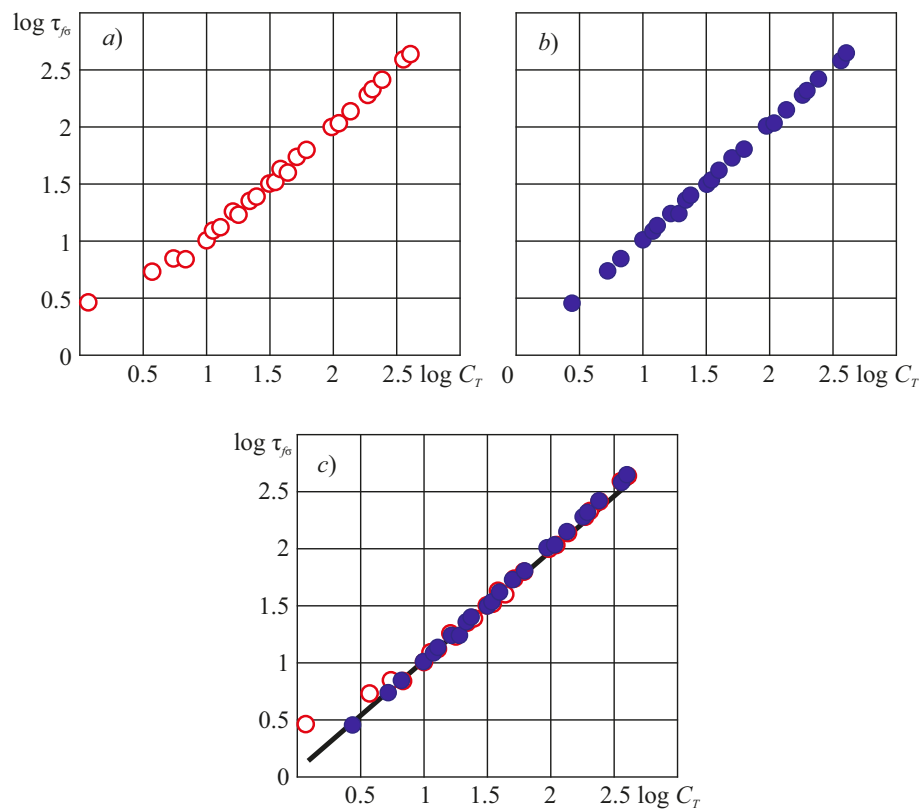


Figure 4. Dependencies of $\tau_{f\sigma}(C_T)$ for Steel 45 (a); silumin (b) and their combined representation (c).

This regularity is in good agreement with the experimental data: correlation coefficients here are $r > 0.95$.

Table 3. Initial test data.

Parameter	Parameter Value	
	Steel 45	Silumin
U_0 , kJ/mole	419	222
T_{melt} , K	1773	830
σ_{-1} , MPa	280	42
τ_{-1} , MPa	170	30
a_T	0.236	0.267
a_σ	0.005	0.126
a_τ	0.014	0.247

It was shown experimentally that the above approach works well for cyclic fatigue (see Figure 5) and static tension (see Figure 6) of various materials like steels, titanium and its alloys, aluminum alloys and polymers. It turns out that dependences $\sigma_{-1T}(C_T)$ and $\sigma_{uT}(C_T)$ lie on a single straight line (see Figure 7).

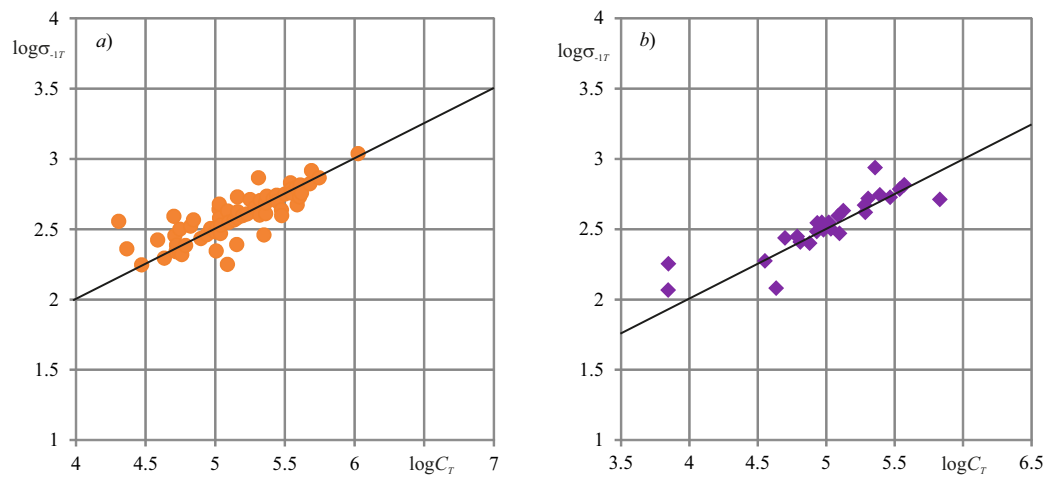


Figure 5. Dependence of $\sigma_{-1T}(C_T)$ for steels (a); titanium and its alloys (b).

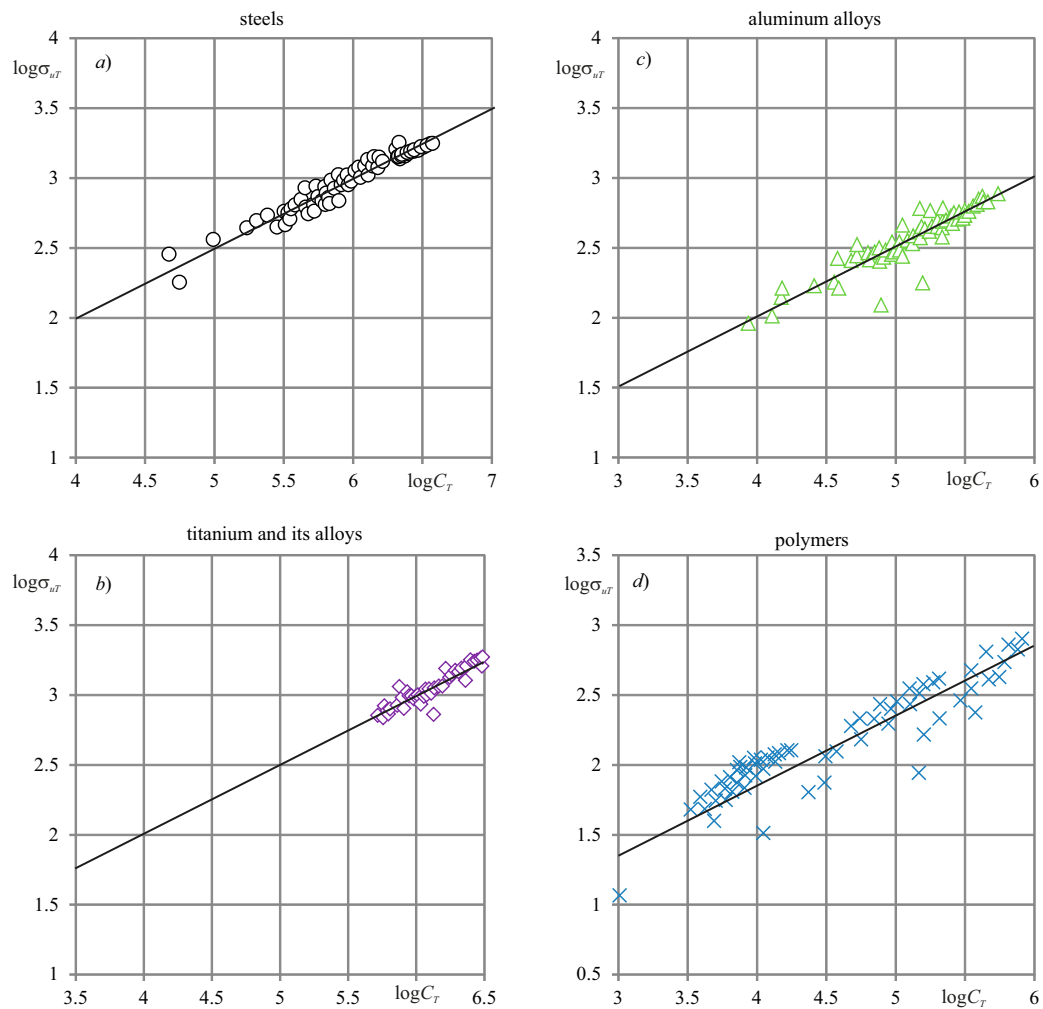


Figure 6. Dependence of $\sigma_{uT}(C_T)$ for steels (a); titanium and its alloys (b); aluminum alloys (c) and polymers (d).

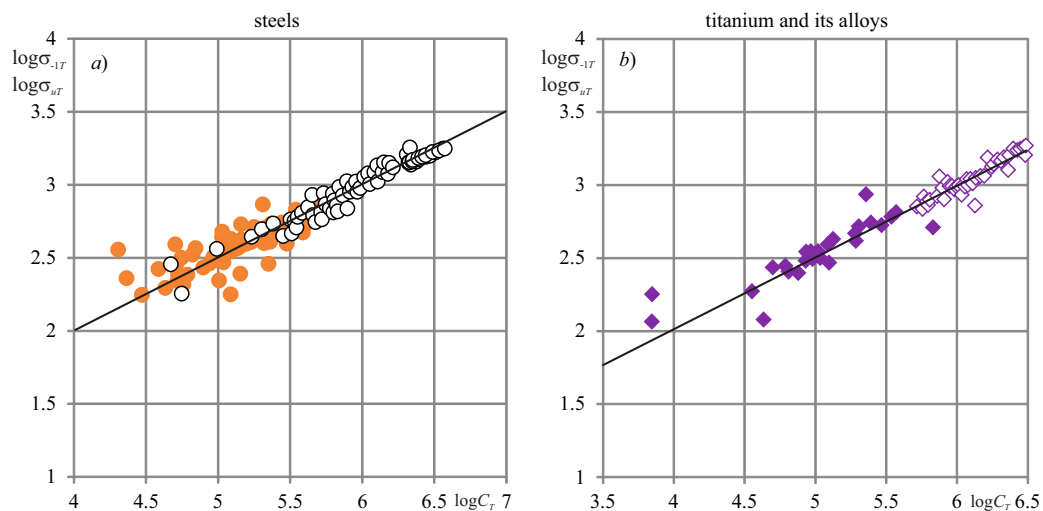


Figure 7. Combined dependencies of $\sigma_{-1T}(C_T)$ and $\sigma_{uT}(C_T)$ for steels (a); titanium and its alloys (b).

Thus the comparative analysis of the results of mechanical and mathematical modeling and the experimental results shows that the developed theory works satisfactorily in conditions of both static loading and fatigue fracture and under surface contact damage of materials of different nature for a wide range of possible changes of temperature ($T_\Sigma \leq T_S$) and stresses ($\sigma \leq \sigma_b$).

The above results were obtained for steels, polymers, titanium and its alloys and aluminum alloys. It seems appropriate to apply the rules in Figures 2 and 3 and Equations (39), (40), (52) to specific polymers, composites and natural materials like seashells for example, which are characterized by high strength and wear resistance [55–59]. Such an application could be realized based on the model of many contact interactions between the elements of the multibody tribo-fatigue system described in [5,60].

9. Conclusions

Two main conclusions follow from the above stated.

- (1) Thermodynamic thermomechanical entropies are specific characteristics of energy dissipation in the environment. In contrast tribo-fatigue entropy is the specific characteristic of absorption of energy in the moving and deformable solid. Mechano-thermodynamic entropy is a generalized characteristic of both energy absorption in a body and dissipation in an environment. It is generated by interacting forces of different nature (mechanical, thermodynamic, electrochemical, etc.).
- (2) The processes of friction, wear and fatigue in a general case generate mechano-thermodynamic entropy, however irreversible damageability of moving and deformable solids caused by loads of different nature (mechanical, thermal, electrochemical, etc.) is most appropriately characterized by (internal) mechano-thermodynamic entropy.

Experimental verification of complex damage functions was made for the processes of sliding friction with wear under sliding and mechano-sliding fatigue. The proposed function is in good agreement with the experimental data, with correlation coefficients of more than 0.95.

Author Contributions: Leonid A. Sosnovskiy and Sergei S. Sherbakov conceived and designed the mathematical formulation of the problem, obtained its solution and analyzed graphical results. Both authors have read and approved the final manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

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