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GEOMETRIZATION FOR A QUANTUM-MECHANICAL PROBLEM OF THE SPIN 1 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT IN THE COULOMB FIELD

In [1] the quantum-mechanical problem for a spin 1 particles with anomalous magnetic in the presence of external Coulomb field was studied and the system of radial equations was obtained. It was shown that the system cannot be solved completely even in the case of ordinary particle without additional electromagnetic moments. To simplify the problem, restriction to non-relativistic equations was performed and the system of two 4-th order ordinary differential equations was found out. Its Frobenius solutions were constructed and transcendental solutions and corresponding energy spectra were found. However, the problem cannot be considered as studied exhaustively.

In this study the problem of spin 1 particle with anomalous magnetic moment in the external Coulomb field are considered in non-relativistic approximation using Kosambi–Cartan–Chern geometrical approach (KCC-theory) [2]. In this approach, one considers a system of second order differential equations

$$\dot{y}^i(r) + 2Q^i(r,x,y) = 0, \tag{1}$$

which corresponds to the the Euler-Lagrange equations for some dynamical system with Lagrangian L. In (1), the symbol x^i designates coordinates, their derivatives in argument r are $y^i = dx^i/dr = \dot{x}^i$, and the quantities Q_i are determined through some Lagrangian L as follows

$$Q^i = \frac{1}{4} g^{il} \left(\frac{\partial^2 L}{\partial x^k \partial y^l} y^k - \frac{\partial L}{\partial x^l} + \frac{\partial^2 L}{\partial y^l \partial r} \right), \qquad g_{ij} = \frac{1}{2} \frac{\partial^2 L}{\partial y^i \partial y^j}. \tag{2}$$

The first and second invariants, $\varepsilon^i(r,x,y)$ and P^i_j are introduced by the definitions

$$\varepsilon^i = \frac{\partial Q^i}{\partial y^j} y^j - 2Q^i, \qquad P^i_j = 2\frac{\partial Q^i}{\partial x^j} + 2Q^s \frac{\partial^2 Q^i}{\partial y^j \partial y^s} - \frac{\partial^2 Q^i}{\partial y^j \partial x^s} y^s - \frac{\partial Q^i}{\partial y^s}\frac{\partial Q^s}{\partial y^j} - \frac{\partial^2 Q^i}{\partial y^j \partial r}. \tag{3}$$

The second invariant P^i_j relates to Jacobi stability of dynamical system. A pencil of geodesic curves from the some point r_0 converges (or diverges) if the real parts of all eigenvalues of the invariant P^i_j are negative (or positive) ones.

We start with the knows radial system [1] of two second-order differential equations for two radial functions, which arises when considering a non-relativistic spin 1 particle with anomalous magnetic moment in the external Coulomb field. In the explicit form the system can be presented as

$$\begin{gathered}
\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + 2m\frac{\alpha + Er}{r} - \frac{2\nu^2}{r^2} \right)\Psi_1(r) - \nu \frac{2r+\Gamma}{r^3}\Psi_2(r) = 0, \\
\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + 2m\frac{\alpha + Er}{r} - \frac{2\nu^2}{r^2} - \frac{2}{r^2} - \frac{4\Gamma}{r^3} - \frac{\Gamma^2}{r^4} \right)\Psi_2(r) - 2\nu \frac{2r+\Gamma}{r^3}\Psi_1(r) = 0.
\end{gathered} \tag{4}$$

We follow the case of bound states, so assuming $\nu = \sqrt{j(j+1)/2},\ j = 1,2,3,...$. Let apply the notations $x^i = \Psi_i(r),\ y^i = (d/dr)\Psi_i(r) = \dot{\Psi}_i(r)$. Then comparing equations (4) and (1), one finds the relevant quantities Q^i :

$$\begin{gathered}
Q^1(r,\Psi_i,\dot{\Psi}_i) = \frac{1}{2}\left(\frac{2}{r}\dot{\Psi}_1 + (2m\frac{\alpha+Er}{r} - \frac{2\nu^2}{r^2})\Psi_1 - \nu\frac{2r+\Gamma}{r^3}\Psi_2 \right), \\
Q^2(r,\Psi_i,\dot{\Psi}_i) = \frac{1}{2}\left(\frac{2}{r}\dot{\Psi}_2 + (2m\frac{\alpha+Er}{r} - \frac{2\nu^2}{r^2} - \frac{2}{r^2} - \frac{4\Gamma}{r^3} - \frac{\Gamma^2}{r^4})\Psi_2 - 2\nu\frac{2r+\Gamma}{r^3}\Psi_1 \right).
\end{gathered} \tag{5}$$

By direct calculation according the formulas (3), we calculate both invariants:

$$\varepsilon^1 = \frac{\nu\Psi_2(\Gamma+2r)}{r^3} + \Psi_1(-2mE + \frac{2\nu^2}{r^2} - \frac{2m\alpha}{r}) - \frac{\dot{\Psi}_1}{r},$$
$$\varepsilon^2 = \frac{2\nu\Psi_1(\Gamma+2r)}{r^3} + \Psi_2(-2mE + \frac{\Gamma^2}{r^4} + \frac{4\Gamma}{r^3} + \frac{2(\nu^2+1)}{r^2} - \frac{2m\alpha}{r}) - \frac{\dot{\Psi}_2}{r}; \quad (6)$$

$$P^i_j = \begin{vmatrix} 2m\frac{\alpha+Er}{r} - \frac{2\nu^2}{r^2} & -\frac{(2r+\Gamma)\nu}{r^3} \\ -\frac{2(2r+\Gamma)\nu}{r^3} & -\frac{\Gamma^2}{r^4} - \frac{4\Gamma}{r^3} + 2m\frac{\alpha+Er}{r} - \frac{2(\nu^2+1)}{r^2} \end{vmatrix}. \quad (7)$$

The eigenvalues Λ_1, Λ_2 of the second invariant P^i_j are given by the formulas

$$\Lambda_{1,2} = 2mE + \frac{1-2\nu^2}{r^2} - \left(\frac{(\Gamma+2r)^2}{2r^4} \pm \frac{\sqrt{\left(\Gamma^2+2r^2+4\Gamma r\right)^2 + 8\nu^2 r^2(\Gamma+2r)^2}}{2r^4} \right) + \frac{2m\alpha}{r}. \quad (8)$$

Typical behavior of eigenvalues at different j is presented in Fig. 1. Let us specify their behavior near the singular points $r=0$, $r=\infty$, and $r=-\Gamma/2$:

$$r \to 0, \Lambda_1 \to \frac{2m\alpha}{r} > 0, \Lambda_2 \to -\frac{\Gamma^2}{r^4} < 0; \qquad r \to \infty, \Lambda_1, \Lambda_2 \to 2mE < 0; \quad (9)$$

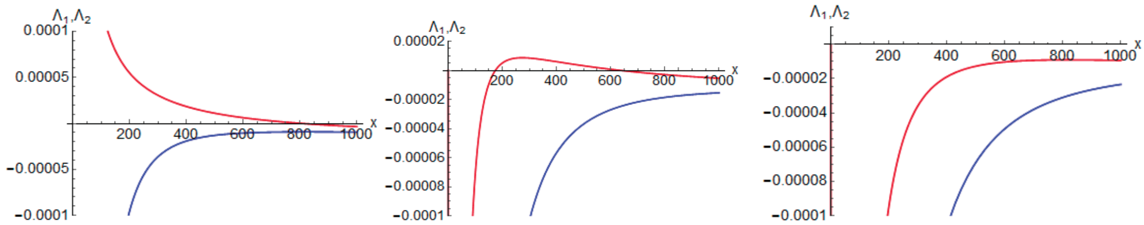

Figure 1. The dependencies of eigenvalues Λ_1 (red) and Λ_1 (blue) on radial coordinate ($x = mr$) at different j : (a) $j=1$, (b) $j=2$, (c) $j=3$. We used the following parameters: $\Gamma m = 1$, $E/m = -0.000009$.

$$r \to -\frac{\Gamma}{2}, \Lambda_1 \to 2mE - \frac{8\nu^2}{\Gamma^2} - \frac{4m\alpha}{\Gamma} < 0, \quad \Lambda_2 \to 2mE - \frac{8\left(\nu^2-1\right)}{\Gamma^2} - \frac{4m\alpha}{\Gamma} < 0. \quad (10)$$

Behavior of the real parts of eigenvalues near the singular points $r = 0, \infty, -\Gamma/2$ correlates with the properties of solutions near these points for quantum mechanical bound states.

The next step is to construct a Lagrangian function L for the phase space $\dot{\Psi}_i, \Psi_i$, defined by (5). We will search for the function in the form

$$L = g_{ij}(r) y^i y^j + b_j(r,x) y^j, \qquad b_j(r,x) = h_{ij}(r)\, x^i. \quad (11)$$

Substituting (11) into (2) and assuming that the tensor g_{ij} is diagonal ($g_{12} = g_{21} = 0$), we derive

$$Q^1 = \frac{1}{4g_{11}} \left(2\dot{g}_{11} y^1 + \frac{\partial b_1}{\partial r} + (\frac{\partial b_1}{\partial x^2} - \frac{\partial b_2}{\partial x^1}) y^2 \right), \quad Q^2 = \frac{1}{4g_{22}} \left(2\dot{g}_{22} y^2 + \frac{\partial b_2}{\partial r} + (\frac{\partial b_2}{\partial x^1} - \frac{\partial b_1}{\partial x^2}) y^2 \right). \quad (12)$$

Equating the terms from (5) to the corresponding terms from (12), we obtain the system of equations with respect to $g_{ij}(r)$ and $b_j(r,x)$:

$$\frac{\partial b_1}{\partial x^2} - \frac{\partial b_2}{\partial x^1} = 0, \frac{\dot{g}_{11}}{2g_{11}} = \frac{1}{r}, \frac{\dot{g}_{22}}{2g_{22}} = \frac{1}{r}, \quad \frac{1}{4g_1}\frac{\partial b_1}{\partial r} = \frac{x^1\left(r(2m\alpha + 2mEr) - 2\nu^2\right)}{2r^2} - \frac{\nu x^2(\Gamma+2r)}{2r^3},$$

$$\frac{1}{4g_2}\frac{\partial b_2}{\partial r} = -\frac{\nu x^1(\Gamma+2r)}{r^3} - \frac{x^2\left(\Gamma^2 + r^2\left(2\nu^2 - 2mr(Er+\alpha)+2\right)+4\Gamma r\right)}{2r^4}.$$

Its solution is given by the formulas:

$$g_{11} = 2C_1 r^2, g_{22} = C_1 r^2,$$

$$b_1 = B_1(x^1, x^2) - 4C_1\left\{rx^1[2\nu^2 - \frac{2mEr^2}{3} - mr\alpha] + \nu x^2(\Gamma \ln r + 2r)\right\},$$

$$b_2 = B_2(x^1, x^2) - 4C_1\left\{x^2[-\frac{\Gamma^2}{2r} + r\left(\nu^2+1\right) + -\frac{mEr^3}{3} - \frac{m\alpha r^2}{2} + 2\Gamma \ln r\ ] + \nu x^1(\Gamma \ln r + 2r)\right\},$$

where C_1 is the arbitrary constant. Functions $B_1(x^1, x^2)$ and $B_2(x^1, x^2)$ obey the restriction

$$\frac{\partial B_1(x^1,x^2)}{\partial x^2} - \frac{\partial B_2(x^1,x^2)}{\partial x^1} = 0. \quad (13)$$

So, 2-dimensional vector field B_1, B_2 can be presented as a gradient of a scalar function

$$B_1(x^1,x^2) = \frac{\partial}{\partial x^1}\varphi(x^1,x^2), \quad B_2(x^1,x^2) = \frac{\partial}{\partial x^2}\varphi(x^1,x^2), \quad B_i = \mathrm{grad}\,\varphi. \quad (14)$$

Therefore, there exist some freedom in choosing the Lagrangian (the constant C_1 may be taken as 1):

$$L = 2r^2(y^1)^2 + r^2(y^2)^2 + \left(\frac{2\Gamma^2}{r} + \frac{4}{3}mEr^3 + 2m\alpha r^2 - 4\left(\nu^2+1\right)r - 8\Gamma \ln r\right)x^2y^2 -$$
$$-4\nu(\Gamma \ln r + 2r)x^1y^2 - 4\nu(\Gamma \ln r + 2r)x^2y^1 - r(8\nu^2 - \frac{8mEr^2}{3} - 4m\alpha r)x^1y^1 + y^1\frac{\partial\varphi}{\partial x^1} + y^2\frac{\partial\varphi}{\partial x^2}. \quad (15)$$

Concluding, we have used a geometrical KCC-based method to study the quantum-mechanical problem of spin 1 particle with anomalous magnetic moment in the external Coulomb field. The KCC-invariants were calculated. It has been shown that the different branches of the solution converges near the singular points $\infty, -\Gamma/2$, and may converge either diverge near the singular points $r = 0$. This correlates with behavior of solutions near these points for quantum mechanical bound states. The Lagrangian corresponding to the geometrical problem has been found, it is demonstrated to have the arbitrariness up to some special term, which may be considered as specific gauge freedom.

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НЕРАЗРУШАЮЩИЙ КОНТРОЛЬ И ДИАГНОСТИКА ДЕФЕКТОВ НА ПОВЕРХНОСТИ МЕТАЛЛОВ

Введение. Оптико-акустические (ОА) источники ультразвуковых (УЗ) волн имеют ряд преимуществ перед традиционными (пьезоэлектрическими и электромагнитно-акустическими), включая отсутствие контакта со средой, возможность легкого изменения геометрических параметров акустической антенны, диагностики объектов, движущихся с любой скоростью [1–5]. Для возбуждения коротких акустических импульсов перспективно применение ОА методов при импульсном лазерном воздействии [2, 3]. Среди режимов генерации поверхностных акустических волн (ПАВ) предпочтительным является именно термоупругий режим, реализуемый в отсутствие абляции материала и минимальном шумовом фоне, создаваемым продольной и сдвиговой составляющими ПАВ [1–3]. Возбуждение ПАВ Рэлея при поглощении лазерных импульсов наносекундной длительности в материале из плавленого кварца исследовано в работе [4]. Рассмотрены особенности диагностики неоднородностей в виде полосок из золота на поверхности плавленого кварца.