Study of polarized effects in Compton scattering

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Polarization effects in Compton scattering are considered. Differential as well as total cross sections of photons by polarized leptons scattering are calculated and discussed. The formulae are obtained in the Lorentz-invariant form without any approximation. They can be used at any kinematic conditions. The polarized asymmetry of processes unpolarized photons scattering as well as polarized ones are obtained.

Introduction

The Standard Model provides the possibility for describing all present experimental data. Nevertheless the linear collider experiments [1]-[2] that run at the center of mass energies up to 1 TeV [3], may finally reveal the deviations of electroweak interaction from the Standard Model predictions. At linear lepton colliders $e\gamma$ and $\gamma\gamma$ processes will be investigated at the same energies and luminosity as in e^+e^- experiments. The intense γ-beams are suggested to be obtained by Compton scattering of laser light focused on the electron beams of e^+e^- accelerators [3].

Processes of γe-collisions

It is expected that some time in the future, International Linear Collider (ILC) will conduct experiments using both a longitudinally polarized electron and positron beams.

Linear colliders will provide possibility to investigate photon collisions at energies and luminosities close to those in e^+e^- collisions. There is a better signal/background ratio in comparison with hadron colliders. The production cross sections at photon colliders are usually larger than hadron colliders. The advance of experimental tools such as highly polarized photon beams,

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polarized targets and more powerful accelerators, makes it feasible to study polarized Compton scattering in detail.

At present it is possible to form beams of polarized initial particles and to determine the polarization of scattered particles. As it is well known, by evaluating the left-right asymmetry, for example, Standard Model parameters can be measured with even greater accuracy than using unpolarized beams [4]. Furthermore, one-loop effects are expected to be observable. In order to achieve this desired accuracy, a precise measurement of the degree of polarization of the incoming electron beam is requires. The errors in the asymmetry and in the polarization P of the electron beam are related as follows, neglecting statistical errors:

$$\frac{\Delta A_{LR}}{A_{LR}} = \frac{\Delta P}{P}, \tag{1}$$

where

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}. \tag{2}$$

Here σ_L and σ_R denote the cross sections for the process $e^+e^- \to f\bar{f}$ (f – is a fermion) with left- and right-hand polarized incoming electrons. P can be measured through Compton scattering. To this purpose the longitudinally polarized electron beam is scattered by left and right circular polarized photons and the energy of the scattered electron is measured. From this, one determines the asymmetry $A_{LR}^{\mathrm{exp}}(e^-\gamma \to e^-\gamma)$ as a function of energy. A_{LR} denotes $A_{LR}(e^-\gamma \to e^-\gamma)$ and L and R stands for left and right circular polarized photons. The relation between A_{LR}^{exp} and A_{LR}^{theor}, where A_{LR}^{theor} is the value for the asymmetry when the electron beam is 100% polarized, is

$$A_{LR}^{\mathrm{exp}} = P A_{LR}^{\mathrm{theor}}. \tag{3}$$

A_{LR}^{theor} can be theoretically evaluated and thus P can be determined.

Therefore necessity appeared to calculate basic characteristic of polarized particle interactions such as cross sections, decay properties, asymmetries etc. The purpose of this paper is to obtain and analyze the cross section and asymmetry of the polarized photon-electron scattering. The calculations are carried out in Lorentz-invariant form without any approximations.

The experimental setup of ILC leads to important consequence for the calculation. Although the high energy of incoming electron beam at ILC, the electron mass cannot be neglected, since the energy of the incoming photon beam is in the electronvolts range. This is explain why the electron scattering angle is close to zero.

The process is defined in Fig.1, p_1, n_1 and p_2, n_2 are the momentum four-vectors and polarization four-vectors of the incoming and outgoing electron. k_1, e_1 and k_2, e_2 are the momentum four-vectors and polarization four-vectors of the incoming and outgoing photon.

$$e^-(p_1, n_1) + \gamma(k_1, \mathrm{e}_1) \to e^-(p_2, n_2) + \gamma(k_2, \mathrm{e}_2) \tag{4}$$

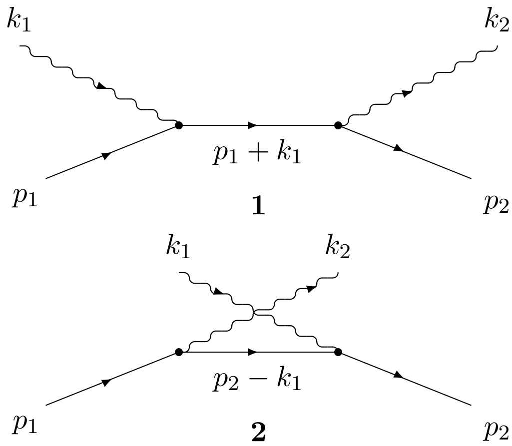

FIG. 1. The lowest-order Feynman diagrams corresponding to photon-electron interaction

The expression of the Compton scattering differential cross section can be written as follows:

$$d\sigma = \alpha^2 \frac{{k_2^0}^2 d\Omega}{(2p_1k_1)^2} |M|^2. \tag{5}$$

Here $\alpha = e^2/4\pi$ and k_2^0 is the energy of scattered photon: $k_2 = (k_2^0, \vec{k}_2)$; $p_1k_1 = p_1^0k_1^0 - \vec{p}_1\vec{k}_1$; and the total cross section considering process

$$\sigma = \frac{e^2}{2(p_1k_1)} \int |M|^2 d\Gamma. \tag{6}$$

The phase volume is defined as

$$d\Gamma = \frac{d^3p_2}{(2\pi)^3 2p_2^0} \, \frac{d^3k_2}{(2\pi)^3 2k_2^0} (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2).$$

In the lowest order there are two diagrams (see Fig.1). The amplitude M is given by the following expressions

$$\begin{aligned} M_1 &= -ie^2 \cdot \bar{u}(p_2) \, \hat{\mathrm{e}}_2 \frac{1}{\hat{p}_1 + \hat{k}_1 - m} \, \hat{\mathrm{e}}_1 \, u(p_1) = -ie^2 \cdot \bar{u}(p_2) \, \hat{\mathrm{e}}_2 \frac{\hat{p}_1 + \hat{k}_1 + m}{(p_1 + k_1)^2 - m^2} \, \hat{\mathrm{e}}_1 \, u(p_1) \,, \\ M_2 &= -ie^2 \cdot \bar{u}(p_2) \, \hat{\mathrm{e}}_1 \frac{1}{\hat{p}_2 - \hat{k}_1 - m} \, \hat{\mathrm{e}}_2 \, u(p_1) = -ie^2 \cdot \bar{u}(p_2) \, \hat{\mathrm{e}}_1 \frac{\hat{p}_2 - \hat{k}_1 + m}{(p_2 - k_1)^2 - m^2} \, \hat{\mathrm{e}}_2 \, u(p_1) \,, \end{aligned} \tag{7}$$

where amplitude $M_{1(2)}$ corresponds to the first (second) Feynman diagram of the photon-electron interaction.

Using standard QED equation the final amplitude formulae can be written as

$$M = ie^2 \left\{ \left[\frac{(p_2\mathrm{e}_1)}{(p_2k_1)} - \frac{(p_1\mathrm{e}_1)}{(p_1k_1)} \right] \cdot \bar{u}(p_2) \, \hat{\mathrm{e}}_2 \, u(p_1) - \frac{1}{2} \cdot \bar{u}(p_2) \left[\frac{\hat{\mathrm{e}}_2 \hat{k}_1 \hat{\mathrm{e}}_1}{(p_1k_1)} + \frac{\hat{\mathrm{e}}_1 \hat{k}_1 \hat{\mathrm{e}}_2}{(p_2k_1)} \right] u(p_1) \right\}. \tag{8}$$

This amplitude is gauge invariant.

To calculate and analyse amplitude in case of polarized interaction particles the method proposed in refs. [5], [6] was used. We present calculated amplitude in following form

$$\bar{u}_f Q u_i = \frac{(Q\ u_i\bar{u}_i\ \mathcal{P}\ u_f\bar{u}_f)}{\sqrt{(\mathcal{P}\ u_i\bar{u}_i)(\mathcal{P}\ u_f\bar{u}_f)}}\ , \tag{9}$$

where

$$\mathcal{P} = \frac{1}{2}(1 \pm \gamma^5)\hat{q}\ , \tag{10}$$

q is an arbitrary massless vector.

Polarization vectors of initial (final) photons can be written as

$$\mathrm{e}^{\pm}_{\mu}(k_1) = \frac{\left[(1\pm\gamma^5)\gamma_\mu \hat{k}_1\hat{q}\hat{k}_2\right]}{8\sqrt{(k_1k_2)(k_1q)(k_2q)}} = \frac{\left[(1\mp\gamma^5)\gamma_\mu \hat{k}_2\hat{q}\hat{k}_1\right]}{8\sqrt{(k_1k_2)(k_1q)(k_2q)}} = \mathrm{e}^{\mp}_{\mu}(k_2)\ ; \tag{11}$$

$$\hat{\mathrm{e}}^{\pm}(k_1) = \frac{(1\pm\gamma^5)\hat{k}_2\hat{q}\hat{k}_1 + (1\mp\gamma^5)\hat{k}_1\hat{q}\hat{k}_2}{4\sqrt{(k_1k_2)(k_1q)(k_2q)}} = \hat{\mathrm{e}}^{\mp}(k_2)\ . \tag{12}$$

As a result squared module of obtained matrix element

$$|M(\tau_1=\pm1,\tau_2=\pm1;\lambda_1,\lambda_2)|^2 = e^4\cdot\frac{m^2(k_1k_2)^2}{2(p_1k_1)^2(p_1k_2)^2}\cdot$$
$$\cdot[(p_1p_2) \pm \lambda_1 m(p_2n_1) \mp \lambda_2 m(p_1n_2) - \lambda_1\lambda_2 m^2(n_1n_2)]\ ;$$

$$|M(\tau_1=\pm1,\tau_2=\mp1;\lambda_1,\lambda_2)|^2 = e^4\cdot\frac{2(p_1k_1)(p_1k_2)-m^2(k_1k_2)}{2(p_1k_1)^2(p_1k_2)^2}\cdot \tag{13}$$
$$\cdot\Big\{\ [(p_1k_1)^2+(p_1k_2)^2-m^2(k_1k_2)] \mp \lambda_1 m(k_1k_2)\cdot[(n_1k_1)+(n_1k_2)]\mp$$
$$\lambda_2 m(k_1k_2)\cdot[(n_2k_1)+(n_2k_2)] - \lambda_1\lambda_2\cdot\Big\langle\ [2(p_1k_1)(p_1k_2)-m^2(k_1k_2)]\ (n_1n_2)+$$
$$+2(p_1k_1)(n_1k_2)(n_2k_1) - 2(p_1k_2)(n_1k_1)(n_2k_2)\ \Big\rangle\Big\}\ ,$$

corresponds to scattering of polarized photons and leptons. Here τ_1, τ_2 – circular polarizations of initial and final photons and λ_1 and λ_2 – helicities of initial and final leptons correspondingly.

Total cross section of $e\gamma$-interaction for different spin configuration are expressed by a set of formulae:

$$\sigma(\pm,\pm;\pm,\pm) = r_0^2\ \frac{\pi}{2}\ \frac{1}{\gamma^5}\Bigg\{\gamma\left[\frac{2\gamma-1}{2} - \frac{5\gamma+2}{2(2\gamma+1)^2} + \frac{2}{\gamma+2}\right]$$
$$+\left(\gamma^2+\frac{2\gamma+1}{4}\right)\ln(2\gamma+1) - \gamma\left(\gamma+2+\frac{1}{\gamma+2}\right)\sqrt{\frac{\gamma}{\gamma+2}}\cdot\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\}\ , \tag{14}$$

$$\sigma(\pm,\pm;\pm,\mp) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\Bigg\{-\gamma\left[\frac{5\gamma-3}{2}+\frac{2}{\gamma+2}\right]-\frac{2\gamma+1}{4}\ln(2\gamma+1)$$
$$+\gamma\left(\gamma+2+\frac{1}{\gamma+2}\right)\sqrt{\frac{\gamma}{\gamma+2}}\cdot\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\}, \tag{15}$$

$$\sigma(\pm,\pm;\mp,\pm) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\Bigg\{\gamma\left[-3\gamma+\frac{\gamma+1}{2(2\gamma+1)}+\frac{\gamma}{\gamma+2}\right]$$
$$-\left[\gamma^2(6\gamma+5)+\frac{1}{4}\right]\ln(2\gamma+1)+\gamma\left(6\gamma^2+11\gamma+2+\frac{1}{\gamma+2}\right)\cdot \tag{16}$$
$$\sqrt{\frac{\gamma}{\gamma+2}}\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\},$$

$$\sigma(\pm,\pm;\mp,\mp) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\Bigg\{\gamma\left[4\gamma^3+9\gamma^2+\frac{7\gamma-1}{2}-\frac{\gamma}{\gamma+2}\right]+\frac{1}{4}\ln(2\gamma+1)$$
$$-\gamma\left(6\gamma^2+11\gamma+2+\frac{1}{\gamma+2}\right)\cdot\sqrt{\frac{\gamma}{\gamma+2}}\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\}, \tag{17}$$

$$\sigma(\pm,\mp;\pm,\pm) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\Bigg\{\gamma\left[5\gamma^2+\gamma\frac{15\gamma+12}{2(\gamma+2)}-\frac{1}{2}\right]+\left(4\gamma^4-2\gamma^2+\frac{1}{4}\right)\ln(2\gamma+1)$$
$$-\gamma\left(4\gamma^3+2\gamma^2+7\gamma-\frac{2\gamma-5}{\gamma+2}\right)\cdot\sqrt{\frac{\gamma}{\gamma+2}}\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\}, \tag{18}$$

$$\sigma(\pm,\mp;\pm,\mp) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\Bigg\{\gamma\left[-4\gamma^3+\gamma^2\frac{8\gamma+5}{2\gamma+1}-\gamma\frac{15\gamma+12}{2(\gamma+2)}+\frac{1}{2}\right]$$
$$-\left(2\gamma^3+3\gamma^2+\frac{1}{4}\right)\ln(2\gamma+1)+\gamma\left(4\gamma^3+2\gamma^2+7\gamma-\frac{2\gamma-5}{\gamma+2}\right)\cdot \tag{19}$$
$$\sqrt{\frac{\gamma}{\gamma+2}}\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\},$$

$$\sigma(\pm,\mp;\mp,\pm) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\Bigg\{\gamma\left[2\gamma^3-\frac{1}{2}(\gamma+1)\frac{\gamma-2}{\gamma+2}\right]-\frac{(2\gamma+1)^3}{4}\ln(2\gamma+1)$$
$$+\gamma\left(5\gamma+2+\frac{1}{\gamma+2}\right)\cdot\sqrt{\frac{\gamma}{\gamma+2}}\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\Bigg\}, \tag{20}$$

$$\sigma(\pm,\mp;\mp,\mp) = r_0^2\,\frac{\pi}{2}\,\frac{1}{\gamma^5}\cdot\left\{\gamma\left[-2\gamma^2+\frac{1}{2}(\gamma+1)\frac{\gamma-2}{\gamma+2}\right]+\left[\frac{(2\gamma+1)^3}{4}+\gamma^2\right]\ln(2\gamma+1)\right.$$
$$\left.-\gamma\left(5\gamma+2+\frac{1}{\gamma+2}\right)\cdot\sqrt{\frac{\gamma}{\gamma+2}}\ln[\gamma+\sqrt{\gamma(\gamma+2)}+1]\right\}, \tag{21}$$

where $\gamma = \dfrac{(p_1k_1)}{m^2} = \dfrac{k_1^0}{m}$.

Conclusion

Detailed numerical analysis of the $e\gamma$ cross section and polarized asymmetry demonstrates the contribution of polarization term decreases with increase of initial electron energy. The value of the cross section for extremely high energies ($\sqrt{s} \geq 200$ GeV) proves to be negligible. The cross section is almost completely determined by unpolarized part and therefore it is clear that the high-energy experiments with polarized particles are excellent instrument to investigate of electromagnetic interaction in Compton process.

It is convenient to analyze polarization effects using polarized left-right asymmetry:

$$A_{LR} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}. \tag{22}$$

Here σ_+ and σ_- is the total cross sections for left and right polarized scattered photons.

The polarized asymmetry (22) as a function of initial electron energy is given in Fig.2.

Since the cross sections and polarized asymmetry have significant value (see Fig.2) it is evident it is important to include in consideration the higher order effects (radiative effects). We calculate them using Helicity Amplitudes Method. The corresponding corrections are evaluated for two different values of the central mass energy interacting particles ($\sqrt{s} = 500$ GeV and 1000 GeV). The correction to the unpolarized differential cross section is very small and varies between 0.5% and 1%. The correction is totally negligible at the lower end of the spectrum. At the other end of the spectrum it seems that the correction becomes quite large, around 10%. The reason for this is that the lowest-order value is already small.

The considered polarized asymmetry is significant ($\geq$ 90%) when the energies of initial particles occupy the same diapason and it gradually decline with the growth of electron energy compared to the energy of a photon (Fig.2). The radiative correction would not very meaningful in the largest part of the kinematic region.

Thus, high polarized $e\gamma$ processes are good instrument for calibration of high energy accelerators and preparation of polarized electron beams with significant degree of polarization.

FIG. 2. Polarized asymmetry as a function of initial central mass energy (a) and as a function of scaling variable y (b) at the energy of interacting particles $\sqrt{s} = 120$ GeV.

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