Multipole moments of gauge bosons

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Models with composite gauge bosons are considered. It is assumed that gauge bosons could possess multipole moments which are different from values predicted by the standard model. The process $f + \overline{f} \to W^+ + W^-$, is investigated. The total cross section is calculated in the second order of the perturbation theory. When the energy of colliding fermion-antifermion pairs in the center of mass system s increases the total cross section grows as a linear function of s. However, under certain values of the MM's, $\sigma(f + \overline{f} \to W^- W^+)$ exposes an interesting property. For example, in the case of annihilation of $e^- e^+$ or $\mu^- \mu^+$ pairs, the total cross section increases until energy 200 GeV or so, then it falls down up to its minimum and only after that it starts to grow linearly on s. The conditions of the minima existence are found.

1. Introduction

At present, in elementary particle physics the very interesting situation has arisen. It bears a strong resemblance to the situation that we had at the beginning of the XXth century before the discovery of nonrelativistic quantum mechanics and the special theory of relativity. During the last decade many impressive experimental achievements have been done: top-quark discovery, measurement of the direct CP violation in K-meson system as well as the CP violation in B-meson system, evidence of accelerated Universe expansion, determination of portions of dark matter and dark energy in the Universe, establishment of relict radiation anisotropy and so on. These results strengthen the status of the standard model (SM) as that which successfully describes the Nature. However, we again see some small clouds in the clear sky of the SM — the experimental facts that have not found satisfactory explanation within the SM. This is first of all a neutrino mass smallness, observation of solar and atmospheric neutrino oscillations, the value of the muon AMM, prediction of equal amount of matter and antimatter in the Universe. Moreover, experiments showed that the density of matter entering into the SM constitutes approximately 5% of the matter density in the Universe. From the theoretical point of view, the SM is also far from perfection because it leaves without answer of many fundamental questions associated with its structure. The SM may be divided into three sectors: a gauge sector, a flavor sector and a sector in which a gauge symmetry is breaking. Whereas the two first sectors have well been studied in accelerating experiments (LEP, SLD, BABAR, BELLE, and so on), now the sector of the spontaneous symmetry breaking attracts rapt attention, and not only because physicists hope to discover the Higgs boson at LHC, but because this sector can give the first hints on the existence of New Physics beyond the SM. The Higgs mechanism in the

SM represents only description of the electroweak symmetry breaking. It does not give any explanations of the symmetry violation. In particular, dynamics which could explain the reason of the Higgs potential instability appearance at zero is absent. Consequently, the standard Higgs mechanism calls in question the contemporary understanding of the SM at the quantum level. There is a need to introduce additional structures (new particles, new symmetries, additional dimensions, and so on) in order to stabilize the electroweak scale. All this stimulate searches and investigations of models which lead to the same results like the SM in the low-energy region, while in the high-energy region their predictions are different from the SM predictions. There are at least three principle ways of the SM extension. The first consists in building of models with a composite Higgs boson and a dynamical symmetry breaking. Non-observation of the Higgs boson the SM predicts generates a class of Higgsless models. The third direction involves the Higgs mechanism and is based on the idea that the SM is considered as low-energy approximation of some grand unification theory (GUT). For a symmetry group of the GUT, $SO(10)-$, E_6-, and higher dimensionality groups may be employed. They lead either to the extension of the electroweak group by the factors $SU(2)$ [1] and $U(1)$ [2], or to the replacement $SU(2)_L$ by $SU(3)_N$ [3]. In this paper the first two classes of models will be discussed.

Models of an extended technicolor are typical representatives of models with a composite Higgs boson. The basic idea consists in the fact that technifermions are coupled by gauge interactions those are built by analogy with the QCD [4]. At that, the existence of a set of new gauge charges, the technicolors, is postulated. The goal is to have a spontaneous symmetry breaking (SSB) theory with gauge interactions alone: there is no elementary scalar with its self-couplings and Yukawa couplings.

The next way of the SM extension is building Higgsless models. Models with additional dimensions can be a good example. Some kinds of such models include the Higgs mechanism. We shall concentrate attention on Higgsless version of models in five-dimensional space-time (5HLM's). The principal idea on which the 5HLM's are based lies in the fact that a particle momentum component along additional dimension is equivalent to a mass in four dimensions [5]. By this reason, one may introduce a mass of a particle imparting a momentum along some additional dimension to it. Similar that, as happens in the quantum mechanics, nonzero momentum along a compact direction may appear as a result of imposing nontrivial boundary conditions. Therefore, the task is to find an additional dimension geometry and select an appropriate boundary conditions, so that the spectrum of SM particles can be reproduced. The 5HLM's are non-renormalizable and become strongly coupled at a cut-off scale Λ. However, Higgsless models was proposed just in order to make the scale Λ sufficiently large and in order to avoid difficulties which might appear when model predictions are confronted with the results of precision measurements of electroweak observables (this means that a sector with a strong coupling can not be observed at LHC). And still increasing Λ needs introduction of weakly coupled states in addition to the spectrum of the SM particles. Just these states represent one of objects of searches at LHC.

There exists many realizations of Higgsless models which are distinguished by the way of introducing fermions or additional dimensions into the theory. However, the fundamental mechanism used for increasing the scale Λ is common for all these models. It is as follows. At the scale few TeV, the existence of new massive particles with the spin 1 and having the same quantum number as the SM gauge bosons (Kaluza—Klein bosons) The KK bosons appear as a result of compactification and their masses usually equal the integer or half-integer number of inverse compactification radii. is needed. Constants determining interactions of the Kaluza—Klein (KK) bosons with $W^{\pm}$, Z and γ have to obey the unitary sum rules to call forth cancelation of contributions which come from longitudinal $W^{\pm}$ and Z bosons and increase versus an energy. Therefore, scattering processes of the vector gauge bosons represent the test for Higgsless scenario which is independent on a model.

One more interesting predictions of Higgsless models lies in the presence of anomalous three- and four-boson interaction constants. Really, in the SM the sum rules those provide for cancelation of terms growing as the fourth power of energy are automatically satisfied because of the gauge invariance. To correctly include contributions of new states, constants of interaction between the SM gauge bosons must be changed. If one assumes the sum rules to be fulfilled already for the first resonance, then the value of these deviations for the WWZ vertices is given by the expression:

$$\Delta = \frac{\delta(g_{WWZ})}{g_{WWZ}} \sim \frac{1}{3}\frac{m_Z^2}{m_{Z'}^2}, \qquad (1)$$

where Δ is the shift in the interaction constant, and the deviation is estimated with the help of the sum rule for elastic scattering of the W bosons. The deviations in the three-boson interaction constants are expected in the range from 1% to 3%. These values are very close to experimental bounds obtained by LEP and they could be tested at LHC. Without question, the linear colliders ILC and CLIC will be able to measure such shifts. It should be stressed that these deviations represent the firm prediction of Higgsless mechanism and do not depend on details of realization of particular Higgsless model.

2. The process $f + \overline{f} \to W^- + W^+$

So, the existence of the fundamental scale Λ is of a common property for models belonging to the first two classes of the SM extensions. When the energy $\sqrt{s}$ exceeds Λ, the deviations from the SM are easily observed. Manifestations of deviations from the SM predictions are less obvious when $\sqrt{s} < \Lambda$. Let us attract attention to looking-for the New Physics signals at $\sqrt{s} < \Lambda$. Since the models in question predict anomalous interactions between W, Z and γ, then it is worthwhile to consider a process in which these interactions play a determinative part.

When investigating processes in the range of energies smaller than Λ, it is enough to use the effective Lagrangian technique. In so doing, we have to determine the content of particles sector and demand fulfillment of particular symmetries. For the sake of simplicity, we assume the following: (i) there are only the SM particles in the theory; (ii) the theory possesses the $SU(2)_L \times U(1)$ global gauge symmetry; (iii) any exotic-fermion contact interactions are absent. This leads to the fact that the charged and neutral currents have the same form as in the SM. As regards the Lagrangian describing trilinear gauge boson couplings (TBC's), in the most general case it will look like [6]:

$$\mathcal{L}_{WWV} = g_{WWV}\Bigg[ig_1^V(W^*_{\mu\nu}W^\mu V^\nu - W^{*\mu}W_{\mu\nu}V^\nu) + ik_V W^*_\mu W_\nu V^{\mu\nu} + \frac{i\lambda_V}{m_W^2}W^*_{\lambda\mu}W^{\mu\nu}V^\lambda_\nu -$$
$$-g_2^V W^*_\mu W_\nu(\partial^\mu V^\nu + \partial^\nu V^\mu) + g_3^V \varepsilon^{\mu\nu\rho\sigma}(W^*_\mu\partial_\rho W_\nu - \partial_\rho W^*_\mu W_\nu)V_\sigma + i\tilde{k}_V W^*_\mu W_\nu \tilde{V}^{\mu\nu} +$$
$$+\frac{i\tilde{\lambda}_V}{m_W^2}W^*_{\lambda\mu}W^{\mu\nu}\tilde{V}_\nu)\lambda\Bigg], \tag{2}$$

where $V = Z,\gamma$, $W_{\mu\nu} = \partial_\mu W_\mu - \partial_\nu W_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\tilde{V}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}V_{\lambda\sigma}$, $g_{WW\gamma} = -e$, $g_{WWZ} = -e\cot\theta_W$. The coefficients k_γ and λ_γ determine the magnetic dipole and electric quadruple moments of the W boson in accordance with the relations

$$\mu_\gamma = \frac{e}{2m_W}(1 + k_\gamma + \lambda_\gamma), \qquad Q_\gamma = -\frac{e}{m_W^2}(k_\gamma - \lambda_\gamma). \tag{3}$$

The expressions for their Z analogues follows from (3) by the replacements

$$e \to e\cot\theta_W, \qquad \gamma \to Z. \tag{4}$$

The parameters $\tilde{k}_\gamma$ and $\tilde{\lambda}_\gamma$ fix the values of electric dipole and magnetic quadruple moments that are determined by the relations:

$$d_\gamma = \frac{e}{2m_W}(\tilde{k}_\gamma + \tilde{\lambda}_\gamma), \qquad \tilde{Q}_\gamma = -\frac{e}{m_W^2}(\tilde{k}_\gamma - \tilde{\lambda}_\gamma). \tag{5}$$

In the SM and its extensions belonging to the third class, we have at the tree level:

$$k_V = 1, \tilde{\lambda}_V = \lambda_V = \tilde{k}_V = g_2^V = g_3^V = 0. \tag{6}$$

The radiative corrections (RC's) increase the value of the electromagnetic and weak multipole moments (MM's). Thus, in the one-loop approximation, these corrections to k_γ give the value less than 10^{-2} [7].

A good test for definition of the MM's is provided by the process

$$f + \overline{f} \to W^- + W^+. \tag{7}$$

In the second order of the perturbation theory the diagrams of this reaction are shown in Fig. 1. Next we shall assume that only k_V and λ_V take anomalous values. In the case when the initial

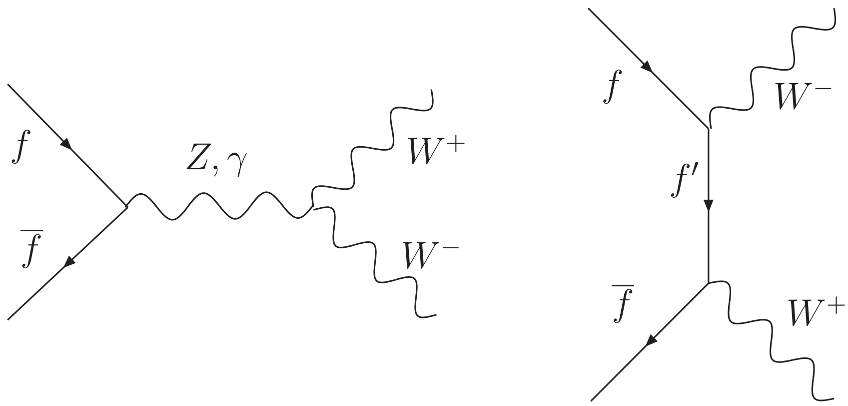

FIG. 1.

and final particles are unpolarized, the total cross section is governed by the expression:

$$\sigma = \frac{a^2\beta}{32 s_W^4 s}\left[M_{\gamma\gamma} + M^f_{ZZ} + M_{f'f'} + M^f_{\gamma Z} + M^f_{f'Z} + M_{\gamma f'}\right], \tag{8}$$

where

$$M_{\gamma\gamma} = 4 s_W^4 D_1(k_\gamma, k_\gamma; \lambda_\gamma, \lambda_\gamma),$$

$$M^f_{ZZ} = \frac{s^2\{1 + 4|q_f| s_W^2 [2|q_f| s_W^2 - 1]\}}{2[(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2]}\, D_1(k_Z, k_Z; \lambda_Z, \lambda_Z),$$

$$M^f_{\gamma Z} = \epsilon(-q_f)\frac{2s(s - m_Z^2)\{s_W^2[1 - 4|q_f| s_W^2]\}}{[(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2]}\, D_1(k_\gamma, k_Z; \lambda_\gamma, \lambda_Z),$$

$$M^f_{Zf'} = \epsilon(-q_f)\frac{s(s - m_Z^2)[2|q_f| s_W^2 - 1)}{2[(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2]}\, D_2(k_Z, \lambda_Z),$$

$$M_{\gamma f'} = -s_W^2 D_2(k_\gamma, \lambda_\gamma), \qquad M_{f'f'} = D_3,$$

$$D_1(a, b; c, d) = \beta^2\left\{\frac{4y^2}{3}(ab + 2cd) + \frac{8y}{3}\left[1 + 2d + 2c + (a + c)(b + d) + \frac{3}{2}(a + b)\right] + 4\right\},$$

$$D_2(a, b) = 2\beta^2 y\left[4\left(2a + 2b + \frac{5}{3}\right) + \frac{4ay}{3}\right] +$$

$$16(a + b + 1)\left(1 - \frac{1}{\beta y}\ln\frac{1+\beta}{1-\beta}\right) - \frac{8}{\beta y^2}\ln\frac{1+\beta}{1-\beta} + 4(1 + \beta^2),$$

$$D_3 = 8 + \frac{4(1+\beta^2)}{\beta}\ln\frac{1+\beta}{1-\beta} + 2\beta^2 y\left(4 + \frac{y}{3}\right)$$

$\beta = (1 - 2/y)^{1/2}$, $y = s/2m_W^2$, q_f is the fermion charge in the electron charge unit, $f' = \nu$ for e^+e^--, $f' = u$ for d^+d^--, and $f' = d$ for u^+u^--annihilation, respectively.

In the range of energies at least less than the fundamental scale Λ, the obtained cross section linearly increases versus an energy

$$\sigma^{MM}(f\overline{f} \to W^-W^+) \simeq s. \tag{9}$$

One would think that at these circumstances a model with anomalous values of the MM's could be easily distinguished from some other models in any range of energies under consideration. Really, in the SM, in the case of lepton-antilepton annihilation, the total cross section having reached its maximum at $\sqrt{s} \simeq 200$ GeV starts to decrease according to

$$\sigma^{CM} \simeq s^{-1} \ln s. \tag{10}$$

The total cross sections of models with extended gauge groups of the electroweak symmetry display the analogous behavior with the only difference that they have one more maximum in the vicinity of the Z' resonance. In a model with the anomalous MM's, we are expecting that σ^{MM} will grow as a linear function of s over the whole range of energies. However, under certain values of the MM's $\sigma^{MM}(f\overline{f} \to W^-W^+)$ exposes an interesting property easily visible from its analytical expression. The total cross section increases until energy 200 GeV or so, then it falls down up to its minimum and only after that it starts to grow linearly on s.

The conditions

$$\Delta\lambda_\gamma = \Delta\lambda_Z = \Delta k_\gamma = 0, \qquad -0.7 < \Delta k_Z < 0.83 \tag{11}$$

is one of the examples of the minimum existence. When $\mid \Delta k_Z \mid$ increases, then the minimum is shifted towards smaller values of $\sqrt{s}$, the minimal value of σ^{MM} increasing. Giving the SM values to the three MM parameters and varying only a single one, we find four simplest conditions for the minimum existence:

$$\left.\begin{array}{l} -0.78 < \Delta k_\gamma < 1.10, \\ -0.61 < \Delta\lambda_\gamma < 0.77, \\ -0.57 < \Delta\lambda_Z < 0.59. \end{array}\right\} \tag{12}$$

Of course, there exist the minimum conditions under deviations of all the four MM's from the SM values, but, in what follows, we shall constrain ourselves by variation of one of them.

It is evident, if we work in the range of energies less than $(\sqrt{s})_{min}$, then the deviations of the σ^{MM} values from the SM predictions are small. To obtain more precise constraints on the MM values, an excess of a collider energy over $(\sqrt{s})_{min}$ is needed because, as this takes place, σ^{MM} turns to the range of a linear rise on s.

Let us introduce the quantity δ which characterizes the experimental sensibility to the deviations from the SM

$$\delta = \frac{\sigma^{MM} - \sigma^{SM}}{\sqrt{\sigma^{SM}}}\sqrt{LT}, \tag{13}$$

where σ^{MM} is the cross section of the process (2) in a model with the anomalous MM's and LT is the integrated luminosity of the collider in units of pb^{-1}. From (13) it is evident that δ is an observable of the effect caused by the New Physics and it gives the deviations from the SM expressed in the standard error units. The notation $\delta(\Delta k_Z)$ means that δ is a function of Δk_Z at $\Delta k_\gamma = \Delta\lambda_Z = \Delta\lambda_\gamma = 0$ and so on. In Fig. 2 we present the graphs of the functions $\delta(\Delta k_\gamma)$, $\delta(\Delta k_Z)$, $\delta(\Delta\lambda_Z)$ and $\delta(\Delta\lambda_\gamma)$ under $LT = 500$ pb^{-1} and $\sqrt{s} = 196$ GeV.

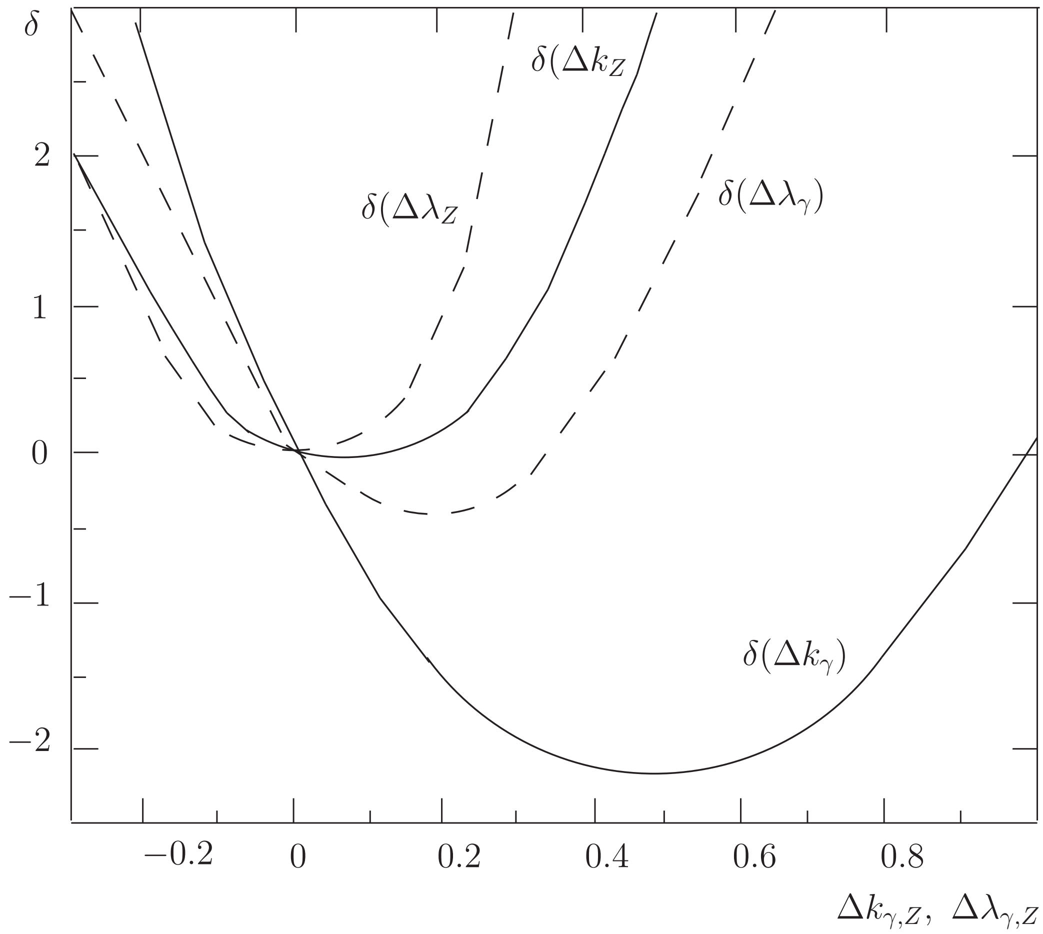

FIG. 2.

3. Conclusion

In this work models where gauge bosons possess arbitrary values of the MMs were considered. The process of the W^-W^+ pair production under fermion-antifermion annihilation was investigated. The total cross section was calculated in the second order of the perturbation theory. When the energy of colliding fermion-antifermion pairs in the center of mass system s increases the total cross section grows as a linear function of s. However, it was shown that under certain values of the MMs, $\sigma(f+\overline{f} \to W^-W^+)$ exposes an interesting property. The total cross section increases until some energy, then it falls down up to its minimum and only after that it starts to grow linearly on s. The conditions of the minima existence were found.

It should be noted that there is a substantial distinction in investigating the MMs at e^+e^- or $\mu^-\mu^+$ and hadron colliders. Since at LEP, ILC and CLIC a detail study of individual helicity amplitudes is possible, then contributions of formfactors could be divided at any energy in the center-of-mass frame. In addition, specificity of these machines is such that the cross section of the W pair production may be measured with the precision of few percents, that is, looking-for of the anomalous MM's corresponds to that of $O(10^{-2})$ deviations of the cross section from the SM predictions. The hadron colliders (Tevatron, LHC) allow to investigate all processes of boson-pair production, namely, W^+W^-–, $W^\pm\gamma$- and $W^\pm Z$-processes. In the last two cases, one may measure independently the $WW\gamma$- and WWZ-couplings. Due to more complicated situation associated with the background isolation, incomplete knowledge of the QCD radiative corrections, effects of structure functions and so on, comparison of measurement results and theory predictions at the level $O(10^{-2})$ is impossible unfortunately. However, on the other hand, at hadron colliders, in comparison with e^+e^- or $\mu^+\mu^-$ machines, larger values of $\sqrt{s}$ are available what allows to explore the MMs behavior under higher energy scales.

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