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КЛАССИЧЕСКИЕ ИНСТАНТОННЫЕ РЕШЕНИЯ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ

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Инстантоны – нетривиальные решения классических уравнений движения с конечным действием. Они обеспечивают точки стационарной фазы в функциональном интеграле для туннельной амплитуды между двумя топологически различными вакуумами. Это обуславливает их полезность во многих приложениях квантовой теории,

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в особенности для описания волновой функции систем с вырожденным вакуумным состоянием в рамках формализма функциональных интегралов. Цель данной работы – введение в современное состояние дел в исследованиях инстантонов и подготовка к экспериментам. Дан обзор инстантонных эффектов в квантовой теории. Найдены инстантонные решения, возникающие в некоторых квантово-механических задачах, а именно в задачах об одномерном движении частицы в двумерном и периодическом потенциалах. Описаны известные инстантоны квантовой теории поля, возникающие, в частности, в двумерной абелевой модели Хиггса и в SU(2)-калибровочных полях теории Янга – Миллса. Найдены инстантонные решения двумерных скалярных полевых моделей с потенциалом синус-Гордона и двумерным потенциалом в ограниченном пространственном объеме. Показано, что учет инстантонов значительно меняет форму потенциала Юкавы для модели синус-Гордона в двух измерениях.

Ключевые слова: инстантон; квантовая теория; квантовая механика; калибровочные поля; скалярное поле; конфайнмент.

CLASSICAL INSTANTON SOLUTIONS IN QUANTUM FIELD THEORY

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Instantons are non-trivial solutions of classical Euclidean equations of motion with a finite action. They provide stationary phase points in the path integral for tunnel amplitude between two topologically distinct vacua. It make them useful in many applications of quantum theory, especially for describing the wave function of systems with a degenerate vacua in the framework of the path integrals formalism. Our goal is to introduce the current situation about research on instantons and prepare for experiments. In this paper we give a review of instanton effects in quantum theory. We find instanton solutions in some quantum mechanical problems, namely, in the problems of the one-dimensional motion of a particle in two-well and periodic potentials. We describe known instantons in quantum field theory that arise, in particular, in the two-dimensional Abelian Higgs model and in SU(2) Yang – Mills gauge fields. We find instanton solutions of two-dimensional scalar field models with sine-Gordon and double-well potentials in a limited spatial volume. We show that accounting of instantons significantly changes the form of the Yukawa potential for the sine-Gordon model in two dimensions.

Keywords: instanton; quantum theory; quantum mechanics; gauge field; scalar field; confinement.

Introduction

Instantons are non-trivial solutions of classical Euclidean equations of motion with a finite action (see, for example, [1]). Euclidean equations of motions are obtained from Lagrange – Euler equations by analytical continuation on imaginary time $t \rightarrow -i\tau$ (so called Wick rotation). This continuation changes Minkowskian space to 4-dimensional Euclidean. Lagrange – Euler equations acquire a new form and can lead to new solutions that can not be reduced to the old ones by formal replacement.

During the Wick rotation, convergence of the path integral is improved because of appearance of falling exponents instead of oscillating. The evolution trajectories of the system, close enough to instantons, make the main contribution to the amplitude of tunnel due to the fact that instantons are local minima of the action.

Instantons in quantum mechanics

Double-well potential. Simplest instanton appears in a one-dimensional mechanical problem of a particle in a double-well potential (fig. 1) [1]:

$$\mathcal{L} = \frac{1}{2} \left(\frac{dx}{dt} \right)^2 - V(x), \quad V(x) = \lambda (x^2 - \rho^2)^2$$

with $\mathcal{L}$ – Lagrange function, λ and ρ – real positive constants.

Well-known classical equations of motion

$$\frac{d^2x}{dt^2} + \frac{dV}{dx} = 0$$

after Wick rotation take the following form

$$\frac{d^2x}{d\tau^2} - \frac{dV}{dx} = 0$$

with $\tau = it$.

Instanton solution given by

$$x^{inst}(\tau) = \pm \rho \tanh \left[\rho \sqrt{2\lambda} (\tau - \tau_0) \right]$$

corresponds to the motion from one maximum of the «inverted» potential (fig. 2) to another during an infinite time. Trivial solutions $x = \pm \rho$ and solutions with infinite action (corresponding to the motion from points $x = \pm \rho$ to $x = \pm \infty$) are not considered to be instantons.

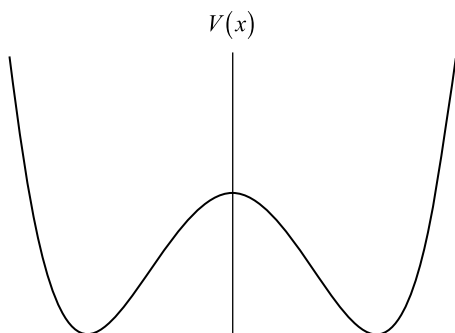


Fig. 1. Double-well potential in Minkowski space

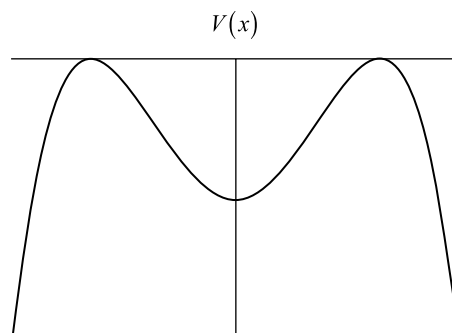


Fig. 2. Inverted by Wick rotation potential

Periodical potential. Another simple example of instanton appears in the problem of a motion in a periodical potential (fig. 3)

$$\mathcal{L} = \frac{1}{2} \left(\frac{dx}{dt} \right)^2 - \lambda (1 - \cos(\rho x)),$$

or in similar problem, particle in a ring, describing behavior of a particle in a one-dimensional closed spatial region

$$\mathcal{L} = \frac{1}{2} \left(\frac{dx}{dt} \right)^2 - V(x), \quad x \in [0, l], \quad V(x+l) = V(x).$$

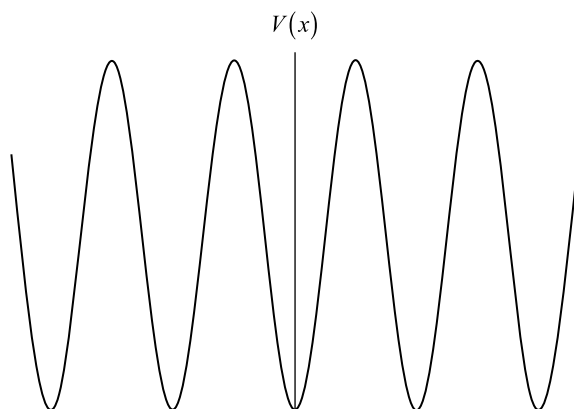


Fig. 3. Periodical potential

Decay of a metastable state. Metastable state (fig. 4) of quantum system can decay through tunneling. Special instanton solution (so-called bounce) provides a main contribution in transition amplitude and describe the decay probability:

$$\Gamma \propto e^{-2S_E[x_b(\tau)]}.$$

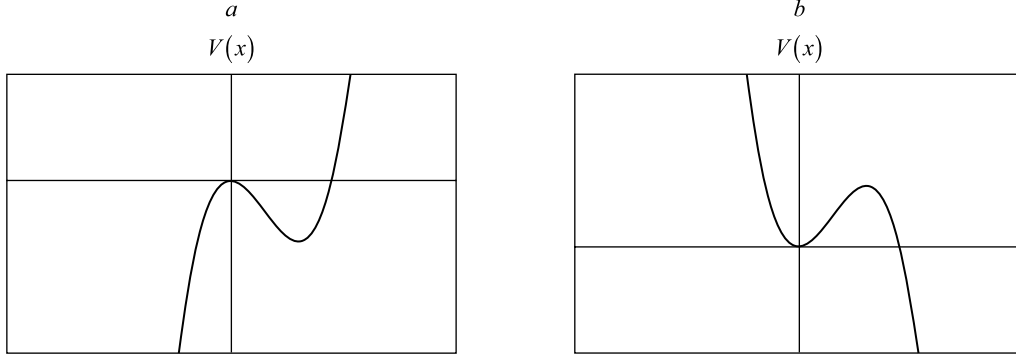


Fig. 4. Potential of a system with metastable state:
a – Lorentzian version; b – Euclidean version

Instantons in quantum field theory

Scalar field theories. *Scalar field theories with classically degenerate vacuum.* Some scalar theories with non-trivial potential consist classically degenerate vacuum. Lagrangian of such theory has a form

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi - V(\phi). \quad (1)$$

For example, scalar theories with double-well and sine-Gordon potentials (fig. 5 and 6) are theories with degenerate vacuum

$$V(\phi) = \lambda (\phi^2 - \rho^2)^2, \quad (2)$$

$$V(\phi) = \lambda (1 - \cos(\rho\phi)). \quad (3)$$

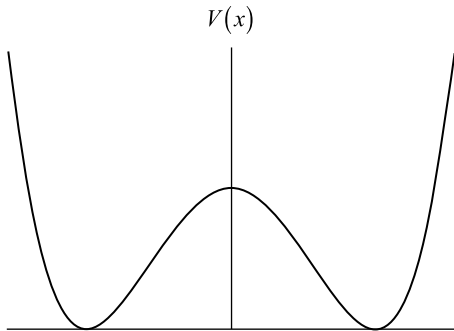


Fig. 5. Double-well potential (2)

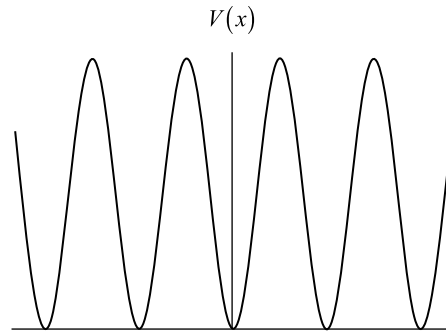


Fig. 6. Sine-Gordon potential (3)

Instanton solutions in d dimensions coincide with static soliton solutions in $d + 1$ dimensions. By Derrick's theorem [2; 3] there are no soliton solutions for scalar field models with Lagrangian of the form (1) in three and more dimensions. Hence there are no instantons in two and more dimensions in theories with Lagrangian of the form (1). Physically this prohibition is due to the fact that vacuum tunneling transitions are impossible because of infinite magnitude of the energy barrier between neighboring vacuums (since considered spatial region is infinite: $-\infty < x < +\infty$).

Scalar field theories with false vacuum. Tunneling causes decay of false vacuum in theories that contain it (fig. 7). As long as field contribution in transition amplitude depends on the action of the field configuration, corresponding instanton solutions (so-called bounces) as local minima of the action are configurations with the greatest contribution and describe tunneling:

$$\Gamma \propto e^{2S_E[\varphi_b(x, \tau)]},$$

where $\varphi_b(x, \tau)$ is the bounce solution.

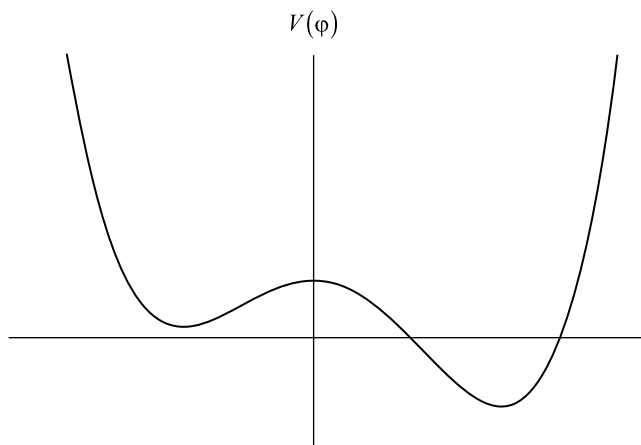


Fig. 7. Example of a potential of a system with a false vacuum

Gauge theories. 2-Dimensional abelian Higgs model. Lagrangian of abelian Higgs model is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}D_\mu\varphi D_\mu\varphi - \lambda(\varphi\varphi^* - \rho^2), \quad D_\mu = \partial_\mu - ieA_\mu.$$

Variety of vacua

$$A_x = \frac{1}{e} \frac{d\alpha(x)}{dx}, \quad \varphi = \rho e^{i\alpha(x)}, \quad A_0 = 0,$$

splits in quantum case into discrete number of classes. They are bounded by Nielsen – Olesen vortices.

SU(2) Yang – Mills theory. The first instanton solution was obtained by A. M. Polyakov and colleagues for the pure SU(2) Yang – Mills gauge field theory [4]:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu},$$

$$A_\mu^a = \frac{2}{g} \frac{\eta_{a\mu\nu}(x-x_0)_\nu}{\rho^2 + (x-x_0)^2},$$

where $\eta_{a\mu\nu}$ is the 't Hooft symbol:

$$\eta_{a\mu\nu} = \begin{cases} \varepsilon_{a\mu\nu}, & \mu, \nu = 1, 2, 3, \\ -\delta_{a\nu}, & \mu = 4, \\ \delta_{a\mu}, & \nu = 4, \\ 0, & \mu = \nu = 4. \end{cases}$$

Experimental status of instantons. Instanton processes are strongly suppressed in physically meaningful gauge theories like electroweak theory (EWT) and quantum chromodynamics (QCD):

$$e^{-\frac{4\pi}{\alpha}} \approx \begin{cases} 10^{-160}, & \text{EWT}, \\ 10^{-10}, & \text{QCD}, \end{cases}$$

but they can become observable at high energies or at high temperatures.

Electroweak theory. Instantons induce a violation of the baryon and lepton number, what may be bounded with the problem of asymmetry of matter and antimatter in visible part of the Universe.

Quantum chromodynamics. Instantons cause processes with non-conservation of chirality.

Six criteria for the search for QCD instantons at the HERA accelerator (DESY, Germany) was proposed [5–8]. The specific behavior of the factorial and cumulant moments can be chosen as one of the criteria for detecting QCD instantons [9].

As a theory based on non-abelian SU(3) gauge group, quantum chromodynamics has a self-interacting gluon fields. This causes a complicated vacuum structure of QCD.

By picking SU(2) subgroup of SU(3) we can note that in singular gauge there is an instanton given by

$$A_\mu^a = \frac{2}{g_s} \frac{\bar{\eta}_{a\mu\nu}(x-x_0)_\nu \rho}{(x-x_0)^2((x-x_0)^2 + \rho^2)}, \quad (4)$$

where $\bar{\eta}_{a\mu\nu}$ is 't Hooft symbol. This solution have finite action

$$S_E = \frac{2\pi}{\alpha_s} |Q|,$$

where Q is topological charge of instanton:

$$Q = \frac{\alpha_s}{8\pi} \int G_{\mu\nu}^\alpha \tilde{G}_{\mu\nu}^\alpha d^4x.$$

Solution (4) connects through imaginary time topologically distinct «pure gauge» vacua.

Instantons leads to a specific multi-quark 't Hooft vertex [10]. It can be described (for $\rho \rightarrow 0$, $N_c = N_f = 3$) by effective Lagrangian

$$\begin{aligned} \mathcal{L}_{eff}^{(3)} = \int d\rho n(\rho) & \left\{ \prod_{i=u,d,s} \left(m_i \rho - \frac{4\pi}{3} \rho^3 \right) + \frac{3}{32} \left(\frac{4}{3} \pi^2 \rho^3 \right)^2 \times \right. \\ & \times \left(\left(j_u^a j_d^a - \frac{3}{4} j_{u\mu\nu}^a j_{d\mu\nu}^a \right) \left(m_s \rho - \frac{4}{3} \pi^2 \rho^3 \bar{q}_{SR} q_{SL} \right) + \right. \\ & + \frac{9}{40} \left(\frac{4}{3} \pi^2 \rho^3 \right)^2 d^{abc} j_{u\mu\nu}^a j_{d\mu\nu}^b j_s^c + 2 perm. \Big) + \frac{9}{320} \left(\frac{4}{3} \pi^2 \rho^3 \right)^2 d^{abc} j_u^a j_d^b j_s^c + \\ & \left. + \frac{ig f^{abc}}{256} \left(\frac{4}{3} \pi^2 \rho^3 \right)^3 j_{u\mu\nu}^a j_{d\nu\lambda}^b j_{s\lambda\mu}^c + (R \leftrightarrow L) \right\}, \end{aligned}$$

where $q_{R,L} = \frac{1 \pm \gamma_5}{2q(x)}$; $j_i^a = \bar{q}_{iR} \lambda^a q_{iL}$; $j_{i\mu\nu}^a = \bar{q}_{iR} \sigma_{\mu\nu} \lambda^a q_{iL}$ and $n(\rho)$ is instanton density.

For massless quarks and number of flavors $N_f = 2$ structure of effective Lagrangian is significantly reduced:

$$\mathcal{L}_{eff}^{(3)} = \int d\rho n(\rho) \left(\frac{4}{3} \pi^2 \rho^3 \right)^2 \left\{ \bar{u}_R u_L \bar{d}_R d_L \left[1 + \frac{3}{32} \left(1 - \frac{3}{4} \sigma_\mu^u \nu \sigma_\mu^d \nu \right) \lambda_u^a \lambda_d^a \right] + (R \leftrightarrow L) \right\}.$$

Previous Lagrangians are obtained from the consideration of quark scattering by so-called zero mode in the instanton field. The quark zero mode was found by 't Hooft, who showed that the Dirac equation

$$\left(i\partial_\mu + g \frac{\lambda^a}{2} A_\mu^a(x) \right) \Psi_n(x) = \epsilon_n \Psi_n(x)$$

in instanton field has a solution with zero energy ($\epsilon_0 = 0$)

$$\Psi_0(x-x_0) = \frac{\rho(1-\gamma_5)}{2\pi((x-x_0)^2 + \rho^2)^{3/2}} \frac{\hat{x}}{\sqrt{x^2}} \varphi \quad (5)$$

with φ is two-component spinor $\varphi_m^a = \frac{\epsilon_m^a}{\sqrt{2}}$.

Let us note some features of function $\Psi_0(x-x_0)$ from (5):

- instanton zero modes have a certain helicity;
- at zero mode, the values of the color of the quark and its spin are strictly correlated through the spinor, so that their sum is zero.

Consequently, only one quark of a certain flavor can be in zero mode, and the helicity of the quark is reversed upon scattering by an instanton.

Tunneling and confinement in scalar field theories

As was said earlier, there are no instantons in scalar theories like (1).

However, this obstacle can be avoided by considering a system in a limited spatial volume.

2-Dimensional sine-Gordon model. Let us consider a system

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi - V(\varphi),$$

$$V(\varphi) = \lambda (1 - \cos(\rho\varphi)), \quad \mu = 0, 1, \quad -\frac{l}{2} \leq x \leq \frac{l}{2}.$$

Instanton solution for this system is given by [11; 12]

$$\varphi^{inst}(\tau, x) = \pm \frac{4}{\rho} \arctan \left(e^{(\tau - \tau_0)\rho\sqrt{\lambda}} \right).$$

Euclidean action of this solution is equal

$$S[\varphi^{inst}(\tau, x)] = \frac{8\sqrt{\lambda}l}{\rho}.$$

In quantum version of theory, instantons describe tunnel transitions between classical vacua

$$\varphi_n^{vac}(x, t) = \frac{2\pi n}{\rho}, \quad n = 0, \pm 1, \pm 2, \dots$$

In order to study the confinement possibility, let us introduce Yukawa interaction

$$\mathcal{L}_{int} = g \bar{\psi} \psi \varphi.$$

Potential obtained from this system

$$V(L_1) = \text{const} L_1 e^{-\frac{8\sqrt{\lambda}}{\rho} L_1}$$

significantly differs from Yukawa potential

$$V(L_1) = \text{const} L_1 e^{-\rho\sqrt{\lambda} L_1}.$$

2-Dimensional double-well potential model. Lagrangian of this model is given by

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi - V(\varphi),$$

$$V(\varphi) = \lambda (\varphi^2 - \rho^2)^2, \quad \mu = 0, 1, \quad -\frac{l}{2} \leq x \leq \frac{l}{2}.$$

Instanton solution is given by

$$\varphi^{inst}(\tau, x) = \pm \tanh \left((\tau - \tau_0) \rho \sqrt{2\lambda} \right).$$

Euclidean action of this solution is equal to

$$S[\varphi^{inst}(\tau, x)] = \frac{4\sqrt{2\lambda}\rho^3 l}{3}.$$

Conclusion

Instantons are associated with many phenomena in quantum theory. For example, they change vacuum structure of quantum chromodynamics. Despite this, instanton processes are strongly suppressed in physically meaningful gauge theories, but they can become observable at high energies or at high temperatures. We have proposed experimental detection criteria for instantons. We obtained exact solutions for two-dimensional models with sine-Gordon and double-well potentials. The presence of instantons changes the structure of the Yukawa interaction in two-dimensional case.

In the future, it is planned to develop criteria for detection of instantons in SPD/NICA (spin physics detector/nucleon-based ion collider facility) experiment, taking into account chaotic behavior of solutions in the presence of external influences (see, for example, [13]).

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