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Deformed Quantum Field Theory, Thermodynamics at Low and High Energies, and Gravity. II. Deformation Parameter

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Abstract The principal objective of this paper is to show that deformation of the General Relativity within the scope of both the Generalized Uncertainty Principle (UV-cutoff) and the Extended Uncertainty Principle (IR-cutoff) in the cases when the corresponding gravitational equations have a thermodynamic interpretation may be studied in terms of a small parameter introduced in previous works of the author. From this viewpoint the parameter is though to be universal. Consideration is given to the possibility for extension of the obtained results to more general cases. Possible generalization of the uncertainty relation for the pair (cosmological constant, "space-time volume"), where the cosmological constant is regarded as a dynamic quantity at high and low energies, is analyzed. This paper is devoted to the 70-th Anniversary of Professor Nikolai Maksimovich Shumeiko, Director of the Belarusian Particles and High Energy Physics National Centre.

Keywords Quantum Field Theory with UV and IR Cutoff, Gravitational Thermodynamics, Deformed Gravity

1. Introduction

In the last decade numerous works devoted to a Quantum Field Theory (QFT) at Planck's scale[1–3] have been published (of course, the author has no pretensions of being exhaustive in his references). This interest stems from the facts that (i) at these scales it is expected to reveal the effects of a Quantum Gravity (QG), and this still unresolved theory is intriguing all the researchers engaged in the field of theoretical physics; (ii) modern accelerators, in particular LHC, have the capacity of achieving the energies at which some QG effects may be exhibited.

Now it is clear that Quantum Field Theory (QFT) at Planck's scales, and possibly at very large scales as well, undergoes changes associated with the appearance of additional parameters related to (i) a minimal length (on the order of the Planck's length) and (ii)a minimum momentum. As this takes place, the corresponding parameters are naturally considered as deformation parameters, i.e. the related quantum theories are considered as a high-energy deformation (at Planck's scales) and a low-energy deformation (IR-cutoff), respectively, of the well-known quantum field theory, the latter being introduced in the corresponding high- and low-energy limits and exact to a high level. The deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition [4].

Most natural approach to the introduction of the above-mentioned parameters is to treat a quantum field theory with the Generalized Uncertainty Principle (GUP)[5–15] and with the Extended Uncertainty Principle (EUP), respectively [12–15]. In the case of GUP we easily obtain a minimal length on the order of the Planck's $l_{min} \sim l_p$ and the corresponding high-energy deformation of well-known QFT–QFT with GUP. It should be noted that QFT with GUP at Planck's scales (Early Universe) is attested in many works (for example[5–11])). Even if we disregard the works devoted to a string theory, still remaining a tentative one, GUP is quite naturally derived from the gedanken experiment[6–9].

On the other hand, GUP has no way in the spaces with large length scales (for example (A)dS). For such spaces, e.g., in[12,14] the Extended Uncertainty Principle has been introduced (find its exact definition below) giving an absolute minimum in the uncertainty of the momentum.

The problem is to find whether there are cases when the deformations generated by GUP and EUP are defined by the same parameter. By author's opinion this is the case for Gravity modified (deformed) within GUP and EUP, when the corresponding initial theory has a "thermodynamic interpretation"[16–21]. Specifically, the deformation parameter $\alpha = l_{min}^2 / x^2$, $l_{min} \sim l_p$, $0 < \alpha \le 1/4$ where x is the measuring scale, introduced by the author in a series of works[22–32] meets the above requirements.

Note that this parameter has been introduced to study the

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deformation of QFT at Planck's scale, although the deformation per se, associated with a high-energy modification of the density matrix, was "minimal" in that it presented no noncommutativity operators related to different spatial coordinates

$$[X_i, X_j] \neq 0, i \neq j \tag{1}$$

and hence "limited" as in the end it failed to lead to GUP. Nevertheless, the corresponding deformation parameter in some way is universal.

This paper continues the studies, described in[33–35] (the latter in particular), of the fundamental quantities in "thermodynamic interpretation" of gravity[16–21] for GUP and EUP deformations of the latter. Compared to the works[35], the results from which are used in this paper, the important results associated with EUP are put forward together with the demonstration that GUP and EUP have the same deformation parameter, at least in this context.

The structure of this work is as follows. In Section 2 it is shown that the deformation of the fundamental thermodynamic quantities for black holes within GUP and EUP may be interpreted with the use of the same parameter. In Section 3, within the scope of a dynamic model for the cosmological constant Λ (vacuum energy density), GUP is studied for the pair (Λ, V)[33,34], where V – is the "space-time volume". In this Section consideration is given to the possible existence of EUP for this pair, i.e. to a possible extension of the Uncertainty Principle to the pair in the IR region, and hence to the possible substantiation of the proper (coincident with the experimental) value for Λ. In Section 4 the results of Section 2 are applied to Einstein's Equations for space with horizon and to Friedmann's Equations. It is demonstrated that in both cases their deformation (in the first case within GUP and in the second case within EUP) may be interpreted with the use of the same small dimensionless parameter having a known variability domain.

And, finally, in Section 5 the problems of further investigations are discussed, some final comments are given.

2. Universal Deformation Parameter in Gravitational Thermodynamics with GUP and EUP

In this Section the Gravitational Thermodynamics (GT) is understood as thermodynamics of spaces with horizon[18, 20].

2.1. Gravitational Thermodynamics with GUP

We use the notation and principal results from[14]. So, GUP is of the form

$$\Delta x_i \Delta p_j \geq \hbar \delta_{ij} [1 + \alpha'^2 l_p^2 \frac{(\Delta p_i)^2}{\hbar^2}] \tag{2}$$

and, since $\Delta x_i \Delta p_j > \hbar \delta_{ij}$, we have

$$\Delta x_i \Delta p_i \geq \hbar \delta_{ij} [1 + \alpha'^2 \frac{l_p^2}{\Delta x_i^2} \frac{(\Delta p_i)^2 \Delta x_i^2}{\hbar^2}] > \hbar \delta_{ij} [1 + \frac{1}{4} \alpha_{\Delta x_i}], \tag{3}$$

where $\alpha_{\Delta x_i}$ – parameter α corresponding to Δx_i, $l_{min} = 2\alpha' l_p$. Besides, as distinct from[14], for the dimensionless factor in GUP, instead of α, we use α' to avoid confusion with the deformation parameter.

In this terms the uncertainty in moment is given by the nonstrict inequality

$$2\hbar(\alpha_{\Delta x_i} \Delta x_i)^{-1}[1 - \sqrt{1 - \alpha_{\Delta x_i}}] \leq \Delta p_i \leq 2\hbar(\alpha_{\Delta x_i} \Delta x_i)^{-1}[1 + \sqrt{1 - \alpha_{\Delta x_i}}]. \tag{4}$$

But for the quantities determining GT in terms of α one can derive exact expressions. Indeed, in terms of α the GUP-modification (or rather GUP-deformation)is easily obtained for the Hawking temperature[36–39,14,15] that has been computed in the asymptotically flat d - dimensional space for a Schwarzshild black hole with a metric given by

$$ds^2 = -N^2 dt^2 + N^{-2} dr^3 + r^2 d\Omega_{d-2}^2, \tag{5}$$

where

$$N^2 = 1 - \frac{16\pi GM}{(d-2)\Omega_{d-2} r^{d-3}}, \tag{6}$$

Ω_{d-2} is the area of the unit sphere S^{n-2}, and r_+ is the uncertainty in the emitted particle position by the Hawking effect, expressed as

$$\Delta x_i \approx r_+ \tag{7}$$

and being nothing else but a radius of the event horizon. In this case the deformation parameter α arises naturally. Actually, modification of the Hawking temperature is of the form, see formula (10) in[14]

$$T_{GUP} = (\frac{d-3}{4\pi}) \frac{\hbar r_+}{2\alpha'^2 l_p^2} [1 - (1 - \frac{4\alpha'^2 l_p^2}{r_+^2})^{1/2}] \tag{8}$$

and may be written in a natural way as

$$T_{GUP} = 2(\frac{d-3}{4\pi}) \frac{\hbar}{r_+} \alpha_{r_+}^{-1} [1 - (1 - \alpha_{r_+})^{1/2}], \tag{9}$$

where α_{r_+} - parameter α associated with r_+. It is clear that T_{GUP} is actually the deformation T_{Hawk} – black hole temperature for a semiclassical case[40]. In such a manner compared to T_{Hawk} T_{GUP} is additionally dependent only on the dimensionless small deformation parameter α_{r_+}.

The dependence of the black hole entropy on α_{r_+} may be derived in a similar way. For a semiclassical approximation of the Bekenstein-Hawking formula[41,40]

$$S = \frac{1}{4} \frac{A}{l_p^2}, \tag{10}$$

where A – surface area of the event horizon, provided the horizon event is of radius r_+, $A \sim r_+^2$ and (10) is clearly of the form

$$S = \sigma \alpha_{r_+}^{-1}, \tag{11}$$

where σ is some dimensionless denumerable factor. The general formula for quantum corrections [38] given as

$$S_{GUP} = \frac{A}{4l_p^2} - \frac{\pi \alpha'^2}{4} \ln\left(\frac{A}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \text{const}, \tag{12}$$

where the expansion coefficients $c_n \propto \alpha'^{2(n+1)}$ can always be

computed to any desired order of accuracy[38], may be also written in the general case as a Laurent series in terms of α_{r_+}

$$S_{GUP} = \sigma\alpha_{r_+}^{-1} - \frac{\pi\alpha'^2}{4}\ln(\sigma\alpha_{r_+}^{-1}) + \sum_{n=1}^{\infty}(c_n\sigma^{-n})\alpha_{r_+}^{n} + \text{const.} \quad (13)$$

In what follows the representation in terms of the deformation parameter α is referred to as **α-representation**.

2. Gravitational Thermodynamics with EUP

Let us consider QFT with EUP[14]. In this case we obtain QFT with p_{min}. Obviously, there is no minimal length l_{min} in QFT with EUP whatsoever but we assume that QFT with GUP is valid. At the present time for such an assumption we can find solid argumentation[6–9]. As will be shown later, in this case the fundamental quantities may be also expressed in terms of α. Hereinafter we use a small dimensionless parameter

$$\alpha_{\tilde{l}} = \frac{l_{or}^2}{\tilde{l}^2}, \quad (14)$$

where $l_{or} \equiv l_{original} = 2\alpha' l_p$, α' –dimensionless constant on the order of unity from GUP (2), and it is suggested that

$$2l_{or} \le \tilde{l} \quad ,i.e \quad 0 < \alpha_{\tilde{l}} \le 1/4. \quad (15)$$

Similar to the previous Section, it is convenient to use the principal results of[14] (sections 3,4). Then EUP in (A)dS space takes the form

$$\Delta x_i \Delta p_j \ge \hbar\delta_{ij}[1 + \beta^2 \frac{(\Delta x_i)^2}{l^2}], \quad (16)$$

where l is the characteristic, large length scale $l \gg l_p$ and β is a dimensionless real constant on the order of unity[14]. From EUP there is an absolute minimum in the momentum uncertainty**:**

$$\Delta p_i \ge \frac{2\hbar\beta}{l}, \quad (17)$$

and EUP (16) may be rewritten as

$$\Delta x_i \Delta p_j \ge \hbar\delta_{ij}[1 + \beta^2 \frac{(\Delta x_i)^2}{l_{or}^2}\frac{l_{or}^2}{l^2}] = \hbar\delta_{ij}[1 + \beta^2\alpha_l\alpha_{\Delta x_i}^{-1}]. \quad (18)$$

Considering that in a theory with fixed $l \gg l_p$

$$\alpha_l = const \ll 1, \quad (19)$$

(16),(18) may be written as

$$\Delta x_i \Delta p_j \ge \hbar\delta_{ij}[1 + \beta^2\alpha_l\alpha_{\Delta x_i}^{-1}] = \hbar\delta_{ij}[1 + \widetilde{\beta}^2\alpha_{\Delta x_i}^{-1}], \quad (20)$$

where β is redetermined as

$$\beta \mapsto \widetilde{\beta} = \sqrt{\alpha_l}\beta. \quad (21)$$

However, in this case β may be left as it is, whereas α may be redetermined because $\alpha_{\Delta x_i}^{-1}$ in (18),(20) is not a small parameter. In consequence we can redeterminate α as

$$\widetilde{\alpha}_{\Delta x_i} = \alpha_l\alpha_{\Delta x_i}^{-1}, \quad (22)$$

where $\widetilde{\alpha}_{\Delta x_i}$ is now a small parameter.

Owing to such a duality, EUP (16), (18) may be rewritten in terms of a new small parameter α similar to α as follows:

$$\Delta x_i \Delta p_j \ge \hbar\delta_{ij}[1 + \beta^2\widetilde{\alpha}_{\Delta x_i}]. \quad (23)$$

Then in analogy with [14] (Section 3), for Hawking temperature of the d-dimensional Schwarzshild-AdS black hole with the metric function we have

$$N^2 = 1 + \frac{r^2}{l_{AdS}^2} - \frac{16\pi GM}{(d-2)\Omega_{d-2}r^{d-3}} \quad (24)$$

in the metric of(5) and the cosmological constant $\Lambda = -(d-1)(d-2)/2l_{AdS}^2$.

Therewith the α-representation of the Hawking temperature T_{EUP} [14] (formula (15)) takes the form

$$T_{EUP(AdS)} = (\frac{d-3}{4\pi})\frac{\hbar}{r_+^2}[1 + \frac{(d-1)}{(d-3)}\alpha_{r_+}^{-1}\alpha_{l_{AdS}}] = (\frac{d-3}{4\pi})\frac{\hbar}{r_+^2}[1 + \frac{(d-1)}{(d-3)}\widetilde{\alpha}_{r_+}]. \quad (25)$$

In the same way we can easily obtain the α- representation of the Hawking temperature for a Schwarzshild-AdS black hole and for a combined case ((formula (28) from the[14])) of GUP and EUP – (GEUP)

$$T_{GEUP(AdS)} = 2(\frac{d-3}{4\pi})\frac{\hbar}{r_+}\alpha_{r_+}^{-1}\left[1 - \sqrt{1 - \alpha_{r_+}[1 + \frac{\alpha_{l_{AdS}}(d-1)}{(d-3)}\alpha_{r_+}^{-1}]}\right] = 2(\frac{d-3}{4\pi})\frac{\hbar}{r_+}\alpha_{r_+}^{-1}\left[1 - \sqrt{\frac{d-3-\alpha_{l_{AdS}}(d-1)}{(d-3)} - \alpha_{r_+}}\right], \quad (26)$$

i.e. in the general case we get a Laurent series from α.

Similarly, we can obtain the α-representation for the corresponding value of $T_{GEUP(dS)}$ ((formula (32) from[14]) in the de Sitter (dS) space by the substitution $l_{AdS}^2 \to -l_{dS}^2$.

Note that, as it has been indicated in[33, 34], $\alpha_{r_+}^{-1}$ has one more interesting feature

$$\alpha_{r_+}^{-1} \sim r_+^{\,2}/l_p^2 \sim S_{BH}. \quad (27)$$

Here S_{BH} is the Bekenstein-Hawking semiclassical black hole entropy with the characteristic linear size r_+. For example, in the spherically symmetric case $r_+ = R$ - radius of the corresponding sphere with the surface area A, and

$$A = 4\pi r_+^2, S_{BH} = A/4l_p^2 = \frac{\pi}{4\alpha'^2}\alpha_{r_+}^{-1}. \quad (28)$$

In [15] GUP and EUP are combined by the principle called the Symmetric Generalized Uncertainty Principle (SGUP):

$$\Delta x\Delta p \ge \hbar\left(1 + \frac{(\Delta x)^2}{L^2} + l^2\frac{(\Delta p)^2}{\hbar^2}\right), \quad (29)$$

where $l \ll L$ and l defines the limit of the UV-cutoff (not being such up to a constant factor as in the case of GUP).Then a minimal length is determined as

$$\Delta x_{\min} = 2l/\sqrt{1 - 4l^2/L^2},$$

whereas L defines the limit for IR-cutoff i. e. we have a minimum momentum

$$\Delta p_{\min} = 2\hbar/(L\sqrt{1 - 4l^2/L^2}).$$

And using the Euclidian action formalism by Gibbons and Hawking[42], in[15] the corresponding correction of the Hawking temperature for an ordinary(not A(dS)) Schwarzshild-black hole is computed. This correction is given as T_{SGUP}. In the notation of this work

$$\Delta x_{\min} = 2l / \sqrt{1-4\alpha_l^{-1}\alpha_L} = 2l / \sqrt{1-4\widetilde{\alpha_l}},$$

where $\widetilde{\alpha_l}$ –small parameter introduced in conformity with (22). We can easily obtain the α-representation for T_{SGUP} that is completely similar to the α-representation of $T_{GEUP(AdS)}$.

It should be noted that in the realistic theories $l \sim l_p$, and it is obvious that $(\sqrt{1-4l^2/L^2}) \approx 1$. Thus, $\Delta x_{\min} \approx 2l \sim 2l_p$ and hence in this case we get a minimal length that is much the same (to within α) as in the case of GUP. It is seen that, with due regard for the requirement $l \ll L$, $\Delta p_{\min}$ is derived close (to within β) to $\Delta p_{\min}$ (17) in a theory with EUP.

The question arises as to what for all these manipulations with writing and rewriting of the already derived expressions in the α-representation are necessary.

1. Owing to this procedure, we can draw the conclusion that all the quantities within the scope of the stated problem are dependent on one and the same deformation parameter α that is small, dimensionless (discrete in the case of GUP), and varying over the given interval. And, provided the infrared cutoff l is defined, we have

$$\alpha_l = \alpha_{min} = l_{or}^2 / l^2 \le \alpha \le 1/4 \quad \text{and} \quad l_{or} \equiv l_{original} \sim l_p .$$

If we primordially consider a theory with GUP only, then $l_{or} \equiv l_{min}$. But in the arbitrary case it is required that $l_{or} = 2\alpha' l_p$, where α is a certain dimensionless constant on the order of unity.

The property of discreteness is retained for α in the cases when only GUP (without generalizations)is valid because in this case the length seems to be quantized, the lengths being considered from $2l_{min}$ rather than from $l_{or} = l_{min}$ as a singularity arises otherwise[23–30].

2. Actually, all the quantities may be represented as a Laurent series in terms of α, and a solution of the problem at hand may be understood as finding of the members in this series.

3. When the problem has separate solutions for the cases including the UV- and IR-cutoffs, we can consider expansion in each of the cases in terms of their own small parameters: α in the case of UV-cutoff and α in the case of IR-cutoff, where α is a duality of α

$$\widetilde{\alpha}_{\tilde{l}} = \alpha_l \alpha_{\tilde{l}}^{-1},$$

l determines, to within a factor on the order of unity, the characteristic system's size, and $l \gg l_p$.

3. The Cosmological Constant Problem and QFT with GUP and SGUP

In this section it is assumed that Λ may be varying in time. Generally speaking, Λ is referred to as a constant just because it is such in the equations, where it occurs: Einstein equations[43]. But in the last few years the dominating point of view has been that Λ is actually a dynamic quantity, now weakly dependent on time[44–46]. It is assumed therewith that, despite the present-day smallness of Λ or even its equality to zero, nothing points to the fact that this situation was characteristics for the early Universe as well. Some recent results[47–50] are rather important pointing to a potentially dynamic character of Λ. Specifically, of great interest is the Uncertainty Principle derived in these works for the pair of conjugate variables (Λ, V):

$$\Delta\Lambda \, \Delta V \sim \hbar, \tag{30}$$

where Λ is the vacuum energy density (cosmological constant). It is a dynamic value fluctuating around zero; V is the space-time volume. Here the volume of space-time V results from the Einstein-Hilbert action S_{EH} [48]:

$$\Lambda \int d^4x \sqrt{-g} = \Lambda V \tag{31}$$

where (31) is the term in the S_{EH}. In this case the notion of conjugation is well-defined, but approximate, as implied by the expansion about the static Fubini–Study metric (Section 6.1 of [47]). Unfortunately, in the proof per se (30), relying on the procedure with a non-linear and non-local Wheeler–de-Witt-like equation of the background-independent Matrix theory, some unconvincing arguments are used, making it insufficiently rigorous (Appendix 3 of[47]). But, without doubt, this proof has a significant result, though failing to clear up the situation.

In [33, 34, 51] the Heisenberg Uncertainty Relation for the pair (Λ, V) (30) has been generalized to GUP

$$\Delta V \ge \frac{\hbar}{\Delta\Lambda} + \alpha'_\Lambda t_p^2 \overline{V}_p^2 \frac{\Delta\Lambda}{\hbar} \tag{32}$$

or that is the same

$$\Delta V \Delta\Lambda \ge \hbar(1 + \alpha'_\Lambda t_p^2 \overline{V}_p^2 \frac{(\Delta\Lambda)^2}{\hbar^2}). \tag{33}$$

where α'_Λ is a new constant and $\overline{V}_p = l_p^3$.

In the case of UV - limit: $t \to t_{min}$, $\Delta\Lambda$ becomes significant

$$\begin{aligned} &\lim_{t \to t_{min}} \overline{V} = \overline{V}_{min} \sim \overline{V}_p = l_p^3; \\ &\lim_{t \to t_{min}} V = V_{min} \sim V_p = l_p^3 t_p, \end{aligned} \tag{34}$$

where V – spatial part of V

The existence of $V_{min} \sim V_p$ directly follows from GUP for the pair (p,x) (2) and GUP for the pair (E,t) [24,30] as well as from solutions of the quadratic inequalities(32),(33).

So, (32) is nothing else but

$$\Delta V \ge \frac{\hbar}{\Delta\Lambda} + \alpha'_\Lambda V_p^2 \frac{\Delta\Lambda}{\hbar}. \tag{35}$$

And in the case of UV – cutoff we have

$$\lim_{t \to t_{min}} \Lambda \equiv \Lambda_{UV} \sim \Lambda_p \equiv \hbar / V_p = E_p / \overline{V}_p. \tag{36}$$

It is easily seen that in this case $\Lambda_{UV} \sim m_p^4$, in agreement with the value obtained using a standard (i.e. without super-symmetry and the like) quantum field theory[52,53]. Despite the fact that Λ at Planck's scales (referred to as Λ_{UV}) is also a dynamic quantity, it is not directly related to the familiar Λ because the latter, as opposed to the first one,

is derived from Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \left(-\Lambda g_{\mu\nu} + T_{\mu\nu} \right). \quad (37)$$

However, Einstein's equations (37) are not valid at the Planck scales and hence Λ_{UV} may be considered as some high-energy generalization (deformation) of the conventional cosmological constant in the low-energy limit.

The problem is whether a correct generalization of GUP for the pair (Λ, V) (33) to the Symmetric Generalized Uncertainty Principle (SGUP) of the form given by (29) is possible. If the answer is positive, a theory also includes Λ_{min} that may be referred to as Λ_{IR} in similarity with Λ_{UV}. Then, similar to T (29), an additional term defining the IR-cutoff must be of the form

$$\Omega_{IR} = \frac{(\Delta V)^2}{\widetilde{V}^2}, \quad (38)$$

where $\widetilde{V}$ - certain space-time volume effectively specifying the IR-limit of the observable part of the Universe with the spatial part $\widetilde{\widetilde{V}} \sim L^3$; L – radius of the observable part of the Universe. Now it is known that $L \approx 10^{28}$. Clearly, the introduction of an additional term of the form (38) into the right-hand side of (33) leads to $\Lambda_{IR} \ll \Lambda_{UV}$ and might lead to the value of Λ close to the experimental value Λ_{exp} [54]. Note that the Holographic Principle[55–61] used to the Universe as a whole[61] gives Λ_{exp} [60]. In[62,63,33,34] it has been demonstrated that the α –representation (α –deformation) of QFT with GUP plays a significant role. In particular, consider

$$\Lambda_{exp} \approx \alpha_L \Lambda_{UV}, \quad (39)$$

where $L \approx 10^{28}$.(39) is like (22). But the Holographic Principle imposes strict restrictions on the number of degrees of freedom in the Universe, and hence for us it is important to study the inferences of introducing the additional term of the form (38) in (33).

4. GUP, EUP, and General Relativity Deformation

In this Section we use the previously obtained results for some cases of high- energy and low-energy deformation of GR. Specifically, we demonstrate that in the cases when the Thermodynamics Approach[16–21] is applicable to the General Relativity the deformation of GR with GUP and EUP may be a natural result of the α-representation.

4.1. α –Representation of Einstein's Equations for Space with Horizon

Let us consider α-representation and high energy α-deformation of the Einstein's field equations for the specific cases of horizon spaces (the point (c) of Section 4). In so doing the results of the survey work ([20] p.p.41,42)are used. Then, specifically, for a static, spherically symmetric horizon in space-time described by the metric

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \quad (40)$$

the horizon location will be given by simple zero of the function $f(r)$, at $r = a$.

It is known that for horizon spaces one can introduce the temperature that can be identified with an analytic continuation to imaginary time. In the case under consideration ([20], eq.(116))

$$k_B T = \frac{\hbar c f'(a)}{4\pi}. \quad (41)$$

Therewith, the condition $f(a) = 0$ and $f'(a) \neq 0$ must be fulfilled.

Then at the horizon $r = a$ Einstein's field equations

$$\frac{c^4}{G}\left[\frac{1}{2} f'(a) a - \frac{1}{2}\right] = 4\pi P a^2 \quad (42)$$

may be written as the thermodynamic identity ([20] formula (119))

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T} \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4} 4\pi a^2\right)}_{dS} \underbrace{-\frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{Pd\left(\frac{4\pi}{3} a^3\right)}_{PdV} \quad (43)$$

where $P = T_r^r$ is the trace of the momentum-energy tensor and radial pressure. In the last equation da arises in the infinitesimal consideration of Einstein's equations when studying two horizons distinguished by this infinitesimal quantity a and $a + da$ ([20] formula (118)).

Now we consider (43) in a new notation, expressing a in terms of the corresponding deformation parameter α. Hereinafter in this Section we write α instead of α_a as we consider the same a. Then we have

$$a = l_{min} \alpha^{-1/2}. \quad (44)$$

Therefore,

$$f'(a) = -2 l_{min}^{-1} \alpha^{3/2} f'(\alpha). \quad (45)$$

Substituting this into (42) or into (43), we obtain in the considered case of Einstein's equations in the " a –representation" the following:

$$\frac{c^4}{G}(-\alpha f'(\alpha) - \frac{1}{2}) = 4\pi P \alpha^{-1} l_{min}^2. \quad (46)$$

Multiplying the left- and right-hand sides of the last equation by a, we get

$$\frac{c^4}{G}(-\alpha^2 f'(\alpha) - \frac{1}{2}\alpha) = 4\pi P l_{min}^2. \quad (47)$$

But since usually $l_{min} \sim l_p$ (that is just the case if the Generalized Uncertainty Principle (GUP) is satisfied), we have $l_{min}^2 \sim l_p^2 = G\hbar / c^3$. When selecting a system of units, where $\hbar = c = 1$, we arrive at $l_{min} \sim l_p = \sqrt{G}$, and then (46) is of the form

$$-\alpha^2 f'(\alpha) - \frac{1}{2}\alpha = 4\pi P \vartheta^2 G^2, \quad (48)$$

where $\vartheta = l_{min} / l_p$. L. h. s. of (48) is dependent on a. Because of this, r. h. s. of (48) must be dependent on a as well,

i. e. $P = P(\alpha)$.

Analysis of a -Representation of Einstein's Equations

Now let us get back to (43). In[20] the low-energy case has been considered, for which ([20] p.42 formula (120))

$$S = \frac{1}{4l_p^2}(4\pi a^2) = \frac{1}{4}\frac{A_H}{l_p^2}; \quad E = \frac{c^4}{2G}a = \frac{c^4}{G}\left(\frac{A_H}{16\pi}\right)^{1/2}, \quad (49)$$

where A_H is the horizon area. In our notation (49) may be rewritten as

$$S = \frac{1}{4}\pi\alpha^{-1}; \quad E = \frac{c^4}{2G}a = \frac{c^4}{G}\left(\frac{A_H}{16\pi}\right)^{1/2} = \frac{\vartheta}{2\sqrt{G}}\alpha^{1/2}. \quad (50)$$

We proceed to two entirely different cases: low energy (LE) case and high energy (HE) case. In our notation these are respectively given by

$$\text{A)}\ \alpha \to 0\ \text{(LE)},\ \text{B)}\ \alpha \to 1/4\ \text{(HE)}$$

C) a complies with the familiar scales and energies.

The case of C) is of no particular importance as it may be considered within the scope of the conventional General Relativity.

Indeed, in point A) $\alpha \to 0$ is not actually an exact limit as a real scale of the Universe (Infrared (IR)-cutoff $l_{max} \approx 10^{28} cm$), and then

$$\alpha_{min} \sim l_p^2/l_{max}^2 \approx 10^{-122}$$

In this way A) is replaced by A1) $\alpha \to \alpha_{min}$. In any case at low energies the second term in the left-hand side (48) may be neglected in the infrared limit. Consequently, at low energies (48) is written as

$$-\alpha^2 f'(\alpha) = 4\pi P(\alpha)\vartheta^2 G^2. \quad (51)$$

Solution of the corresponding Einstein equation – finding of the function $f(\alpha) = f[P(\alpha)]$ satisfying(51). In this case formulae (49) are valid as at low energies a semiclassical approximation is true. But from (51)it follows that

$$f(\alpha) = -4\pi\vartheta^2 G^2 \int \frac{P(\alpha)}{\alpha^2} d\alpha. \quad (52)$$

On the contrary, knowing $f(\alpha)$, we can obtain $P(\alpha) = T_r^r$.

Possible High Energy a -Deformation of General Relativity

Let us consider the high-energy case B). Here two variants are possible.

I. First variant.

In this case it is assumed that in the high-energy (Ultraviolet (UV))limit the thermodynamic identity (43) is retained but now all the quantities involved in this identity become a -deformed. This means that they appear in the a -representation with quantum corrections and are considered at high values of the parameter a , i.e. at a close to 1/4. In particular, the temperature T from equation (43) is changed by T_{GUP} (9), the entropy S from the same equation given by semiclassical formula (49) is changed by S_{GUP} (13), and so forth:

$$E \mapsto E_{GUP}, V \mapsto V_{GUP}.$$

Then the high-energy a -deformation of equation (43) takes the form

$$k_B T_{GUP}(\alpha) dS_{GUP}(\alpha) - dE_{GUP}(\alpha) = P(\alpha) dV_{GUP}(\alpha). \quad (53)$$

Substituting into (53) the corresponding quantities $T_{GUP}(\alpha), S_{GUP}(\alpha), E_{GUP}(\alpha), V_{GUP}(\alpha), P(\alpha)$ and expanding them into a Laurent series in terms of a , close to high values of a , specifically close to $\alpha = 1/4$, we can derive a solution for the high energy a -deformation of general relativity (53) as a function of $P(\alpha)$. As this takes place, provided at high energies the generalization of (43) to (53) is possible, we can have the high-energy a -deformation of the metric. Actually, as from (43) it follows that

$$f'(a) = \frac{4\pi k_B}{\hbar c} T = 4\pi k_B T \quad (54)$$

(considering that we have assumed $\hbar = c = 1$), we get

$$f'_{GUP}(a) = 4\pi k_B T_{GUP}(\alpha). \quad (55)$$

L.h.s. of (55) is directly obtained in the a -representation. This means that, when $f' \sim T$, we have $f'_{GUP} \sim T_{GUP}$ with the same factor of proportionality. In this case the function f_{GUP} determining the high-energy a -deformation of the spherically symmetric metric may be in fact derived by the expansion of T_{GUP} , that is known from (9), into a Laurent series in terms of a close to high values of a (specifically close to $\alpha = 1/4$), and by the subsequent integration.

It might be well to remark on the following.

1.1 As on going to high energies we use (GUP), ϑ from equation (48)is expressed in terms of a –dimensionless constant from GUP (2): $\vartheta = 2\alpha'$.

1.2 Of course, in all the formulae including l_p this quantity must be changed by $G^{1/2}$ and hence l_{min} by $\vartheta G^{1/2} = 2\alpha' G^{1/2}$.

1.3 As noted in the end of subsection 6.1, and in this case also knowing all the high-energy deformed quantities $T_{GUP}(\alpha), S_{GUP}(\alpha), E_{GUP}(\alpha), V_{GUP}(\alpha)$, we can find $P(\alpha)$ at a close to 1/4.

1.4 Here it is implicitly understood that the Ultraviolet limit of Einstein's equations is independent of the starting horizon space. This assumption is quite reasonable. Because of this, we use the well-known formulae for the modification of thermodynamics and statistical mechanics of black holes in the presence of GUP[36–39,14,15].

1.5 The use of the thermodynamic identity (53) for the description of the high energy deformation in General Relativity implies that on going to the UV-limit of Einstein's equations for horizon spaces in the thermodynamic representation (consideration) we are trying to remain within the scope of equilibrium statistical mechanics[64] (equilibrium thermodynamics)[65]. However, such an assumption seems to be too strong. But some grounds to think so may be found as well. Among other things, of interest is the result from[36] that GUP may prevent black holes from their total evaporation. In this case the Planck's remnants of black holes will be stable, and when they are considered, in some approximation the equilibrium thermodynamics should be valid. At the same time, by author's opinion these arguments are rather weak to think that the quantum gravitational effects in this

context have been described only within the scope of equilibrium thermodynamics[65].

II. Second variant.

According to the remark of 4.1.5, it is assumed that the interpretation of Einstein's equations as a thermodynamic identity (43) is not retained on going to high energies (UV–limit), i.e. at $\alpha \to 1/4$, and the situation is adequately described exclusively by non-equilibrium thermodynamics [65,66]. Naturally, the question arises: which of the additional terms introduced in (43) at high energies may be leading to such a description?

In the[51,33] it has been shown that in case the cosmological term Λ is a dynamic quantity, it is small at low energies and may be sufficiently large at high energies. In the right-hand side of (48) in the a –representation the additional term $GF(\Lambda(\alpha))$ is introduced:

$$-\alpha^2 f'(\alpha)-\frac{1}{2}\alpha = 4\pi P\vartheta^2 G^2 - GF(\Lambda(\alpha)), \qquad (56)$$

where in terms of $F(\Lambda(\alpha))$ we denote the term including $\Lambda(\alpha)$ as a factor. Then its inclusion in the low-energy case (42)(or in the a -representation (48)) has actually no effect on the thermodynamic identity (43)validity, and consideration within the scope of equilibrium thermodynamics still holds true. It is well known that this is not the case at high energies as the Λ -term may contribute significantly to make the "process" non-equilibrium in the end[65,66].

Is this the only cause for violation of the thermodynamic identity (43) as an interpretation of the high-energy generalization of Einstein's equations? Further investigations are required to answer this question.

4.2. α –Representation for Friedmann Equations with GUP and EUP

Thermodynamic interpretation of Section 4 has been also developed for Friedmann Equations (FEs) of the Friedmann-Robertson-Walker (FRW) Universe in[21]. In the process it is taken into consideration that in the FRW space-time, where the metric is given by the formula

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{n-1}^2\right), \qquad (57)$$

and $d\Omega_{n-1}^2$ denotes a line element of the $(n-1)$-dimensional unit sphere, a is the scale factor, k is the spatial curvature constant, there is a dynamic apparent horizon, the radius of which is as follows:

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \qquad (58)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter.

FEs in[21] have been derived proceeding from the assumption that apparent horizon is endowed with the associated entropy and temperature such the event horizon in the black hole case

$$S = \frac{A}{4G}, \quad T = \frac{1}{2\pi\tilde{r}_A}, \qquad (59)$$

and from the validity of the first low of thermodynamics, we have

$$dE = TdS. \qquad (60)$$

In[67] with the use of this thermodynamic interpretation of FEs the modifications of GUP and EUP (or more precisely the GUP and EUP deformations) of FEs have been obtained. It is clear that these (GUP and EUP)–deformed FEs may be written in the form of the a –representation. For simplicity, let us consider the case $n=3$.

Then for GUP the formula (26) from[67] takes the form

$$(\dot{H} - \frac{k}{a})[1 + \pi\alpha'^2 l_p^2 \frac{1}{A} + 2(\pi\alpha'^2 l_p^2)^2 \frac{1}{A^2} + \sum_{d=3} c_d (4\pi\alpha'^2 l_p^2)^{2d} \frac{1}{A^d}] = -4\pi G(\rho + p), \qquad (61)$$

whereas in the a –representation its form is more elegant

$$(\dot{H} - \frac{k}{a})[1 + \frac{1}{16}\alpha_{\tilde{r}_A} + \frac{1}{32}\alpha_{\tilde{r}_A}^2 + \sum_{d=3} \frac{c_d}{4^d}\alpha_{\tilde{r}_A}^d] = -4\pi G(\rho + p),. \qquad (62)$$

Also, more elegant is a –representation of the second Friedmann Equation (formula (27) from [67])

$$\frac{8\pi G^2}{3}\rho = \frac{\alpha_{\tilde{r}_A}}{4\pi\alpha'^2}[\pi + \frac{1}{32}\alpha_{\tilde{r}_A} + \frac{1}{96}\alpha_{\tilde{r}_A}^2 \backslash + \sum_{d=3} \frac{c_d}{4^d(d+1)}\alpha_{\tilde{r}_A}^d], \qquad (63)$$

with the assumption that $\hbar = c = 1$.

It is obvious that therewith familiar FEs appear at low energies, i.e. at $\alpha_{\tilde{r}_A} \ll 1/4$.

In the nontrivial high-energy case one can obtain the solution for FE and, in particular ρ, H, p as a series in terms of a close to 1/4.

In the case of EUP the a –representation of the deformed FE[67] seems to be even simpler. Specifically, using (18) – (23), one can derive deformed first Friedmann equation of the form

$$\begin{aligned}(\dot{H} - \frac{k}{a^2})(1 + \frac{\beta^2}{\pi l^2}A) &= (\dot{H} - \frac{k}{a^2})(1 + \frac{\beta^2}{\pi l^2}\frac{4\pi\tilde{r}_A^2}{l_{or}^2}l_{or}^2) \\ &= (\dot{H} - \frac{k}{a^2})(1 + 4\beta^2\alpha_{\tilde{r}_A}^{-1}\alpha_{l_{or}}) \\ &= (\dot{H} - \frac{k}{a^2})(1 + 4\beta^2\tilde{\alpha}_{\tilde{r}_A}) = -4\pi G(\rho + p),\end{aligned} \qquad (64)$$

where, as expected, the deformation parameter $\tilde{\alpha}_{\tilde{r}A}$ is small.

In a similar way we can obtain the a –representation of the EUP-deformation for the second Friedmann equation.

5. Some Comments and Problems of Interest

In this Section some comments are given and some problems are stated.

C1. The Laurent series expansion in terms of a is asymmetric for UV and IR cutoffs. Indeed, as in the general case the variability domain $0 < \alpha \le 1/4$, in the UV-cutoff

when $\alpha \approx 1/4$ the contribution is made by a-terms both with positive and with negative powers, while in the IR-cutoff ($\alpha \ll 1/4$) only the a-terms with negative powers will be significant.

C2. The external constant a in the cases, where $l_{or} \neq l_{min}$ (EUP or SGUP is the case), is not found in the final expressions, being reduced due to the substitution of (22).

Several questions remain to be answered and necessitate further investigations.

Q1) How far the a-representation may be extended for the General Relativity? As shown in this work, such a representation exists for the General Relativity at High and Low Energies when the Thermodynamic Approach[16–21] is applicable or, that is the same, the Thermodynamic Interpretation is the case. It is interesting whether the extension of the a-representation to the general case both at High and Low Energies is possible. The problem is whether, in some or other way, the general case may be reduced to the well-known ones.

Q2) Considering Q1), for High Energy the problem is whether there is an effective description of the space-time foam [68–70] in terms of a. The results of[71] suggest that such a description should be existent.

Q3) Proceeding from the results of E. Verlinde[72], the problem is whether the High-Energy deformation of the Entropic Force is obtainable. Provided the answer of Q1) positive, the problem concerns the form of this deformation in terms of a: we must find its a-representation.

Note that the notion of Entropic Force, however, without the introduction of the term per se has been proposed by T. Padmanabhan in Conclusion of his paper[73] earlier than by E. Verlinde.

6. Conclusions

The essence of the principal result obtained in this work may be stated as follows.

In the cases when Einstein equations have an explicit thermodynamic interpretation, their limiting transition to UV and IR cutoffs may be described within the scope of the same dimensionless deformation parameter that is naturally introduced in a quantum field theory with the fundamental length.

In the case the problems stated in the previous Section will be solved positively, the small dimensionless discrete parameter a must be at once introduced in High-Energy Thermodynamics and Gravity, without its appearance in the low-energy limit at the scales under study. At the same time, at large scales GR has not been subjected to verification too [74]. The availability of Dark Matter and Dark Energy is a strong motivation for the IR-modification of GR [75–77]. The deformation of the General Relativity due to EUP seems to be one of the IR-modifications of Gravity possible. In this case an analysis of such a deformation in terms of the parameter a, of the corresponding variability domain, and the like may be important for studies of the IR-modified (IR-deformed) General Relativity.

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