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MATHEMATICAL LIFE

Vladimir Petrovich Platonov (on his 70th birthday)

On 1 December 2009 Vladimir Petrovich Platonov, academician of the Russian Academy of Sciences and of the National Academy of Sciences of Belarus, turned 70.

V. P. Platonov was born in the village Staiki of the Orshan district of the Vitebsk region. In 1961 he graduated with distinction from Belarus State University. In 1963 he defended his Ph.D. dissertation, and in 1966 his D.Sc. dissertation. He has been a professor since 1968. In 1969 he was elected a corresponding member and in 1972 an academician of the Academy of Sciences of the BSSR, and in 1987 an academician of the Academy of Sciences of USSR. He is the author of 161 research publications.

From 1963 to 1971 he worked as a docent, professor, and head of the Department of Algebra at Belarus State University. From 1971 to 1995 he headed the Laboratory of Algebraic Geometry and Topology, was the director from 1977 to 1992, and was a principal research fellow at the Institute of Mathematics of the Academy of Sciences of the BSSR. From 1987 to 1992 he served as the president of the Academy of Sciences of the BSSR. He worked at universities and research centres in the USA, Canada, and Germany in the period from 1992 to 2004. At present he is a principal research fellow at the Research Institute of System Research of the Russian Academy of Sciences.

Platonov is an internationally known mathematician, an expert in the area of algebra, algebraic geometry, and number theory. He has solved a number of well-known research problems which had long withstood the efforts of many mathematicians. Among these is the strong approximation problem in algebraic groups, the Tannaka–Artin problem and the Kneser–Tits conjecture on the structure of isotropic algebraic groups, the problem of rationality of group algebraic varieties, and others. He has made major contributions to the arithmetic theory of algebraic groups — a direction of research that lies at the junction of group theory, algebraic geometry, and algebraic number theory.

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One of the first major achievements of Platonov was the determination and detailed study of basic classes of locally compact topological groups from both the group-theoretic and the topological viewpoint. His interests were then connected for a long time with the study of algebraic groups. He investigated the structure of linear algebraic groups and their automorphisms, and discovered new applications of the theory of algebraic groups to the theory of linear groups. The use of the methods of the theory of algebraic groups enabled him to determine the structure of periodic and finitely generated linear groups.

Major achievements are due to Platonov in the arithmetic theory of algebraic groups. He obtained a complete solution of the strong approximation problem in algebraic groups over number fields (posed in 1937), solved the genus problem for arithmetic groups and integral representations of finite groups, classified the maximal arithmetic subgroups, and obtained together with A. S. Rapinchuk and A. A. Bondarenko a description of the class numbers and class groups of algebraic groups. The problems and conjectures formulated by Platonov in his talk at the International Congress of Mathematicians in Vancouver served as an important impetus for further investigations. Among these conjectures was a new local-global principle describing the projective simplicity of groups of rational points of algebraic groups over global fields.

A special role among the mathematical achievements of Platonov is occupied by the cycle of papers published in the mid-1970s in which a number of fundamental problems were solved in the theory of algebraic groups and algebraic K-theory. This cycle began with a negative solution of the old Tannaka–Artin problem, which demonstrated that the reduced Whitehead group of a finite-dimensional central simple algebra is non-trivial in the general case (this also refuted the Kneser–Tits conjecture on the structure of simple simply connected isotropic algebraic groups). In subsequent papers he constructed a reduced K-theory containing methods for computing the reduced Whitehead group and establishing connections with other key properties of algebraic groups, such as the birational structure of the corresponding group varieties and the weak approximation property. The intensive development of reduced K-theory in the mid-1970s was summarized in J. Tits' talk at the Bourbaki seminar in 1977 and in Platonov's talk at the 1978 International Congress of Mathematicians in Helsinki.

In his further creative work Platonov has returned several times to various aspects of the problem of the structure of finite-dimensional simple algebras and the classical groups connected with them. Together with V. I. Yanchevskii he established the non-triviality of the reduced unitary Whitehead group for finite-dimensional central simple algebras with an involution of the second kind. The joint work with Yanchevskii also resulted in the construction of a theory of finite-dimensional valued division algebras which is based on the classification of Henselian division algebras (the completion of the investigations of local division algebras that had been actively going on from the beginning of 1930s). In papers of Platonov and Rapinchuk they studied the multiplicative arithmetic of finite-dimensional division algebras over number fields, and as a consequence obtained results on the normal structure of anisotropic algebraic groups of type $\mathrm{SL}(1, D)$.

In subsequent years Platonov's attention was drawn to a circle of questions connected with the study of linear representations and profinite completions of finitely generated groups. Jointly with O. I. Tavgen' he constructed a counterexample to Grothendieck's problem on profinite completions of residually finite groups. In the study of varieties of representations Platonov and V. V. Benyash-Krivets obtained a complete solution to the problem of finite generation of character rings of linear representations for finitely generated groups. At the end of the 1980s Platonov stated the conjecture on the arithmeticity of linear groups of finite representation type. For a number of years this conjecture stimulated research in this direction, and a special conference was devoted to its discussion: "Representation varieties of finitely generated groups and Platonov's conjecture", organized by the University of Oklahoma (USA, 1992).

In 1991 Nauka published the monograph *Algebraic groups and number theory*, the first systematic exposition of the theory lying at the junction of group theory, algebraic geometry, and number theory (an English translation was published by Academic Press in 1993, and Cambridge University Press is presently working on the second edition, in 2 volumes).

In January of 1992 Platonov resigned from the post of president of the Academy of Sciences of Belarus, with the aim of concentrating on his research work. From 1992 to 2004 he was at the Institute for Advanced Study (Princeton, USA), the Max Planck Mathematical Institute (Bonn, Germany), and the Universities of Michigan (USA), Waterloo (Canada), Bielefeld (Germany), and Düsseldorf (Germany).

In 1997–1999 Platonov together with F. Grunewald produced a major cycle of papers devoted to the study of the properties of arithmetic groups, in particular, to the study of finite extensions of arithmetic groups and to the generalization of rigidity theorems to arithmetic subgroups of algebraic groups with a radical. One of the surprising results obtained in their papers was that a finite extension of an arithmetic group is not always an arithmetic group. Among other results, we should mention a proof of the finiteness of the number of conjugacy classes of finite subgroups in finite extensions of arithmetic groups and a complete solution of the arithmeticity problem for polycyclic groups.

In 2003 Platonov and Grunewald studied lattices in Lie groups with finitely many connected components and obtained a criterion for a finite extension of a lattice in a Lie group to be again a lattice. For example, it was proved that a finite extension of a cocompact lattice is again a lattice. At the same time it was proved that for every non-cocompact lattice in $\mathrm{SL}(2,\mathbb{R})$ there exists a finite extension that is not a lattice.

In papers with V. I. Chernousov, Platonov solved the rationality problem for semisimple group varieties over local number fields. Jointly with D. Djoković he developed general approaches, based on the theory of algebraic groups, to the so-called invariance problem for linear transformations and obtained a complete solution of it in a number of cases which had long resisted the efforts of others. And jointly with Rapinchuk, he developed a new approach to the congruence problem based on an analysis of abstract (in particular, combinatorial) properties of arithmetic groups. Using this approach, he and B. Sury proved a conjecture of A. Lubotzky on the congruence subgroup property for arithmetic groups with an adelic profinite completion.

Since 2005 Platonov has been a principal research fellow at the Research Institute of System Research of the Russian Academy of Sciences. In 2006–2009 he developed new methods for investigating S-units in function fields with a finite field of constants, and he (together with Benyash-Krivets) constructed the fastest algorithms for computing S-units in elliptic and hyperelliptic fields. This is the most significant advance since the classical paper of E. Artin in 1924. These results are also important for modern cryptography. In 2010 Platonov published a paper in which he discovered and proved a new local-global principle connecting the problem of existence of non-trivial units in quadratic extensions of function fields with a field of algebraic numbers as the field of constants, and the problem of torsion in the Jacobians of hyperelliptic curves.

The broad international recognition of Platonov's achievements is evident from his invitations to speak at the International Congresses of Mathematicians in Vancouver (1974) and Helsinki (1978) and at the European Mathematical Congress in Budapest (1996), and his talks at Harvard, Princeton, Yale, Cambridge, Paris, Bonn, Göttingen, and many other universities in the USA, Great Britain, Germany, France, and other countries.

During the 15 years from 1977 to 1992 Platonov headed the Institute of Mathematics of the Academy of Sciences of Belarus. Under his leadership the institute grew into a large research centre, well known in the Soviet Union and abroad, carrying out mathematical investigations in a wide range of theoretical and applied problems. He did much scientific-organizational work in the Division of Mathematics of the Academy of Sciences of the USSR, the Committee on the Lenin and the State Prizes of the USSR, and the Higher Certification Committee of the USSR. For a number of years he was a member of the editorial boards of the journals *Izv. Akad. Nauk SSSR Ser. Mat.* (translated as *Izvestiya*: *Mathematics*), *Functsional. Anal. i Prilozhen.* (translated as *Functional Analysis and its Applications*), and *Dokl. Akad. Nauk BSSR* (as editor-in-chief from 1987 to 1992). At present he is a member of the editorial boards of the journals *Uspekhi Mat. Nauk* (translated as *Russian Mathematical Surveys*) and *Dokl. Ross. Akad. Nauk* (partially translated as *Doklady Mathematics*, *Doklady Physics*, and so on). Platonov has consistently devoted much attention to the training of new researchers: among his students eight have D.Sc. degrees and eighteen have Ph.D. degrees. His achievements in research have been recognized by the awarding of a number of prestigious prizes, among which are the Leninskii Komsomol Prize (1968), the Lenin Prize in the area of science and technology (1978), and the Humboldt Prize (Germany, 1993). He has also been elected as a member of several foreign academies and scientific societies.

We wish Vladimir Petrovich good health and new creative accomplishments.

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