An effective method for calculation of a Brownian ratchet driven by small sinusoidal perturbations of particle potential energy

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In a number of Brownian ratchet systems, nanoparticle potential energy has the form $U(x,t)=u(x)+\sigma(t)w(x)$, where $u(x)$ is a main part, and $\sigma(t)w(x)$ is its small perturbation. The choice $w(x)=w_0\sin(kx-c)$ allows one to control the average ratchet velocity, $\langle v\rangle$, by changing a phase shift, c. This fact has been found for low-energy ratchets ($\beta u(x)<<1, \beta w(x)<<1; \beta\equiv 1/k_BT$) [1]. In [2], a general analytical expression for $\langle v\rangle$ has been obtained. It is a double integral containing the quadratic form of $w(x)$, the equilibrium distribution functions $\rho^{(\pm)}(x)$ in potential $\pm u(x)$, the function $S(x,y)=\int_0^\infty dt\, g(x,y,t)K(t)$ including the retarded Green's function $g(x,y,t)$ of diffusion in $u(x)$, $K(t)$ is the second-order autocorrelation function. Finding $S(x,y)$ is the most difficult subtask here, additional difficulties are from the integration in the adiabatic region. We suggest a method reducing the complexity, based on the obtained Fourier analog for the integral representation:

$$\langle v\rangle = LJ\,;\quad J=-i(\beta DL)^2\sum_{pp'p_1p_2} k_{p'}k_{-p-p_1}k_{-p_2+p'}\rho^{(+)}_{p_1}\rho^{(-)}_{p_2}S_{pp'}w_{-p-p_1}w_{-p_2+p'}\,, \qquad (1)$$

$$\left(\frac{\partial}{\partial t}+Dk_p^2\right)g_{pp'}(t)+\beta Dk_p\sum_{\tilde{p}}k_{p-\tilde{p}}u_{p-\tilde{p}}g_{\tilde{p}p'}(t)=-\frac{1}{L}\delta_{pp'}\delta(t)\,, \qquad (2)$$

For $\sigma(t)$ describing deterministic periodic or stochastic dichotomous processes, the problem is reduced to finding the matrices $\left[A_{pp'}\pm \mathrm{i}\omega_j\delta_{pp'}\right]^{-1}$ or $\left[A_{pp'}+\Gamma\delta_{pp'}\right]^{-1}$, where $A_{pp'}$ are determined by the components u_p; ω_j and Γ is the frequency of Fourier component σ_j and the inverse correlation time for stochastic dichotomous $\sigma(t)$, respectively. The proposed method significantly optimizes the model and is used for calculation of $\langle v\rangle$ with different $u(x)$, $w(x)$ and $\sigma(t)$.

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1. V.M. Rozenbaum, *et al*., Phys. Rev. E. **99** (2019) 012103.
2. V.M. Rozenbaum, *et al*., JETP Letters, **105** (2017) 542.