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CLASSICAL SOLUTION OF THE BOUNDARY PROBLEMS FOR THE NONSTRONGLY HYPERBOLIC EQUATIONS OF THE SECOND ORDER

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The classical solution to boundary value problems for nonstrongly hyperbolic equation of the second order. It is known that classical solution is the basis of the theory of numerical methods for boundary value problems for partial differential equations. In the report, the method of characteristics [1] is considered to construct the classical solution to boundary value problems for general hyperbolic equations of the second order.

In the half-string $Q = (0, \infty) \times (0, l)$ the linear partial differential equation with independent variables t, x is considered

$$(\partial_t - a\partial_x + b_1)(\partial_t - a\partial_x + b_2)\, u(t,x) = f(t,x), \tag{1}$$

where $a > 0, b_1, b_2 \geq 0, b_1 \neq b_2$.

The Cauchy conditions

$$u|_{t=0} = \varphi(x), \quad \partial_t u\Big|_{t=0} = \varphi_j(x), \quad x \in (0,l). \tag{2}$$

and the boundary conditions

$$u|_{x=0} = \mu_1(t), \quad t \in (l/a, \infty), \quad u|_{x=l} = \mu_2(t), \quad t \in (0,\infty); \tag{3}$$

or

$$\partial_x u|_{x=0} = \mu_1(t), \quad t \in (l/a, \infty), \quad \partial_x u|_{x=l} = \mu_2(t), \quad t \in (0,\infty); \tag{4}$$

or

$$(\partial_x u + u)|_{x=0} = \mu_1(t), \quad t \in (l/a, \infty), \quad (\partial_x u + u)|_{x=l} = \mu_2(t), \quad t \in (0,\infty) \tag{5}$$

are added to the equation (1).

Conditions (3)–(5) are chosen so that each of the boundary value problems was well-posed. Matching conditions for the functions φ, ψ, μ_1, μ_2 providing the uniqueness of the classical solution are obtained. It is noticed that formulation of well-posed problems in the sense of Hadamard for nonstrongly hyperbolic equation increases demands on the smoothness of the initial data as compared with strictly hyperbolic equation.

REFERENCES

[1] Korzyuk V. I., Cheb E. S. and Karpechina A. A. Classical solution of the first boundary problem in the half-string for the linear hyperbolic equations of the second order. *Trudy IM NAN Belarusi.*, **20** (2), 2012, 64 – 67.