The study of thermodynamic values of the quantum Rabi model based on the operator method

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The interaction of a two-level system with a single-mode quantum field in a cavity is described by the model - the quantum Rabi model (RM). This is the most basic and convenient model for studying the interaction of radiation with matter. To study the dynamic properties of RM, results are often used based on the rotating-wave approximation (RWA), which will lead to the Janes-Cummings (JCM) model. The advantages of RWA are obvious, however the field of application of RWA is strongly limited by the coupling constant of the TLS field f and $\varepsilon = \sqrt{\langle \bar{n} \rangle}$ ($\bar{n}$ – average number of photons). Therefore in this report, in the study of statistical characteristics (RM), on the basis of OM, an approximation with a sufficiently high accuracy is constructed in the entire range of variation of f and ε. This approximation is called the uniformly suitable approximation (UAA).

For RM, the Hamiltonian and integral of motion usually take the following forms [1]:

$$\hat{H} = \frac{1}{2}\Delta\hat{\sigma}_3 + \hat{a}^+\hat{a} + f(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a} + \hat{a}^+);\ \hat{P} = \hat{\sigma}_3\hat{S} = \hat{\sigma}_3 e^{i\pi\hat{a}^+\hat{a}};\ \left[\hat{H}, \hat{P}\right] = 0 \quad (1)$$

The eigenvectors and eigenvalues of the RM satisfy the following system of equations.

$$\hat{H}\left|\psi_{np}\right\rangle = E_{np}\left|\psi_{np}\right\rangle;\quad \hat{P}\left|\psi_{np}\right\rangle = p\left|\psi_{np}\right\rangle \quad (2)$$

where $p = \pm 1$ determines the parity and $n = 0,1,2,3, \ldots$ is the quantum number for the excited states of the field. Exact and approximate solutions of system (2) (in RWA) can be found in works [2,3]. To go beyond the framework of the RWA, in the work [4], eigenvalues within the framework of UAA were obtained:

$$E_{np}^{UAA} \approx \left(n + \frac{1}{2}q\right) - f^2 + \frac{1}{4}p\Delta(S_{nn} + S_{n+q,n+q}) - \frac{1}{2}qM;$$

$$q = p(-1)^n; M = \left\{\left[1 - \frac{1}{2}\Delta(-1)^n(S_{nn} + S_{n+q,n+q})\right]^2 + \Delta^2 S_{n,n+q}^2\right\}^{1/2} \quad (3)$$

$$S_{km} = (-1)^m \exp(-2f^2)\sqrt{\frac{m!}{k!}}(2f)^{k-m} L_m^{k-m}(4f^2);\ k \geq m;\ S_{km} = S_{mk}$$

In the resonator, thermal equilibrium occurs and the field amplitude experiences thermal fluctuations. The characteristic temperature of a statistical ensemble consisting of a two-level atom and electromagnetic radiation is determined by the average number of photons $\bar{n}$ introduced into the resonator from an external source. One can obtain various thermodynamic formulae in the framework of the RWA and UAA, such as:

$$S_{RWA}(\bar{n}) = \left(\ln \frac{\bar{n}+1}{\bar{n}} \right) \left[E_{RWA}(\bar{n}) - F_{RWA}(\bar{n}) \right], \quad (4)$$

$$S_{UAA}(\bar{n}) = \ln \frac{\bar{n}+1}{\bar{n}} \left[E_{UAA}(\bar{n}) - F_{UAA}(\bar{n}) \right] \quad (5)$$

where *E*, *F* can be found through the partition function $Z(\beta) = \sum_{np} \exp(-\beta E_{np})$.

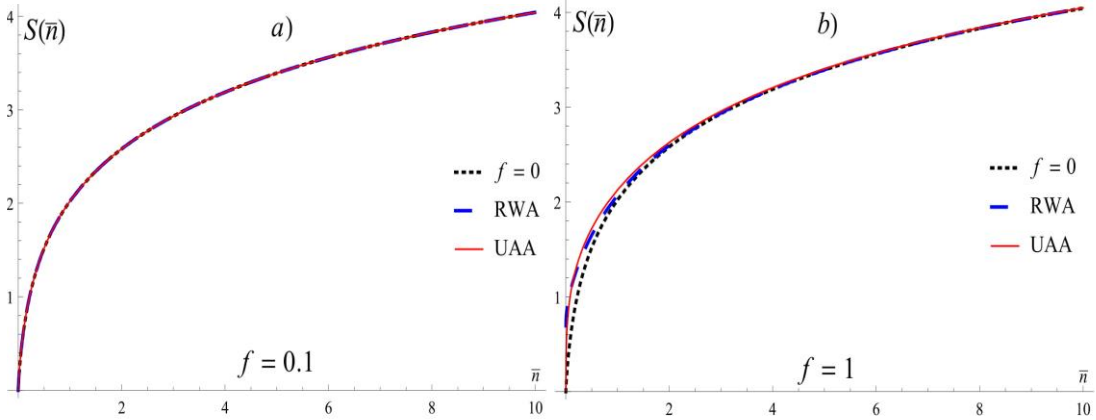

Fig. 1. Entropy as a function of the average number of field quanta in the case of exact resonance ($\Delta = 1$). *a*) – small value of the coupling constant (*f* =0.1); *b*) – great value of the coupling constant (*f* = 1)

In the report, using the operator method, a solution was found to the stationary state of the RM in and as well as beyond the framework of the RWA. Applicability area of the RWA is limited, but not limited for the UAA.

Based on the solutions of the system of equation (2), the thermodynamic quantities in both the approximations RWA and UAA are calculated in comparison with the case of the free system (*f = 0*).

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