EMPIRICAL LIKELIHOOD CONFIDENCE INTERVALS FOR CENSORED INTEGRALS

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Abstract

The problem of finding empirical likelihood confidence intervals is considered for censored integrals.

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Let $X_1, X_2, ...$ (survival times) and $Y_1, Y_2, ...$ (censoring times) be two independent sequences of random variables (r.v.-s) on the real line with marginal distribution function (d.f.-s) F and G respectively. Under the right random censoring model, instead of observing X_i, we observe the pairs $(Z_i, \delta_i), i = 1, 2, ..., n$ where $Z_i = min(X_i, Y_i)$ and $\delta_i = I(X_i \leq Y_i)$ with $I(\cdot)$ the indicator function. Let H denote d.f. of Z_i. Then $H(t) = 1 - (1 - F(t))(1 - G(t))$. Let F and G are continuous. We are interested in constructing a nonparametric confidence interval for a integral functional of the form

$$\theta = \theta(F) = \int \varphi(t) dF(t)$$

where φ is some given Borel measurable function. Let F_n denote the Relative Risk Power estimator of F proposed [1] as

$$F_n(t) = 1 - [1 - H_n(t)]^{R_n(t)},\ t \in R, \tag{1}$$

where $H_n(t) = \frac{1}{n}\sum\limits_{i=1}^{n} I(Z_i \leq t)$ be empirical estimator of $H(t)$ and

$$R_n(t) = \sum_{i=1}^{n} \delta_i(Z_i \leq t)\left[1 - H_n(Z_i) + \frac{1}{n}\right]^{-1}\left\{\sum_{i=1}^{n} I(Z_i \leq t)\left[1 - H_n(Z_i) + \frac{1}{n}\right]^{-1}\right\}^{-1}$$

is the relative-risk function estimator. Note that estimator (1) is a correct estimator of d.f. $F(t)$ than the Product-Limit estimator of Kaplan-Meier and Exponential-Hazard estimator of Altschuler-Breslow (see[2]). Since estimator (1) have same good properties such that representation as sum of independent and identically distributed (i.i.d) r.v.-s up to point $T < T_H = \inf\{t : H(t) = 1\}$, then instead of $\theta(F)$ we consider $\theta_T(F) = \int \varphi^*(t) dF(t)$ where $\varphi^*(t) = \varphi(t) I(t \leq T)$. We prove for plug-in estimator of $\theta_T(F)$ the asymptotic representation

$$\theta_T(F_n) = \int \varphi^*(t) dF_n(t) = \frac{1}{n}\sum_{i=1}^{n} U_i + O_p\left(n^{-\frac{1}{2}}\right) \tag{2}$$

where $U_i = U_i(F, G)$ are i.i.d. r.v.-s. with $EU_i = \theta_T(F)$. Following Owen's [3] idea we propose empirical likelihood confidence interval for truncated integral functional $\theta_T(F)$.

References

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