NONPARAMETRIC MODELLING OF MULTIDIMENSIONAL MEMORYLESS PROCESSES

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Abstract

The report considers the case when multidimensional memoryless objects have an unknown stochastic dependence between output variables, a training sample is available. Such processes are called T-processes. Constructing a model for such a process leads to solve a system of implicit functions. Moreover, the form of these functions is unknown up to parameters. Therefore, practical application of generally accepted parametric identification theory is not possible. In this case, we will use a piecemeal approach to predict output variables from known input variables.
Keywords: data science, memoryless process, nonparametric modeling

1 Introduction

Suppose the object input variables vector is $\vec{u} = (u_1, u_2, \ldots, u_m)$, output vector is $\vec{x} = (x_1, x_2, \ldots, x_n)$ and training sample – $\{u_i, x_i, i = \overline{1, s}\}$. In this case, mathematical description of the object can be represented as analogue of an implicit functions system $F_j(u, x) = 0, j = \overline{1, n}$. The main feature of this case is that the dependency $F(\cdot)$ form is unknown. In this way, it is advisable to use nonparametric methods [1]. Thus, the identification is reduced to solving a system of nonlinear equations $F_j(u, x) = 0, j = \overline{1, n}$, where $\vec{u_s}, \vec{x_s}$ - time vectors of components $x = (x_1, x_2, \ldots, x_n)$ with known u.

2 T-process nonparametric identification

In general, the studied multidimensional system that implements a T-process can be represented in Figure 1.

In Figure 1, the following notations are used: $u(t) = (u_1(t), \ldots, u_m(t))$ - m-dimensional vector of input variable, $x(t) = (x_1(t), \ldots, x_n(t))$ - n-dimensional vector of output variables, $\xi(t)$ - random noise acting on the object , t – time. For various interaction channels, the dependence of j-th component vector x can be represented as some dependence on certain components of the vector u.

A feature of the T-process modelling is that it is described by a system of implicit stochastic equations 1:

$$F_j(u(t - \tau), x(t), \xi(t)) = 0, j = \overline{1, n}, \tag{1}$$

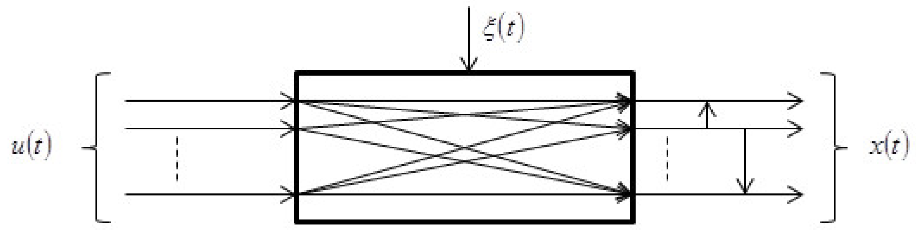

Figure 1: T-process

where $F_j(\cdot)$ – unknown, τ – delay of a multidimensional system's various channels. Since the delay vector is different through various channels, but it is known, we will omit it for simplicity.

Note that when constructing the models of a real technological and production processes (complexes), we often deal with the vectors x and u in a form of certain compound vectors [1]. Compound vector means a vector composed of some components of the corresponding vector, in particular $u^{<j>} = (x_2, x_5, x_7, x_8)$, or another set. In this case, the system of equations takes the form $\hat{F}_j(u^{<j>}, x^{<j>}) = 0, j = \overline{1, m}$

3 T-models

Analyzing the previous considerations, it is easy to see that mathematical description of the process, shown on figure 1, can be taken as a system of implicit functions:

$$F_j(u^{<j>}, x^{<j>}) = 0, j = \overline{1, m} \quad (2)$$

where $u^{<j>}, x^{<j>}$ - compound vectors. The main feature conserned with modelling such a process in the conditions of nonparametric uncertainty, is the fact that the type of functions (2) is unknown. The system of models can be represented as follows:

$$\hat{F}_j(u^{<j>}, x^{<j>}, \vec{u}_s^{<j>}, \vec{x}_s^{<j>}) = 0, j = \overline{1, m} \quad (3)$$

where $\vec{u}_s^{<j>}, \vec{x}_s^{<j>}$ - time vectors (data array received at the s-th time point), in particular ,$\vec{x}_s = (x_1, \ldots, x_s) = (x_{11}, x_{12}, \ldots, x_{1s}, \ldots, x_{21}, x_{22}, \ldots, x_{2s}, \ldots, x_{m1}, x_{m2}, \ldots, x_{ms})$, but $\hat{F}_j(\cdot), j = \overline{1, m}$ also unknown. In the identification theory, such problems are not only considered, but also not posed. Usual way is choosing the parametric structure (3), but, unfortunately it takes a long time to determine the parametric structure.

So, let input variables be inputted to the object u'. In this case, the evaluation of the components of output variables vector x with known values u=u', as it was already noted, leads to solving the system of equations (3). Since the dependence of output variables x on the components of input variables vector is not known, it is reasonable to apply the methods of non-parametric estimation [1].

The problem is reduced to the fact that for a given value of input variables vector u=u', it is necessary to solve the system (3) with respect to output variables vector. The general scheme for solving such a system:

- first, the residual is calculated by the formula:

$$\varepsilon_{ij} = F_j(u^{<j>}, x^{<j>}(i), \vec{u}_s^{<j>}, \vec{x}_s^{<j>}), j = \overline{1,m} \tag{4}$$

where $F_j(u^{<j>}, x^{<j>}(i), \vec{u}_s^{<j>}, \vec{x}_s^{<j>})$ is taken in the form:

$$\varepsilon_j(i) = F_{sj}(u^{<j>}, x_j(i)) = x_j(i) - \frac{\sum_{i=1}^{s} x_j[i] \prod_{k=1}^{<n>} \Phi(\frac{u'_k - u_k[i]}{c_{su_k}})}{\sum_{i=1}^{s} \prod_{k=1}^{<n>} \Phi(\frac{u'_k - u_k[i]}{c_{su_k}})}, \tag{5}$$

where $j = \overline{1,m}$, $< n >$ is dimension of a compound vector u_k . Further, this notation is used for other variables.

- next step is to evaluate the mathematical expectation:

$$x_j = M\{x|u^j, \varepsilon = 0\}, j = \overline{1,m} \tag{6}$$

As an estimate (6), we take non-parametric Nadaraya-Watson regression:

$$\hat{x}_j = \frac{\sum_{i=1}^{s} x_j[i] \prod_{k_1=1}^{<n>} \Phi(\frac{u_{k_1} - u_{k_1}[i]}{c_{su}}) \prod_{k_2=1}^{<m>} \Phi(\frac{\varphi_{k_2}[i]}{c_{s\varphi}})}{\sum_{i=1}^{s} \prod_{k_1=1}^{<n>} \Phi(\frac{u_{k_1} - u_{k_1}[i]}{c_{su}}) \prod_{k_2=1}^{<m>} \Phi(\frac{\varphi_{k_2}[i]}{c_{s\varphi}})}, j = \overline{1,m} \tag{7}$$

where kernel functions $\Phi(*)$ satisfy certain conditions [1].

Realizing this procedure, we obtain the values of output variables x when input action on the object is u=u'. This is the main purpose of the constructed model, which can be applicable in various control systems.

4 Application of the developed method

An object was taken, with five input variables $u(t) = (u_1(t), u_2(t), u_3(t), u_4(t), u_5(t))$, taking random values in the interval $u(t) \in [0, 3]$, and three output variables $x(t) = (x_1(t), x_2(t), x_3(t))$ [2]. For this object, let us generate a sample of input and output variables based on the system of equations, that are unknown to the researcher (they are necessary only to obtain training samples):

$$\begin{cases} x_1(t) - 2u_1(t) + 1.5\sqrt{u_2(t)} - u_5^2(t) - 0.3x_3(t) = 0; \\ x_2(t) - 1.5u_4(t) - 0.3\sqrt{u_5(t)} - 0.6 - 0.3x_1(t) = 0; \\ x_3(t) - 2u_2(t) + 0.9\sqrt{u_3(t)} - 4u_5(t) - 6.6 + 0.5x_1(t) - 0.6x_2(t) = 0. \end{cases} \tag{8}$$

As a result, we obtain a sample of measurements $\vec{u}_s, \vec{x}_s$, where $\vec{u}_s, \vec{x}_s$ - are time vectors.

Let us present the results of a computing experiment with 5% noise acting on the object output. In this case, the elements of training sample $\vec{u_s}, \vec{x_s}$ are used in the algorithms (5) and (6), and for exam input variables values u'_k from the training sample are submitted to the object input. The kernel smoothing c_s will be an adjustable parameter, which in this case we take equal to 0.3 (the value was determined as a result of numerous experiments in order to reduce a quadratic error between an output of the model and object), the kernel smoothing will be the same when calculated in formulas (5) and (6), sample size is s=2000. Below figures show the object outputs $x_1(t), x_2(t)$ and $x_3(t)$.

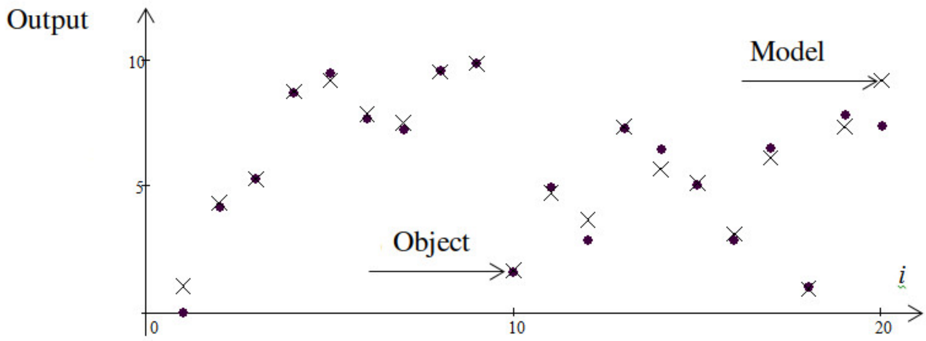

Figure 2: Predicted output x_1 , without noise

In Figure 2 the "point" denotes the output variables values, and the "cross" denotes the model output value. The figures demonstrate a comparison of the true output vector components values of the test sample and their predicted values obtained using the algorithm (5) - (7) (For convenience, the first 20 sample points were shown in the figures).

5 Conclusion

In this paper, we considered the identification problem of memoryless multidimensional objects with delay while components of the output vector have unknown stochastic interactions. Such features are characteristic of complex technological processes with many output variables. The identification algorithm here is the above described procedures, which allows one to predict the output variables with known input variables.

The performed computing experiments showed high efficiency of T-modeling in the presence of random noise acting both on the object and in the measurement channels.

References

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