ABOUT THE PROCESSES IN THE CHANGING FRACTIONAL DIMENSION SPACE

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Abstract

Processes in which the components of the vector of input actions are in stochastic dependence are considered in the report. Such processes are called H-processes. Computational studies have shown that H-processes occur in a space of fractional dimension. An algorithm for estimating the dimension of the space in which the process proceeds is proposed. Computational experiments were carried out on the basis of the proposed algorithm. Experiments have shown that the dimension of the space in which the H-process proceeds is not only fractional, but also variable.
Keywords: data science, fractional dimension space, H-process

1 Introduce

While multidimensional memoryless processes are studying, $\vec{u} \in \Omega(\vec{u}) \subset R^n$ – control variables vector, $\vec{x} \in \Omega(\vec{x}) \subset R^k$ - output variables vector, the following case can arise. When the model is constructed according to the sample $x_i, u_i, i = \overline{1, s}$, as an example $A^\alpha(\vec{u}(t), \vec{x}(t), \vec{\alpha})$, then if $\vec{u} \in \Omega(\vec{u}) \subset R^n$ we can achieve an estimate $\vec{x}_s \notin \Omega(\vec{x})$, i.e. outside the technological rules, even physically impossible $\vec{x}(\vec{u})$ values. This fact can be explained by following considerations.

Let the process proceeds in a single cube $\Omega(\vec{u}, x) = \Omega(u_1, u_2, x) \subset R^3$. The process range is $\Omega^H(\vec{u}, x) \subset \Omega(\vec{u}, x)$, representing the surface in the area $\Omega(\vec{u}, x)$ (Fig. 1), we omit the influence of noise $\xi(t)$ and measurement errors, for simplicity.

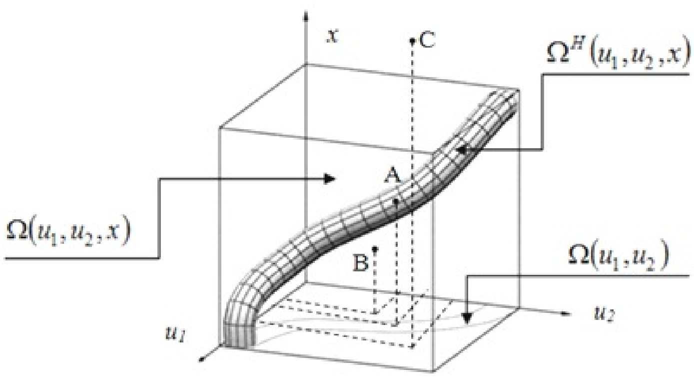

Figure 1: Scheme of the H-process

Real values of the process variables may be known from the technological rules, for example. Thus process takes place in some area $\Omega(u_1, u_2, x)$, in particular, a single cube. If there are a stochastic dependence of input variables the process does not take place in the area $\Omega(u_1, u_2, x)$, but only in its subarea $\Omega^H(u_1, u_2 u, x) \subset \Omega(u_1, u_2, x)$. True process area $\Omega^H(\vec{u}, x)$ is always unknown, precisely this is the main complexity when modelling this kind of processes, named H-processes, in other words, pipe process (Fig.1). An example of such processes can be any processes in which the components of the vector of input actions are stochastically dependent.

2 The processes in the changing fractional dimension space

Let us give an example is concerned with the identification of the memoryless system. Let the object be described by the equation

$$x(\vec{u}) = f(u_1, u_2, u_3), \tag{1}$$

$f(u_1, u_2, u_3) \in R^3$ – input variable, $x \in R^1$ – output variable. The traditional way to build a model of this process is to define a class of parametric operator $\hat{x}(\vec{u}) = \hat{f}(u_1, u_2, u_3, \alpha)$ and further estimate α parameters using a sample of observations $(\vec{u}_i, x_i), i = \overline{1, s}$. s – sample size.

Let the components of input variables vector $\vec{u} = (u_1, u_2, u_3)$ are independent. In this case, the process takes place in 4-dimensional space.

We assume that the components of input vector are in function dependence. For example:

$$u_2 = \phi_1(u_1), u_3 = \phi_2(u2) = \phi_2(\phi_1(u_1)). \tag{2}$$

Obviously, the researcher does not know about dependencies (2). Otherwise, the researcher can make the substitution (2) in (1) and get the dependence of x on one variable u_1:

$$x(\vec{u}) = f(u_1, \phi_1(u_1), \phi_2(\phi_1(u_1))). \tag{3}$$

Equation (3) is easily reduced to the two-dimensional (4), when there is no dependence of u_3 on u_2.

$$x(\vec{u}) = f(u_1, \phi_1(u_1), u_3). \tag{4}$$

From this we can conclude that, in the presence of functional dependencies between the components of the vector, we obtain the dependences of x on u, in this case one-, two-, three-dimensional.

Let us analyze the case related to H-processes. Let u_3 and u_2, have a stochastic dependence. Remind, if the components of input vector are independent, the process is described by a function of three variables. If two components of input vector are functionally dependent, the process is described by a function of two variables. If two components of input vector are stochastically dependent the process is described by a function of more than two variables, but less than three.

When the stochastic dependence of process input variables exists, it occurs in pipe space with fractional dimension F^{λ}.

Dimension calculation can be estimated F^{λ} as (5), for example [1]:

$$dimF^{\lambda} = (n+1) - \sum_{k=1}^{n-1} \lambda_{k,k+1}, \;\; \lambda_{k,k+1} = \frac{\sum_{j=1}^{s}(u_s^k(u_j^{k+1}) - u_j^k)^2}{\sigma^2(u^k)} \tag{5}$$

n – vector $\vec{u}$ dimension; $\lambda_{k,k+1}$ is the "strength" of the stochastically dependence between u_k and u_{k+1} (for example correlation between u_k and u_{k+1}); $\sigma^2(u^k)$ – random variance u^k; $u_s^k(u_j^{k+1})$ – non-parametric estimation of Nadaraya-Watson $M\{u^k|u^{k+1}\}$

Dimension calculation F^{λ} can be estimated other ways.

3 The numerical studies of H-processes

The process is described by the function $x = f(\vec{u})$. Dimension of the vector $\vec{u}$ is 10.

The components of the vector of input actions are functionally dependent. It is important to note that the dimension of the space in which the process proceeds depends on the strength of the stochastic connection between the components of the vector of input variables (this is the second term in (5)). To change the strength of the stochastic connection, between the components of the vector of input variables on the components of the vector of input variables is superimposed noize $\xi(t)$. Considered 2 cases in the first $\xi(t) = 0\%$, and in the second $\xi(t) = 10\%$.

We construct a graph of the dependence between the sample size of observations and the estimate of the dimension of the space in which the process takes place.

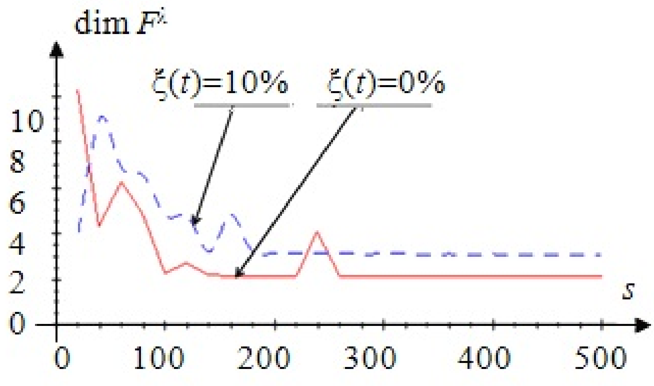

Figure 2: Estimation of dim F^{λ} depending on the sample size

As can be seen from figure 2, for small volumes of a sample of observations, the algorithm does not accurately estimate the dimension of the space in which the process proceeds. With an increase in the sample size of observations $(\vec{u}_i, x_i), i = \overline{1, s}$, the estimation accuracy increases, approaching 2, when $\xi(t) = 0\%$ or 3 when $\xi(t) = 10\%$.

We construct a graph of the dependence between the sample size of observations and the estimate of the dimension of the space in which the process takes place for the case when the dimension of the vector $\vec{u}$ is 2. The components of the vector of input actions are also in a functional dependence.

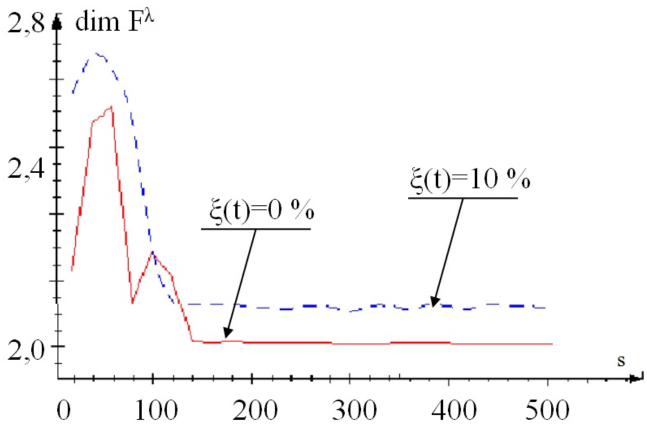

Figure 3: Estimation of dim F^λ depending on the sample size

Figure 3 also shows that with an increase in the sample size of observations, the accuracy of estimating the dimension of the space in which the process proceeds increases, approaching 2, when $\xi(t) = 0\%$ or 2,1 when $\xi(t) = 10\%$.

It should be noted that it is important for the researcher not the ability to determine the dimension of the space in which such processes take place, but the ability to build models of such processes.

4 Conclusion

Numerical researches proved that dimension of the H-process space is a variable quantity. This may mean that in addition to formula (5), other procedures for the dimension calculating can be proposed. Obviously, different formulas will give similar results under identical conditions. This is due to the following factors: the influence of interference, sample sizes, the location of samples in the observation space [1].

References

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