APPLICATION OF OPEN MARKOV NETWORKS WITH VARIOUS FEATURES AT MODELING REAL OBJECTS

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Abstract

In article investigates an open Markov networks with multiple types of customers at a non-stationary regime. For finding state probabilities of this networks generalized the system of difference-differential equations (DDE) was compiled, for the solution of which a modified method of successive approximations was proposed, combined with the method of the series. The properties of successive approximations are described. Concrete examples of the use of open networks in modeling the behavior of the Internet system when it is attacked by various viruses are also presented.
Keywords: Markov network, data science, non-stationarity, difference-differential equation

1 Introduction

The method of successive approximations is used to find the probabilities of the states and expected revenues of the network systems in the case when the revenues from transitions are deterministic and depend only on the state of the network. For the first time in the area of queueing such a method was introduced by Harrison in [1] for a closed Jackson network. Then this method was used to analyze other Markov networks with various features [2-3]. As a result of the analysis of various networks, it was noted that the Kolmogorov DDE system for the probabilities of network states in the general case can be written as:

$$\frac{P(\vec{d},\vec{k},\vec{l},t)}{dt} = -\Lambda(\vec{d},\vec{k},\vec{l})P(\vec{d},\vec{k},\vec{l},t) + \sum_{i^*,j^*=0}^{n} \sum_{\alpha,\beta,\gamma,\theta,\eta=0}^{\Psi r} \sum_{m=0}^{\infty} \sum_{b=0}^{1} \Phi_{i^*j^*\alpha m\beta b\gamma\theta\eta}(\vec{d},\vec{k},\vec{l}) \times$$
$$\times P(\vec{d}+I_{i^*}-I_{j^*},\vec{k}+\tilde{I}_\alpha+m\tilde{I}_\beta-b\tilde{I}_\gamma,\vec{l}+\tilde{I}_\theta-\tilde{I}_\eta,t), \quad (1)$$

where $\tilde{I}_\alpha$ - zero-vector of dimension Ψr, except for the component with the number α, which equal 1, Ψ – some integer positive number , r – number of type customer, I_α - zero-vector of dimension n, except for the component with the number α, which equal 1, $\vec{d}$ - vector of dimension n with component d_i, where d_i - the number of service lines in the i-th QS, $\vec{k}$ - vector of dimension Ψr with component k_{ic}, где k_{ic} – the number of positive customers of type in the i-th QS, $\vec{l}$ - vector of dimension Ψr with component l_{ic}, где l_{ic} - the number of signals of type in the i-th queueing system (QS), $i = \overline{1,n}, c = \overline{1,r}$.

$\Phi_{i^* j^* \alpha m \beta b \gamma \theta \eta}(\vec{d}, \vec{k}, \vec{l})$ – transition intensity of the queueing network (QN) from state $(\vec{d}, \vec{k}, \vec{l})$ in other, and this transition is characterized by the following parameters: i^*, j^* characterize the number of QS in which the service line is broken or restored, respectively, m – size of deleted group, β – number of component of vector $\vec{k}$, which there is a removal group of positive customers of the corresponding type from the corresponding QS, α – number of corresponding component of vector $\vec{k}$, in which there was a receipt of a positive customer of the corresponding type in the corresponding QS and in the corresponding queue, γ – number of corresponding component of vector $\vec{k}$, in which there was a leave of a positive customer of the corresponding type from the corresponding QS, θ, η – number of corresponding component of vector $\vec{l}$ in which arriving or deleting of signal after delete positive customer of corresponding type in corresponding QS, $\Lambda(\vec{d}, \vec{k}, \vec{l})$ – intensity of exit of state $(\vec{d}, \vec{k}, \vec{l})$. $P(\vec{d}, \vec{k}, \vec{l}, t)$ – state probability $(\vec{d}, \vec{k}, \vec{l})$ in moment time t.

Series $\sum\limits_{i^*, j^*=0}^{n} \sum\limits_{\alpha,\beta,\gamma,\theta,\eta=0}^{\Psi r} \sum\limits_{m=0}^{\infty} \sum\limits_{b=0}^{1} \Phi_{i^* j^* \alpha m \beta b \gamma \theta \eta}(\vec{d}, \vec{k}, \vec{l})$ is converge for all Markov QN. This follows from that, what network state in all moment time is Markov chain with countable number states, i.e. system of DDE (1) is special case of system of differential equations for states probabilities of the Markov chain with continuous time, in which $\Lambda(\vec{d}, \vec{k}, \vec{l})$ intensity of exit of state $(\vec{d}, \vec{k}, \vec{l})$, and sum of the series - sum of entry intensity of this state.

2 Solution of the DDE system

The solution of the system (1) is:

$$P(\vec{d}, \vec{k}, \vec{l}, t) = e^{-\Lambda(\vec{d}, \vec{k}, \vec{l})t} \Bigg(P(\vec{d}, \vec{k}, \vec{l}, 0) + \int_0^t e^{\Lambda(\vec{d}, \vec{k}, \vec{l})x} \times$$

$$\times \sum_{i^*, j^*=0}^{n} \sum_{\alpha,\beta,\gamma,\theta,\eta=0}^{\Psi r} \sum_{m=0}^{\infty} \sum_{b=0}^{1} \Phi_{i^* j^* \alpha m \beta b \gamma \theta \eta}(\vec{d}, \vec{k}, \vec{l}) \times$$

$$\times P(\vec{d} + I_{i^*} - I_{j^*}, \vec{k} + \tilde{I}_\alpha + m\tilde{I}_\beta - b\tilde{I}_\gamma, \vec{l} + \tilde{I}_\theta - \tilde{I}_\eta, x) dx. \qquad (2)$$

Denote by $P_q(\vec{d}, \vec{k}, \vec{l}, t)$ - respectively approximation $P(\vec{d}, \vec{k}, \vec{l}, t)$ in the q-th iteration, and $P_{q+1}(\vec{d}, \vec{k}, \vec{l}, t)$ — solution of system (1) obtained by the method of successive approximations. Then from (2) it follows that

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$$P_{q+1}(\vec{d}, \vec{k}, \vec{l}, t) = e^{-\Lambda(\vec{d}, \vec{k}, \vec{l})t} \Bigg(P(\vec{d}, \vec{k}, \vec{l}, 0) + \int_0^t e^{\Lambda(\vec{d}, \vec{k}, \vec{l})x} \times$$

$$\times \sum_{i^*,j^*=0}^{n} \sum_{\alpha,\beta,\gamma,\theta,\eta=0}^{\Psi r} \sum_{m=0}^{\infty}\sum_{b=0}^{1} \Phi_{i^*j^*\alpha m\beta b\gamma\theta\eta}(\vec{d},\vec{k},\vec{l})\times$$

$$\times P_q(\vec{d}+I_{i^*}-I_{j^*},\vec{k}+\tilde{I}_\alpha+m\tilde{I}_\beta-b\tilde{I}_\gamma,\vec{l}+\tilde{I}_\theta-\tilde{I}_\eta,x)dx. \quad (3)$$

We take the stationary solution as the initial approximation $P_0(\vec{d},\vec{k},\vec{l},t) = \lim_{t\to\infty} P_q(\vec{d},\vec{k},\vec{l},t) = = P(\vec{d},\vec{k},\vec{l})$ which satisfies the relation :

$$\Lambda(\vec{d},\vec{k},\vec{l})P(\vec{d},\vec{k},\vec{l}) = \sum_{i^*,j^*=0}^{n} \sum_{\alpha,\beta,\gamma,\theta,\eta=0}^{\Psi r} \sum_{m=0}^{\infty}\sum_{b=0}^{1} \Phi_{i^*j^*\alpha m\beta b\gamma\theta\eta}(\vec{d},\vec{k},\vec{l})\times$$

$$\times P(\vec{d}+I_{i^*}-I_{j^*},\vec{k}+\tilde{I}_\alpha+m\tilde{I}_\beta-b\tilde{I}_\gamma,\vec{l}+\tilde{I}_\theta-\tilde{I}_\eta). \quad (4)$$

For successive approximations, the following statements are valid.

Theorem 1. *Successive approximations $P_q(\vec{d},\vec{k},\vec{l},t), q = 0,1,2,\dots$, converge at $t\to\infty$ to the stationary solution of the system of equations (1).*

Theorem 2. *The sequence $P_q(\vec{d},\vec{k},\vec{l},t), q = 0,1,2,\dots$, constructed according to scheme (4), for any zero approximation bounded in t $P_0(\vec{d},\vec{k},\vec{l},t)$ converge at $q\to\infty$ to the unique solution of the system (1).*

Theorem 3. *Any approximation $P_q(\vec{d},\vec{k},\vec{l},t), q \geq 1$ representable as a convergent power series*

$$P_q(\vec{d},\vec{k},\vec{l},t) = \sum_{l=0}^{\infty} g_{ql}(\vec{d},\vec{k},\vec{l})t^l, \quad (5)$$

whose coefficients satisfy the recurrence relations:

$$\vec{g}_{q+1l}(\vec{d},\vec{k},\vec{l}) = \frac{-\Lambda(\vec{d},\vec{k},\vec{l})^l}{l!}\left\{P(\vec{d},\vec{k},\vec{l},0)+\sum_{u=0}^{l-1}\frac{(-1)^{u+1}u!}{\Lambda(\vec{d},\vec{k},\vec{l})^{u+1}}D_{qu}(\vec{d},\vec{k},\vec{l})\right\} \quad (6)$$

$$\vec{g}_{q0} = P(\vec{d},\vec{k},\vec{l},0), \vec{g}_{0l} = P(\vec{d},\vec{k},\vec{l},0)\delta_{l0}.$$

The proof of Theorems 1-3 is similarly as in [4].

3 Example of the queueing network

Consider examples of the QN, the DDE system for has the form (1).

First network is network with signals and batch removal customers. The network description, it parameters and the DDE system for state probability is present in [4]. This system is a special case of the system (1)

$$\Lambda(\vec{d},\vec{k},\vec{l}) = \lambda^+ + \lambda^{(s)} + \sum_{i=1}^{n}\mu_i,$$

$$\Phi_{i^*j^*\alpha m\beta b\gamma\theta\eta}(\vec{d},\vec{k},\vec{l}) = \Bigg(\delta_{i*j*}\delta_{\vec{d}\vec{1}_n}\delta_{\theta\eta}\delta_{\vec{l}\vec{0}}(\delta_{b1}\delta_{\alpha i}\delta_{\gamma j}(\lambda^+_{0j}u(k_j) + (\mu_j p^+_{ji}u(k_i))(1-\delta_{ij})+$$

$$+\sum_{s=1}^{n}(1-\delta_{si})(\mu_i p_{i0}+\mu_i p^-_{is}(1-u(k_s)))+\sum_{j=1}^{n}(1-\delta_{ij})\lambda^{(c)}_{0j}q_{j0}\pi_{jm}+\sum_{s=1}^{n}(1-\delta_{js})\mu_j p^-_{js}q_{s0}\pi_{sm}+$$

$$+u(m+1)\mu_i p^-_{is}(1-u(k_s))q_{s0}\pi_{sm} + \delta_{b1}\delta_{\alpha i}\delta_{\beta j}\delta_{\gamma s}\mu_i p^-_{ij}q_{js}u(k_s)\Bigg),$$

where $\delta_{ij} = \begin{cases} 1, i=j, \\ 0, i\neq j. \end{cases}$, $u(x)$ – Heaviside function.

This network applied at modeling of the operation of the Internet server, which is prone to attacks of exploits.The QS is understood as the processor of user requests in this server. Positive customers – the requests themselves, and signals (act as a negative customer or the exploit) which transfer requests to the phishing site. Signals that act as triggers are developer exploits that move requests from one developer to another.

Second network is G-network with unreliable nodes. The Network description, him parameters and the DDE system for state probabilities is present in [5]. The system is a special case of the system (1) has the form

$$\Lambda(\vec{d},\vec{k},\vec{l}) = \sum_{i=1}^{n}\Bigg(\lambda^+_{0i}u(k_i) + u(d_i)(\lambda^-_{0i}+\mu_i+\gamma_i)+\beta_i(d_i+1)\Bigg),$$

$$\Phi_{i^*j^*\alpha m\beta b\gamma\theta\eta}(\vec{d},\vec{k},\vec{l}) = \delta_{0b}\delta_{0\alpha}\delta_{\theta\eta}\delta_{\vec{1}\vec{0}}(\delta_{0m}\delta_{0i^*}\gamma_i u(d_j)+$$

$$+\delta_{0m}\delta_{0j^*}\beta_i u(1-d_i)) + \delta_{i^*j^*}(\delta_{0m}\delta_{0\alpha}\delta_{1b}\delta_{\gamma j}\lambda^+_{0i}u(k_i) + \delta_{0m}\delta_{i\alpha}\delta_{0b}\times \times u(d_{i*})(\mu_i p_{i0} + \lambda^-_{0i} + \mu_i\sum_{j=1}^{n}p^-_{ij}(1-u(k_i))) + \delta_{0m}\delta_{i\alpha}\delta_{1b}\delta_{\gamma j}\mu_i u(d_i)p^+_{ij}u(k_j)+$$

$$+\delta_{1m}\delta_{i\alpha}\delta_{0b}\delta_{\beta j}\mu_i u(d_i)p^-_{ij}.$$

This model is also used at modeling Internet servers. Positive customer are user requests, negative customer are the effect of a Botnet, which destroys the request in the developer's queue. Under the influence of the DDOS-attack the handler fails, and after repair by an engineer it recovers after a random time.

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