ON PARAMETER ESTIMATION OF DOUBLE-CENSORED STATIONARY GAUSSIAN TIME SERIES

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Abstract

Double-censored stationary Gaussian time series are considered. Statistical estimators of the model parameters are constructed by using the method of moments for special auxiliary time series. Consistency of constructed estimators are proved under some additional general conditions.

Keywords: data science, censored data, Gaussian time series, estimation

Consider double-censored Gaussian time series x_t. It means that instead of the exact values $x_1, \ldots, x_T$ at the time moments $T_C^+ = \{t : x_t \geq c_+\}$ and $T_C^- = \{t : x_t \leq c_-\}$ only random events are observed [2, 3]:

$$A_t^+ = \{x_t \in [c_+, +\infty)\}, t \in T_C^+,$$
$$A_t^- = \{x_t \in (-\infty, c_-]\}, t \in T_C^-,$$

where c_- and c_+ are the lower and upper censoring levels ($c_- < c_+$), T is the length of the observation process. In other words term "double-censored" means that at the same time we have left and right censoring.

Let $X = (x_1, \ldots, x_T)' \in \mathbf{R}^T$ be the vector of the exact observations. Then for Gaussian time series the vector X has a normal distribution $\mathcal{L}(X) = \mathcal{N}(\mu, \Sigma)$, where the mathematical mean μ and the covariance matrix Σ depend on some unknown parameter $\theta \in \Theta \subseteq \mathbf{R}^m$ of the time series model (e.g. for AR(p) model $\theta = (\varphi_1, \ldots, \varphi_p, \sigma^2)$, where $\varphi_1, \ldots, \varphi_p$ are the coefficients of the autoregression and σ^2 is the variance of the Gaussian innovation process) [1].

Define the auxiliary time series y_t for the double-censored time series x_t [3]:

$$y_t = \begin{cases} x_t, & x_t \in (c_-, c_+), \\ c_-, & x_t \leq c_-, \\ c_+, & x_t \geq c_+. \end{cases}$$

By using the method of moments for auxiliary time series y_t the m values of the second moments $\sigma_\tau = \mathrm{E}\{x_t x_{t+\tau}\}$ for the initial time series x_t can be estimated, i.e. estimators $\hat{\sigma}_\tau$, ($\tau \in \{0, 1, \ldots, m-1\}$) can be found. By using $\hat{\sigma}_\tau$ and the method of moments for initial time series x_t estimators of the model parameters $\hat{\theta}$ can be calculated. The consistency of the constructed estimators $\hat{\theta}$ are proved. Proposed procedure obviously can be used for only left or only right censored data.

The example of this estimation procedure is proposed for the AR(2) model. Results of this example is illustrated on figure 1. As we can see empirical variances of the constructed estimators tend to zero and this result is agreed with consistency of the constructed estimators.
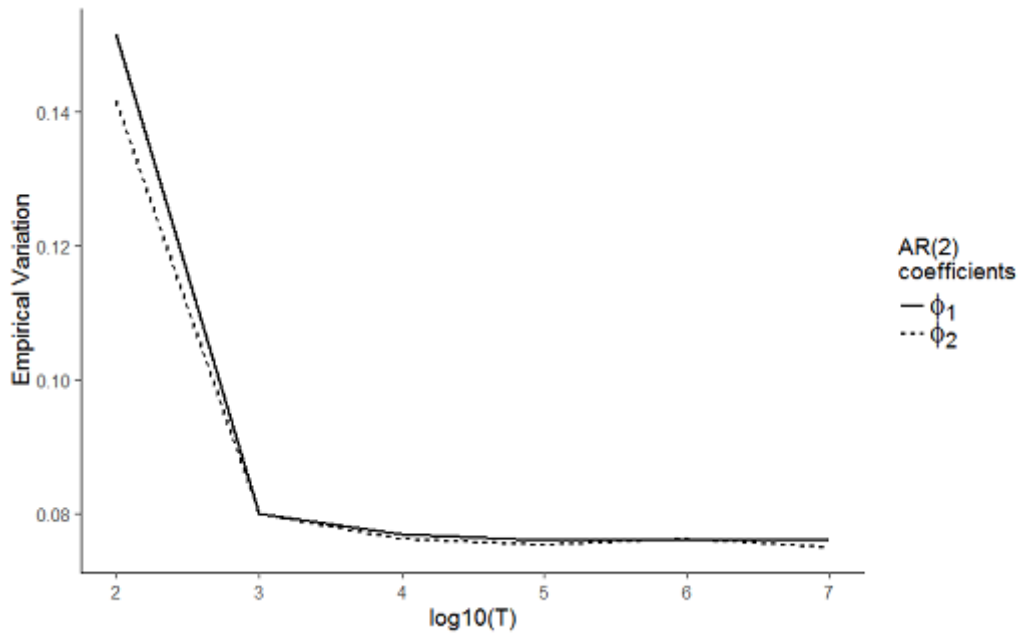
Figure 1: Dependence of empirical variances for the AR(p) coefficients on time series length T ($c_- = 0$, $c_+ = 2$)

References

[1] Anderson T.W. (1971). *The Statistical Analysis of Time Series.* John Wiley & Sons, New York.

[2] Kharin Yu.S. (2013). *Robustness in Statistical Forecasting.* Springer, Heidelberg/Dordrecht/New York/London.

[3] Park J.W., Genton M.G., Ghosh S.K. (2007). Censored time series analysis with autoregressive moving average models. *The Canadian Journal of Statistics.* Vol. **35**, No. 1, pp. 151-168.