рис. 5, *б* представлена временная зависимость вертикального смещения того же узла. Резонансный характер поведения массива УНТ при воздействии с частотой 430 МГц очевиден.

Существенным результатом этих расчетов является отсутствие собственных мод массива до 280 МГц и наличие «окон прозрачности» в более высокочастотной области, в которых собственные колебания тоже отсутствуют. В таких «окнах прозрачности» применимо описание массива УНТ как эффективной сплошной среды. В резонансных участках спектра должно наблюдаться резонансное поглощение и переизлучение электромагнитных и акустических волн, и массив УНТ должен вести себя как «фононный кристалл».

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MATHEMATICAL SIMULATION OF PULSE-DRIVEN JOSEPHSON JUNCTION

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Since its inception, the Josephson junction has been applied in a number of scientific and technological areas in which fast switching coupled with low power dissipation is a desiratum [1]. The standard model of a Josephson superconducting junction consists of a junction shunted by a resistance. We denote ϕ the phase difference between electron pairs across the junction, τ dimensionless time, i_0 and i_1 dimensionless current. The resulting equation is

$$d\phi / d\tau + \sin\phi = i_0 + i_1 f(\tau / b), \tag{1}$$

where parameter b has a dimension of inverse time, f is periodic function. Most analyses of equation (1) are based on the choice of the periodic term proportional to a single sinusoid $\sin \omega t$. In such a case one is forced either to use some form of perturbation theory or a numerical solution of equation to derive an approximate solution. When either of these procedures is used, one can demonstrate, among other things, the existence of so-called Shapiro steps, which is equivalent to phase locking. When Shapiro steps are present there is a net average voltage across the junction which is proportional to mean value ϕ , ϕ_{mv}, where the average is over a complete cycle. An explanation of the phenomenon of Shapiro steps can be given in mechanical terms in terms of an overdamped pendulum driven by the sum of a constant torque i_0 and a time-depended torque $i_1 f(\tau)$. The average voltage across the junction is analogous to the frequency of rotation of such a pendulum provided that the torque is sufficiently large. Thus, if $i_0 < i_1$, and there is no external torque, the pendulum performs small periodic oscillation around $\phi = 0$ with the consequence satisfy $(d\phi / dt)_{mv} = 0$. In contrast, when $i_0 > i_1$, the oscillation is replaced by rotation with a characteristic frequency $\omega_J = b(i_0^2 - i_1^2)^{1/2}$. The motion is further complicated when an oscillatory forcing term is added to the dynamics equation, in which case the system can became phase locked in the sense that the average angular velocity can equal to an integer multiple of ω_J. A complete mathematical analysis of these effects is complicated by the fact that equation (1) cannot be solved in any convenient form when $f(\tau)$ is pure sinusoid. The main aim of the present paper is to present an algorithm that converts the problem of solving equation (1) when $f(\tau)$ is a periodic train of δ functions into the solution of a set of recursion relations which is to easy to solve numerically.

We consider the case when Josephson junction is driven by a set of δ-function pulses in the form

$$f(t) = i_0 + 2\pi i_1 \sum_{k>1} \delta(t - t_0 - kT), \tag{2}$$

Here t_0 is initial moment of time, i_0 is the constant component of current, $2\pi i_1$ is integral amplitude of pulses. The condition of existence of Shapiro steps which is equivalent to phase locking can be written as $\lim_{t\to\infty}(\phi(t + \omega_c \tau_a) - \phi(t) = 2\pi k(\omega_c \tau_a / T) + o(\tau_a)$, where τ_a is the time of averaging, if $\tau_a \to \infty$, $o(\tau_a) \to 0$. The function $\phi(t)$ satisfies the condition of jumping

$$\phi_{n+1}(t_{n+1} + 0) - \phi_n(t_n - 0) = 2\pi i_1 . \tag{3}$$

For existence of phase locking it is necessary and sufficient that following condition holds

$$\lim_{n\to\infty} \phi_{n+1}(t + T) - \phi_n(t) = 2\pi k , \tag{4}$$

here $k = 0, \pm 1, \pm 2 \ldots$ is the order of Shapiro step [1].

Following [2] the equation (1) can be replaced by the system of linear impulsive equation

$$\begin{cases} \dot{x}(t) = 0{,}5(x(t) + i(t)y(t)), \\ \dot{y}(t) = -0{,}5(i(t)x(t) + y(t)), \end{cases} \tag{5}$$

here

$$i(t) = i_0 + 2\pi i_1 \sum_n \delta(t - nT)$$

or

$$i(t) = i_0 + 2\pi i_1 \sum_n \{\delta(t - (T + \tau)/2 - nT) - \delta(t - (T - \tau)/2 - nT)\}$$

i.e. monopolar or bipolar train of impulses.

The solution of the nonlinear impulsive equation (1) may be transformed to a linear impulsive system of second order [3]. Rewrite the equation (1) as the system

$$\begin{cases} d\phi / dt + \sin\phi(t) + q(t) = 0, \\ d\xi / dt - \cos\phi(t) = 0. \end{cases} \tag{6}$$

Following [3] ($(\phi(t), \xi(t))$ with the initial conditions $\phi(0) = \phi_0$, $\xi(0) = 0$ is the solution of the system (6) if and only if the functions

$$x(t) = e^{\xi(t)/2} \cos(0.5(\phi(t) - \phi(0)),$$

$$y(t) = e^{\xi(t)/2} \sin(0.5(\phi(t) - \phi(0))$$

is the solution of the system (5) with the initial conditions $x(0) = 1$, $y(0) = 0$. The solution $\phi(t)$ of the equation (1) satisfies the condition (4) if and only if the sequence $\chi_n(t) = \chi(t + nT)$ convergence of point wise, $i^2 = -1$, $\chi(t) = (x(t) + iy(t))/(x(t) - iy(t))$. For a periodic bipolar train of δ-functions the system (4) can be rewritten as impulsive linear system for vector $z^T = (x, y)$ on interval $[0, T)$

$$\dot{z} = Az - \pi i_1 J\delta(t - t_1) + \pi i_1 J\delta(t - t_2) \tag{7}$$

where $t_1 = (T - \tau)/2$, $t_2 = (T + \tau)/2$, $t \in [0, T), \tau < T/2$, $A = \begin{pmatrix} 1 & 0{,}5 t_0 \\ -0{,}5 t_0 & -1 \end{pmatrix}$,

$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is symplectic matrix. The condition of jumping for vector $z(t)$ has the form

$$\Delta z(t_k) = D_k z(t_k), \ (k = 1,2),$$

where $D_1 = I - \pi i_1 J$, $D_2 = I + \pi i_1 J$. The monodromy matrix of the system (6) has the form

$$U(T) = e^{A(T - t_2)} D_2 e^{A(t_2 - t_1)} D_1 e^{At_1},$$

where $T - t_2 = (T - \tau)/2$, $t_2 - t_1 = \tau$, $t_1 = (T - \tau)/2$. Thus, $\det e^{At} = 1$ then $\det U(T) = \det D_1 \det D_2 = \det U(T) = (1 + \pi^2 i_1^2)^2 > 1$. The impulsive periodic system can be reduced by Lyapunov transformation into the system with constant matrix $\Gamma = (1/T)\ln U(T)$ without impulses. The egenvalues of monodromy matrix $\rho_k, k = 1,2$ are named multiplies. They are connected with egenvalues of matrix Γ by formula $\lambda_k(\Gamma) = (1/T)\ln \rho_k$. The characteristic equation for monodromy matrix has the form

$$\rho^2 - Tr(U(T))\rho + \det(U(T)) = 0,$$

where $Tr(U(T)) = u_{11}(T) + u_{22}(T)$. It has two solutions. The monodromy matrix is also the matrix of the difference equation corresponding to the system (4), $C_{n+1} = U(T)C_n$, where $C_n = z(t_n + 0)$. The condition (4) is equivalent to the condition of convergence of the sequence $z_n, n \to \infty$, with initial condition $z_0^T = (1,0)$. It corresponds the multiplier which satisfy the condition $\rho_k < 1$. So, the condition of phase locking is corresponding to the condition of existence of real value of multiplier for which the recursion relation is convergence.

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ЛАЗЕРНО-ПЛАЗМЕННОЕ ОСАЖДЕНИЕ ДИЭЛЕКТРИЧЕСКИХ АЛМАЗОПОДОБНЫХ УГЛЕРОДНЫХ ПОКРЫТИЙ

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Известно, что безводородные алмазоподобные углеродные пленки (АУП) с высоким (более 80 %) содержанием sp^3-связанного углерода обладают близкими к алмазу физическими характеристиками, среди которых можно отметить высокое удельное сопротивление, высокую механическую, химическую и радиационную стойкость, высокое напряжение пробоя [1, 2]. Перечисленные свойства делают АУП одним из наиболее перспективных диэлектрических материалов современной микроэлектроники и стимулируют исследования в области совершенствования технологий получения таких покрытий. Одним из наиболее перспективных методов получения алмазоподобных пленок является метод лазерно-плазменного осаждения [3, 4]. Настоящая работа посвящена исследованию влияния условий осаждения на электрические характеристики углеродных пленок, получаемых данным методом.

Для осаждения углеродных пленок применялся $YAG:Nd^{3+}$ лазер фирмы Lotis-TII. Лазерное излучение длиной волны λ = 1064 нм и длительностью импульса τ = 20 нс фокусировалось на графитовую мишень, расположенную в вакуумной камере при давлении около $2.6 \cdot 10^{-3}$ Па. Диаметр лазерного пучка во всех экспериментах составлял 2 мм. Плотность мощности излучения варьировалась в диапазоне $(1–5) \cdot 10^8$ Вт/см^2 при постоянном диаметре лазерного пучка. Частота следования лазерных импульсов изменялась от 1 до 5 Гц. Количество лазерных импульсов во всех экспериментах равнялось 4000. Мишени были изготовлены из графита МГ1ОСЧ и устанавливались под углом 45° к оси лазерного пучка. Для предотвращения образования на поверхности мишени эрозионного кратера производилось сканирование